Parameter identification in fresh-saltwater flow based on borehole resistivities and freshwater head data

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(Received 20 May 1997; revised 18 January 1998; accepted 20 November 1998)

By means of a series of borehole resistivity measurements and a resistivity–salinity relation, a particular salt-freshwater inversion was found under the shore with semi-diurnal tides at the French–Belgian border. These resistivity data provide valuable information about the vertical variation of the saltwater percentage in different boreholes. At different places and depths fluctuations of freshwater heads are observed. A regression modelling procedure is proposed in which the hydraulic parameters for density dependent flow and solute transport can be simultaneously considered with the parameters of the resistivity–salinity relation. The object function comprises resistivity residuals and freshwater head residuals along with saltwater percentage residuals and parameter residuals.

First, a synthetic problem is elaborated with this regression modelling procedure. It is followed by the application of the procedure on the observed fresh-saltwater flow problem under the shore. In the synthetic problem the identification of the hydraulic parameters was demonstrated without the inclusion of prior information about these parameters. The resistivity–salinity relation was slightly adjusted in this regression. During the regression modelling of the observations made in one of the shore-normal cross sections, the horizontal and vertical conductivity are identified along with the effective porosity and the longitudinal and transverse dispersivity. The optimal values of the dispersivities are very small. Finally, it is shown that the high waters on the back shore forms the main threat of saltwater enchroachment from the sea side of the dunes and that the isolated fresh-brackish lens under the lower part of the shore before the build up area of De Panne can be explained by overexploitation.

Key words: regression modelling, coupled density flow and transport, resistivity–salinity relation, dispersivities, saltwater enchroachment, overexploitation.

1 INTRODUCTION

The saltwater percentage of groundwater can be deduced from resistivities measured in a borehole with the long normal device. Therefore, these resistivities can be used along with the observed freshwater head data for the calibration of a solute transport model. Two kinds of parameters are then involved in the simulation-regression procedure: the hydraulic parameters that govern the groundwater flow and solute transport and the parameters in the relation between the observed resistivities in the borehole and the saltwater percentage in the sediments. This simulation-regression procedure is first demonstrated on a synthetic problem. This synthetic problem is the flow of fresh and saltwater under a shore with semi-diurnal tides. In this synthetic problem the number of generated input data of freshwater heads and of borehole resistivities is large whereas the simulated time span is relatively small. After the synthetic problem, the proposed simulation regression procedure is applied on the field problem of fresh-saltwater flow and distribution under the shore with semidiurnal tides at the French–Belgian border.
The groundwater flow, solute transport and dispersion are simulated with the numerical model of Korknikow and Bredehoeft. This model was adjusted by Lebbe so that density difference is taken into account by using freshwater heads. This model will be called further the adapted MOC model. Here, a simple linear relation was assumed between the density and the salt-water percentage. The output of this model is a freshwater head configuration and a saltwater percentage distribution in a cross section perpendicular to the shoreline. In the synthetic as well as in the field problem, the treated cross section comprises that part of the dunes where the infiltrated freshwater flows towards the sea, the shore and a small part of the subtidal area (Fig. 1). In both problems, there occurs an important saltwater infiltration on the back shore and saltwater seepage on the fore shore. This shallow saltwater cycle above the deeper freshwater cycle causes an important water density inversion. The presence of this saltwater lens above the freshwater was first observed by means of borehole resistivity measurements.

2 OBSERVED GROUNDWATER FLOW AND DISTRIBUTION

2.1 Geographical and geological description of the studied area

The observations were made on a sandy runnel and ridge shore. At the studied area, near the French–Belgian border, the average slope of the shore is 1.1%. The shore is covered by semi-diurnal tides. The difference between the high and low tide is approximately 5 m at spring tