Numerical simulation of 2D buoyant jets in ice-covered and temperature-stratified water

Ruochuan Gu
Department of Civil and Construction Engineering, Iowa State University, Ames, IA 50011, USA

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A two-dimensional (2D) unsteady simulation model is applied to the problem of a submerged warm water discharge into a stratified lake or reservoir with an ice cover. Numerical simulations and analyses are conducted to gain insight into large-scale convective recirculation and flow processes in a cold waterbody induced by a buoyant jet. Jet behaviors under various discharge temperatures are captured by directly modeling flow and thermal fields. Flow structures and processes are described by the simulated spatial and temporal distributions of velocity and temperature in various regions: deflection, recirculation, attachment, and impingement. Some peculiar hydrothermal and dynamic features, e.g., reversal of buoyancy due to the dilution of a warm jet by entraining cold ambient water, are identified and examined. Simulation results show that buoyancy is the most important factor controlling jet behavior and mixing processes. The inflow boundary is treated as a liquid wall from which the jet is offset. Similarity and difference in effects of boundaries perpendicular and parallel to flow, and of buoyancy on jet attachment and impingement, are discussed. Symmetric flow configuration is used to de-emphasize the Coanda effect caused by offset. © 1998 Elsevier Science Ltd. All rights reserved.

Key words: buoyancy, cold regions, lakes, reservoirs, stratification, temperature, water jet discharges.

1 INTRODUCTION

When the density of discharged water is different from that of ambient water, a buoyant jet is formed and driven to rise or fall by a buoyant force. A submerged water discharge into a cold and stratified waterbody is often encountered in environmental and hydraulic engineering problems. Examples include municipal and industrial wastewater and cooling water discharges into river impoundments or reservoirs, water quality improvement by discharging oxygen-enriched water into large lakes in winter, and river ice control by warm water jets in cold regions. Temperatures of natural waters in cold regions during winter usually cool to 4°C or below. After an ice cover forms, water temperature typically varies from 0°C at the ice-water interface to 2–4°C at the bottom (water–sediment interface), producing a weakly stable stratification. Submerged warm water discharges (4–20°C) into cold waterbodies with ice covers have peculiar flow and mixing features. A warm jet can rise due to the positive initial buoyancy at the discharge point (Fig. 1). As the discharged water moves along the jet trajectory it is diluted by entrainment of cold ambient water and cooled down. The decreased jet temperature causes a loss and eventually a reversal in buoyancy which turns the rising jet into a sinking one.

An integral method was used in previous investigations by Robillard and Vasseur to analyze and describe jet trajectories of horizontal discharges into uniform ambient water at low temperatures (0–4°C). Complexity of flow characteristics of buoyant jet in a stratified cold waterbody makes laboratory experiments costly and difficult. It may be almost impossible for engineers to labor through laboratory experiments on a case-by-case basis. Attempts were made to use numerical models to solve the problem of a buoyant jet in cold water by Gu and Stefan. An one-dimensional (1D) integral simulation model was developed. However, the integral model needs assumption of appropriate velocity and temperature profiles and an empirical entrainment coefficient which is expected to vary. It is obvious that a two-dimensional (2D) simulation model can eliminate the assumptions and empirical relations. The 2D model is expected to be capable of directly predicting the behavior of an plane buoyant jet in
a stratified cold waterbody as a whole flow domain, including mixing processes, recirculation patterns, thermal fields, and pressure and velocity distributions.

In this study a 2D buoyant jet discharged horizontally into a bounded, stationary, and temperature-stratified cold waterbody with an ice cover. The effect of buoyancy and stratification on jet behavior, mixing processes, and circulation patterns in a stratified cold ambient is investigated. The peculiar hydrodynamic and hydrothermal features are identified and illustrated through numerical modeling. Flow processes including deflection, attachment, impingement, and spreading caused by the adjacent boundary and driven by buoyant forces are examined and described using the simulation results. The effect of boundaries perpendicular and parallel to flow on the flow processes is analyzed and compared with the offset jet phenomenon.

2 MODEL DESCRIPTION

2.1 Governing equations

The 2D model employed in this study was based on an earlier one introduced by Patankar.\textsuperscript{13,14} The earlier model was modified with the incorporation of buoyancy, a numerical solution scheme for unsteady and non-linear problems, and appropriate boundary conditions by Gu and Stefan.\textsuperscript{7} The present model is capable of directly simulating incompressible turbulent plane jets discharged vertically or horizontally. The governing equations include four mean flow equations (continuity, horizontal momentum, vertical momentum, and temperature), two $\kappa$-$\varepsilon$ turbulent transport equations,\textsuperscript{10} and an equation of state in the Cartesian coordinate system

$$
\frac{\partial p}{\partial t} + \frac{\partial (pu)}{\partial x} + \frac{\partial (pv)}{\partial y} = 0
$$

$$
\frac{\partial (pu)}{\partial t} + \frac{\partial (pu^2)}{\partial x} + \frac{\partial (puv)}{\partial y} = - \frac{\partial p}{\partial x} + 2 \frac{\partial}{\partial x} \left( \mu_e \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left[ \mu_e \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right]
$$

$$
\frac{\partial (pv)}{\partial t} + \frac{\partial (pvu)}{\partial x} + \frac{\partial (pve)}{\partial y} = - \frac{\partial p}{\partial y} + 2 \frac{\partial}{\partial y} \left( \mu_e \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial x} \left[ \mu_e \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + g \rho_{fs} - \rho
$$

$$
\frac{\partial (pT)}{\partial t} + \frac{\partial (pTu)}{\partial x} + \frac{\partial (pTv)}{\partial y} = \frac{\partial}{\partial x} \left( \alpha_e \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \alpha_e \frac{\partial T}{\partial y} \right)
$$

$$
\frac{\partial (p\kappa)}{\partial t} + \frac{\partial (p\kappa u)}{\partial x} + \frac{\partial (p\kappa v)}{\partial y} = \frac{\partial}{\partial x} \left( \mu_e \frac{\partial \kappa}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu_e \frac{\partial \kappa}{\partial y} \right) + \frac{\partial}{\partial x} \left( \mu \frac{\partial \kappa}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial \kappa}{\partial y} \right) + (C_1 P + C_3 B - C_2 \rho_e) \frac{\varepsilon}{\kappa}
$$

$$
\rho = \rho(T)
$$

where $x$ and $y$ are horizontal and vertical coordinates, respectively, $t$ is time, $u$ and $v$ are velocities in $x$ and $y$ directions, respectively, $T$ is temperature, $\rho$ is density, $\rho_{fs}$ is reference (freestream) ambient density, $p$ is pressure, $P$ is production by shear, and $C_1$, $C_2$, $C\mu$, $\sigma_e$, and $\sigma_\kappa$ are empirical constants which take values 1.44, 1.92, 0.09, 1.00 and 1.30. $\mu_e$ is effective viscosity defined as $\mu_e = \mu + \mu_{\varepsilon}$ in which $\mu_{\varepsilon}$ is turbulent viscosity ($\mu_t = \rho C_{\mu} \kappa^2 / \varepsilon$) and $\mu$ is molecular viscosity. In eqn 4 $\alpha_e$ is effective diffusivity, $\alpha_e = \alpha_t + \mu / \sigma_t$ in which $\alpha_t$ is thermal conductivity and $\sigma_t$ is the turbulent Prandtl number. In eqns (5)–(6) $\mu_{\varepsilon} = \mu + \mu_{\varepsilon} / \sigma_{\varepsilon}$, $\mu_{\varepsilon} = \mu + \mu_{\varepsilon} / \sigma_{\varepsilon}$, and $B = (-g \mu_t) / (\rho \sigma_t)$. When the model is applied to a large lake with the timescale of a day, Coriolis force needs to be added to the momentum equations (eqns (2)–(3)).

2.2 Boundary and initial conditions

Boundary conditions need to be specified on all surfaces of the computational domain. Boundaries presented in this study include free boundary (line AB and CD), inflow (BC) and outflow (FE), and solid wall (AF and ED) as shown in Fig. 1. A free boundary is defined as the location where velocity and a scalar quantity is nearly equal to its free-stream ambient value (like a liquid wall). At a solid surface, the no-slip condition is applied, i.e. both mean and fluctuating velocities are zero. The dependent variables at the near-wall point are connected to the wall conditions, i.e. wall shear
stress, heat fluxes, and wall temperature, by applying a linear function for the viscous sublayer and the log-law of the wall just outside the region. The boundary conditions are specified as follows.

1. \( u = v = 0 \) on lines, AF, ED, AB and CD, \( \partial u/\partial x = \partial v/\partial x = 0 \) and \( \int_0^1 u \, dy = Q_j \) (\( u \geq 0 \)) on line FE, and \( u = U_j \) and \( v = 0 \) on line BC;

2. \( T = 0^\circ C \) on AF, \( \partial T/\partial y = 0 \) on ED, \( T = T_a \) on AB and CD, \( \partial T/\partial x = 0 \) on FE, and \( T = T_j \) on BC; and

3. \( \partial \kappa/\partial y = 0 \) and \( \varepsilon = C_\mu^{1/4} \kappa^{3/2} / 0.4 \rho U_j^2 \) on lines AF and ED, \( \kappa = \varepsilon = 0 \) on AB and CD, \( \partial \kappa/\partial x = \partial \varepsilon/\partial x = 0 \) on FE, and \( \kappa = f_1 \rho U_j^2 \) and \( \varepsilon = f_2 \kappa \) on BC;

where \( U_j \), \( Q_j \), and \( B_0 \) are jet velocity, flow rate, and thickness at the discharge, respectively. \( y_1 \) is the total depth of the flow domain. \( T_a \) is freestream ambient water temperature. \( \delta y \) and \( \delta x \) are thicknesses of near-wall grids. \( f_1 \) and \( f_2 \) are empirical constants for the kinetic energy and dissipation rate, respectively, resulting from inlet velocity fluctuations, i.e., a fraction of that calculated from the discharge velocity, \( U_j \).

The flow field is initially a quiescent fluid with zero initial flow velocity components \( u \) and \( v \). Initial \( \kappa \) and \( \varepsilon \) are set to be zero for the stationary ambient water. Constant initial temperatures are assumed in horizontal planes. A linear or non-linear variation of temperature in the vertical direction can be specified as the initial condition.

2.3 Numerical scheme

The equations governing the flow field (eqns (1)–(7)) are solved numerically by the method given by Patankar.\(^{13,14}\) A modified version of the computer program based on the SIMPLER algorithm\(^5\) is employed. A fully-implicit scheme for the unsteady term was used. Variable timesteps were employed and determined by the speed of convergence in previous timesteps. The original computer code is extended for the solutions of an unsteady, non-linear flow problem and for incorporation of buoyancy and appropriate boundary conditions. The non-linearity of convection, diffusion, and source terms in the governing equations is handled by iteration within a time step.

3 MODEL APPLICATION AND SIMULATION METHOD

In the numerical simulations and analyses of multi-dimensional jet flows, the ambient water on the discharge side of the flow field have been assumed undisturbed or stationary by previous investigators.\(^{10,19,12,5,7}\) Under this assumption, the undisturbed water can be considered as a quiescent free boundary which prohibits mixing between jet flow region and ambient. Therefore, the jet flow bounded by an ice cover, a sediment bottom, and a free boundary on the inflow side can be analyzed as a buoyant offset jet whose behavior is governed by buoyancy as well as boundary.

Lines AB and CD on the inflow side in Fig. 1 were treated as a stationary free boundary. The jet hypothetically becomes an offset jet from a liquid wall. After the offset jet leaves the nozzle, it curves towards the boundary and attaches to it, enclosing an eddying region of separated flow.\(^8\) This phenomenon, known as the Coanda effect,\(^{16}\) is caused by the reduction of pressure on the inner side of the jet. If the discharge is located close to both top and bottom boundary surfaces, attachment of the jet to either boundary is possible in a non-symmetric situation. Generally, the jet will attach to the nearer surface but external forces such as ambient turbulence, surface turbulence, jet buoyancy, and slope of the bottom can alter the jet trajectory. For symmetric geometry, where the discharge is located at an equal distance from the top and from the bottom, and for symmetric external forces about the initial jet centerline, the jet remains straight and attachment does not occur.\(^5\) This discharge configuration can be used to achieve farther distance and longer time for dilution of discharged wastewaters when attachment to boundary needs to be prevented.

To simulate the effects of buoyancy and stratification on the behavior of a buoyant jet in stratified ambient, the Coanda effect should be de-emphasized. The effect of buoyancy on flow pattern can be separated from that of the boundary by setting the discharge centerline at the mid-point between the top boundary and the bottom boundary and by specifying the same conditions at both boundaries except temperature. No deflection occurs to a non-buoyant jet with symmetric boundaries\(^6\) because the reduced pressures caused by the presence of confined vortices on both sides of the jet are symmetrical. The symmetric non-attachment flow is stable if the jet is not very close to the boundaries and if no disturbances are introduced.\(^9\) The vertical deflection of a buoyant jet in the cases to be presented is therefore caused primarily by the buoyant and ambient stratification.

Vertical ambient temperature stratification is assumed initially linear with 0°C at the top, 4°C at the bottom, and 2°C at the mid-depth. This is a typical temperature structure in natural lakes during winter. The undisturbed ambient water along the inflow (free) boundary is assumed calm and linearly stratified.

A non-uniform grid system was used with finer grids for the jet region and boundary layers, and coarser grids for other regions. The size of computational domain was characterized by a ratio of length to depth (\( x_j/y_j \)) and a ratio of depth to jet width (\( y_j/B_0 \)). Numerical experiments and sensitivity analysis were carried out to eliminate the effects of size of computational domain and number and distribution of grid
points on model results. The selected domain was a $34 \times 26$ mesh with a length-to-depth ratio of 3.0 ($x_1/B_0 = 150$ and depth $y_1/B_0 = 50$). A discharge thickness or jet width ($B_0$) of 0.2 m was used. The actual physical dimensions are $x_1 = 30$ m and $y_1 = 10$ m. This relatively coarse grid system did not result in significant numerical effects, which did not require a great amount of computational time. Much finer grids may be needed for fully grid-independent solutions.

The 2D simulation model was validated against existing experimental data for non-buoyant vertical jets in homogeneous waters and for jets discharged parallel to horizontal boundaries and offset from a vertical wall into uniform ambients. The vertical jet data were from experimental studies by Gutmark and Wygnanski and Andreopoulos et al. Laboratory experiments for plane offset jets were conducted by Sawyer, Ali and Salehi-Neyshaboury, and Rajaratnam and Subramanya.

4 CHARACTERIZING PARAMETERS

The major parameters characterizing the discharge of a
buoyant jet are the jet nozzle Reynolds number defined as $Re = U/jB_0/\nu$ and the densimetric Froude number defined as $Fr_d = U/jB_0/\gamma_0^{1/2}$ characterizing the ratio of momentum to buoyancy. The Richardson number, defined as $Ri = g' B_0/U_j^2$, describes the strength and sign of buoyancy ($Ri = 1/Fr_d$). The sign of jet buoyancy (positive or negative) is determined by $g' = g(\rho_a - \rho_j)/\rho_j$, where $\rho_a$ and $\rho_j$ are densities of ambient water and injected water, respectively, at the jet origin. If the jet fluid does not contain dissolved or suspended material, the density is only related to the fluid temperature. Therefore, the injected water temperature, $T_j$, determines the initial direction of the buoyant force through the density–temperature relation with the maximum density at 4°C. A dimensionless time is defined as the real time $t$ multiplied by the ratio of flow rate per unit length of the slot jet to the volume of water body per unit width, i.e., $t^* = tU/jB_0/(\gamma_jx_j)$. At $t^* = 1$, $t$ is equivalent to the time required to completely withdraw the water at a flowrate of $Q_j = U/jB_0$.

The evolution of flow and thermal fields of a submerged jet in an ice-covered lake and the effects of buoyancy and stratification on flow pattern were examined through numerical simulations using the 2D model. Cases with different injection temperatures, $4°C \leq T_j \leq 20°C$, were simulated and analyzed. Distributions of velocity and temperature and details of the flow pattern and mixing process were obtained for each set of conditions. Comparison of jet behavior was made by changing any one of the parameters $t^*$, $T_j$ and $Fr_d$ (or $Ri$) and fixing the other two. Due to the peculiar variation of water density as a function of temperature at low temperatures ($0-8°C$) with the maximum water density at $4°C$, the buoyancy and stratification effects on the jet flow in an ice-covered lake are complicated. A heated jet discharged with a temperature $T_j > 4°C$ into ambient water with a linear vertical temperature distribution between $0°C$ at the surface and $4°C$ at the bottom may rise initially but sink down later. When cooled to $4°C$ by the ambient water, the leading edge of

Fig. 4. Computed velocity vectors for $T_j = 8°C$ ($Re = 35000$ and $Fr_d = 13$).

Fig. 5. Computed velocity vectors and temperature contours for $T_j = 10°C$ ($Re = 35000$ and $Fr_d = 8$).
the jet is always sinking until the ambient water warms up to 4°C and above. The spatial and temporal reversal of flow direction is a unique feature of the low ambient water temperature situation. A jet always rises due to positive buoyancy or sinks all the time due to negative buoyancy if $T_j > 4^\circ C$ and $T_a > 4^\circ C$ as seen in lakes during summer and if both $T_j$ and $T_a$ are lower than 4°C.

5 RESULTS AND DISCUSSION

At the discharge point a jet can be positively or negatively buoyant depending on ambient temperature relative to its own temperature and density. The jet will rise if the initial buoyancy is positive and sink if negative. The behavior of a buoyant jet with a discharge temperature of 6°C is illustrated in Figs 2 and 3 in which $Fr_d = 60$ and $Re = 35,000$. The jet is almost neutral at the jet origin ($Re = 0.00028$) because the density of discharged water (6°C) is slightly lower than that of ambient water (at 2°C). Recirculation due to jet entrainment and ambient water replenishment in the
regions above and below the jet create a vortex on each side of the jet. As the discharged water follows the jet trajectory, it is diluted by entrainment of ambient cold water and hence its temperature drops to 4°C and density increases to its maximum. The increase in density causes negative buoyancy and turns the slightly rising jet at \( t^* = 0.01 \) into a strongly sinking one and attaches to the bottom at \( t^* \geq 0.1 \). The upper vortex moves downstream and stretches in the horizontal direction during the flow attachment and wall jet propagation. A back flow is expected to occur at the upper portion of the outflow boundary to complete the large upper recirculation. Due to limitations of the model, negative velocities at the outflow boundary are prevented by specifying positive or zero velocities, which are calculated from the near-boundary velocities.\(^{15}\) The velocity distribution over the outlet boundary is then adjusted to satisfy the continuity, i.e. flow through the outlet boundary = flow across the inlet boundary.\(^{15}\) The outflow velocity treatment causes the fluid to rise at the outlet boundary to close the upper recirculation. However, as the outflow boundary is located at a sufficiently long distance from the discharge,

\[ T_j = 4 \, ^\circ C \quad T_j = 8 \, ^\circ C \]
\[ T_j = 10 \, ^\circ C \quad T_j = 20 \, ^\circ C \]

\[ t^* = 0.005 \]
\[ t^* = 0.025 \]
\[ t^* = 0.05 \]

**Fig. 8.** Computed velocity vectors for \( T_j = 15^\circ C \) (\( Re = 48 \, 000 \) and \( Fr_d = 6 \)).

**Fig. 9.** Simulated isotherms at \( t^* = 0.01 \) for \( T_j = 4, 8, 10 \) and 20°C (\( Fr_d = 6 \)).
Fig. 10. Simulated isotherms at $t^* = 0.05$ for $T_j = 4, 8, 10$ and $20^\circ C$ ($Fr_d = 6$).

its influence on the flow pattern is expected to be insignificant.

Figures 4 and 5 present flow fields in the form of velocity vectors and thermal fields in the form of isotherms at various times ($t^* = 0.01, 0.1$ and $0.3$) for $T_j = 8$ and $10^\circ C$ with $Fr_d = 13$ and $8$, respectively. These time points were selected to represent three mixing stages, i.e. initial, intermediate, and long-term. As the jets are initially cooled quickly down to $4^\circ C$ (maximum density) by dilution due to entrainment, a reversal in buoyancy (from positive to negative) turns the rising jets into falling ones ($t^* = 0.01$). As time progresses more mixing between the jet and the ambient occurs. The ambient becomes warmer and more uniform. Rapid cooling of the jet is slowed down. A reversal in buoyancy from negative to positive drives the jet upwards to the ice cover ($t^* = 0.1$), impinging on the top surface and becoming a wall jet. Impingement of the jet with $T_j = 10^\circ C$ occurs at $t^* = 0.1$, while the $8^\circ C$ jet impinges on the ice cover at $t^* = 0.3$. This is because the former has stronger positive buoyancy than the latter resulting from a larger difference in jet-ambient temperatures.

The jet with an injection temperature of $15^\circ C$ has strong initial positive buoyancy, driving the jet to rise up because of the significant density difference between the jet and the ambient (Figs 6–8). A significant buoyancy effect on flow behavior can clearly be seen from comparison of the flow fields shown in Fig. 6 ($Fr_d = 4$) with those in Fig. 8 ($Fr_d = 6$) at $t^* = 0.005, 0.025$ and $0.05$, respectively. A smaller $Fr_d$ results in a more curved jet trajectory and a shorter attachment distance.

Flow patterns and trajectories of jets with different discharge temperatures under the same buoyancy condition ($Fr_d$) at a specific time are compared further. Presented in Figs 9 and 10 are the simulated isotherms of four buoyant jets with $T_j = 4, 8, 10$ and $20^\circ C$ at $t^* = 0.01$ and $0.5$, respectively, in which $Fr_d = 6$ and $Re = 8000–70000$. The $4^\circ C$ discharge is driven to the bottom by the negative buoyant force all the time. As cooled down to $4^\circ C$ by the ambient water, the leading edge of a warm jet is always sinking until the ambient water warms up to $4^\circ C$ or higher in all cases of $T_j = 4^\circ C$. The jet with $8^\circ C$ water is first reversed from a positively buoyant jet shortly after the discharge is initiated to a negatively buoyant one at $t^* = 0.01$ and is then turned from sinking to rising at $t^* = 0.05$. This
Table 1. Summary of near-field jet characteristics and flow processes

<table>
<thead>
<tr>
<th>$T_j$ (°C)</th>
<th>Re</th>
<th>Ri</th>
<th>$t^*$</th>
<th>0</th>
<th>0-005</th>
<th>0-010</th>
<th>0-025</th>
<th>0-050</th>
<th>0-1</th>
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<td>4</td>
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<td>S</td>
<td>S</td>
<td>S, A</td>
<td>S, A</td>
<td>N</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>6</td>
<td>35000</td>
<td>0-00028</td>
<td>$L_a/B_0$</td>
<td>R $\geq$ N</td>
<td>S</td>
<td>S</td>
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<td>$L_a/B_0$</td>
<td>R</td>
<td>R $\rightarrow$ S</td>
<td>R $\rightarrow$ S</td>
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<tr>
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<td>0-0278</td>
<td>$L_a/B_0$</td>
<td>R</td>
<td>R $\rightarrow$ S</td>
<td>S</td>
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<td>R, A</td>
<td>N</td>
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</tr>
</tbody>
</table>

Note: 'R', 'S', 'R $\rightarrow$ S', 'A', and 'N' denote Rise, Sink, change from Rise to Sink along a jet trajectory due to spatial reversal of buoyancy, Attachment to and impingement on a boundary, and Neutral or non-buoyant, respectively. $L_a$ is jet attachment distance from the inflow boundary.

is the so-called temporal reversal of buoyancy. In the situation of $T_j = 10^\circ$C, the jet behavior is similar to that of $T_j = 8^\circ$C. At $t^* = 0-01$ the 10°C jet is driven up by a positive buoyant force in the region near the jet origin and deflected down towards the bottom boundary in the downstream field due to buoyancy reversal, the so-called spatial reversal of buoyancy. The jet with a discharge temperature of 20°C is dominated by positive buoyancy. It is seen that the behavior of a buoyant jet is very sensitive to the discharged water temperature, $T_j$. Strong spatial and temporal buoyancy reversals occur at $T_j = 8-10^\circ$C. Negative buoyancy dominates a jet of low discharge temperature (4–5°C). Positive buoyancy plays a very important role in a high discharge temperature situation (≥15°C). The effect of $Fr_d$ on flow pattern becomes very evident if one compares the flow fields of jets with $T_j = 8$ and $10^\circ$C at $t^* = 0-01$ shown in Fig. 9, where $Fr_d = 6$, with that in Figs 4 and 5 where $Fr_d = 13$ and 8, respectively. Higher $Fr_d$ (lower buoyancy) results in straighter jet trajectory and longer attachment distance.

Table 1 summarizes discharge conditions ($T_j$, $Fr_d$ and $Ri$), jet behavior, and flow patterns in the near-field ($0 < x/x_1 < 0-5$) at various time points. The initial jet buoyancy strength is represented by the Richardson number ($Ri$). Strong negative buoyancy drives the jets of 4 and 6°C to sink and attach to the bottom, whereas strong positive buoyant forces drive the jets of 15 and 20°C to rise and attach to the ice cover. The jets with $T_j = 8-15^\circ$C are characterized by a change from rising to sinking due to spatial buoyancy reversal at a given time. During the flow processes of $T_j = 8-10^\circ$C, temporal buoyancy reversals lead to transitions from rising to sinking and, in turn, from sinking to rising, and results in attachment to the upper boundary. In all cases studied there is no attachment until $t^* = 0-025$. The time of first attachment is proportional to $T_j$ and $Ri$ for $T_j = 4–8^\circ$C and inversely related to $T_j$ and $Ri$ for $T_j > 8^\circ$C. Stronger buoyancy (larger $Ri$) yields a shorter attachment distance ($L_a$) for a given $T_j$. A higher discharge temperature results in a shorter attachment length in the situations of $T_j > 6^\circ$C for a given $Ri$. During jet attachment the attachment location in all the jets considered moves upstream, i.e. $L_a$ decreases, until a critical time point at which buoyancy reaches its maximum. However, the attachment point is expected to travel downstream ($L_a$ increases) after the critical time when momentum starts to dominate. As time tends to infinity, all jets will become neutral or non-buoyant and detach from their boundary.

It is shown in Figs 2–10 and Table 1 that the flow pattern and mixing process at a specific time point largely depends on the discharged temperature, $T_j$, and buoyancy, $g B_0/U_z^2$. If the water body has a finite volume, as is often the case, the ambient cold water will eventually be replaced by warmer water recirculating from the jet. If all of the ambient water warmed to 4°C after a certain period of time, the jet discharge ($T_j > 4^\circ$C) would become positively buoyant everywhere and rise continuously. After the ambient water is completely replaced by the discharged water, the jet becomes neutral or non-buoyant in all cases, reaching an equilibrium state and forming a horizontal jet. However, it would take an extremely long time for a large natural lake to be completely filled with the discharged water under a small jet flow rate. It is also possible that only a part of the lake or semi-open waterbody is involved in the circulation induced by the jet.
6 COMPARISON WITH OFFSET JETS

Similar flow patterns can be observed if one compares the processes of jet attachment, boundary impingement and wall-jet flow after the jet is deflected towards the boundary by a buoyant force, with that of a non-buoyant offset jet close to a boundary and deflected by the Coanda effect.\(^{7,8}\) Attachment to either the bottom or the top boundary may occur after a buoyant jet is driven up or down by a positive or negative buoyant force, followed by impingement and wall-jet flow. However, a non-buoyant offset jet remains attached to the boundary once it is deflected by reduced pressure. The driving force for the deflection of a buoyant jet results from density difference, which turns the jet towards its boundary. Buoyancy reversal and non-uniform ambient water density together make jet attachment and boundary impingement unsteady. Attachment length is inversely related to the buoyancy strength \((Ri = g' \rho_0 / U_0^2)\). Since all jets studied are highly turbulent at \(Re = 8000 – 70,000\), flow patterns are not very sensitive to Reynolds number.

Jet detachment from the boundary may occur because of buoyancy reversal, as seen in Fig. 7 at \(t^* = 0.0328\) \((T_i = 15°C)\) and in Fig. 9 \((T_i = 20°C)\) and Fig. 10 \((T_i = 10°C)\). Therefore, wall-jet flow following jet impingement may not appear in the buoyant jet case. In contrast, jet separation from the boundary does not exist in the non-symmetric and non-buoyant offset jet cases.\(^8\) In a symmetric and buoyant situation, jet attachment to and impingement on either the top or the bottom boundary can be caused by a buoyant force or the Coanda effect, or by a combination of the two. Jet attachment and impingement eventually disappear if the jet is located sufficiently far from the boundary and if no disturbances appear. The jet becomes a non-buoyant one, i.e. its equilibrium state after the ambient water is completely mixed by continuous discharge, entrainment and dilution.

7 CONCLUSIONS AND RECOMMENDATIONS

Plane turbulent buoyant jets in stratified cold water with an ice cover were investigated using a 2D simulation model. Velocity, pressure, temperature, density and turbulence, their distributions in the whole domain and their variations with time, were simulated. A detailed description of flow characteristics and mixing processes was provided. Jet behavior and flow patterns including deflection, attachment, impingement, and recirculation were captured through numerical simulations. Peculiar hydrodynamic and hydrothermal features of the jets caused by the temporal and spatial reversals in buoyancy were identified. The flow pattern is very sensitive to the temperature at the discharge due to jet buoyancy and ambient stratification. A warm water jet can be sinking, or it can permanently rise to the surface where it may melt the ice cover depending on the state of mixing achieved in the ambient water. For \(T_j > 4°C\) the leading edge of a jet is always sinking, due to the cooling effect of the ambient water, until the ambient warms up to 4°C or higher. Strong negative buoyancy drives the 4–6°C jets to sink and attach to the bottom, while 15–20°C jets are driven to rise and attach to the ice cover by strong positive buoyant forces. Strong spatial and temporal buoyancy reversals appear in the situations of \(T_j = 8–10°C\). Dilution of a warm discharge by entrainment of cold ambient water causes the reversal in buoyancy which alters the jet’s flow direction. There is no attachment until \(t^* = 0.025\) in all cases. The time of first attachment depends on \(T_j\) and \(Ri\). A higher jet densimetric Froude number results in straighter jet trajectory and longer attachment distance. \(T_j\) also influences attachment distance. Attachment disappears once the jet becomes neutral and after the finite volume of ambient water is completely replaced by discharged water as \(t^* \to \infty\).

The jet was simulated as an offset jet by assuming a liquid wall, i.e. the free boundary on the inflow side of the flow domain. The effect of buoyancy on jet behavior was studied by using a symmetric geometry to decouple it partially from that of the Coanda effect, caused by the boundary. The flow characteristics shown by the processes of deflection, attachment, impingement and wall-jet decay exhibit some similarities of the buoyant jet with the boundary-affected jet. However, some specific deviations are expected because of different driving forces, the buoyant force in the former and reduced pressure in the latter. Attachment may not occur in the case of a symmetric and non-buoyant offset jet in uniform ambient. Detachment of a buoyant jet in stratified cold waters from a boundary may occur due to spatial buoyancy reversal. Attachment of a non-symmetric and non-buoyant offset jet is persistent, while intermittent attachment can result from temporal buoyancy reversal in the buoyant and symmetric case. Both the buoyancy and Coanda effect should be considered in any attempts to analyze buoyant offset jets.

Validation of the model application to buoyant jets in stratified cold ambients is not possible at the present time since there have been no data reported from laboratory experiments. However, a basis and some useful information are provided for further studies, including validation of simulation models for jets with initial buoyancy in cold waters with ice covers. Future studies are recommended in quasi-steady situations \((t^* > 0.3)\) and eventual environmental impacts, the times when the ambient temperatures are raised to 4°C or greater everywhere, the time of shortest attachment distance, and the jet detachment time. More systematic analyses of the time of first attachment and the \(Fr_0\) effect on attachment length are needed. If the flow is not
fully turbulent, it may also be necessary to investigate the effect of \( Re \) on flow patterns.

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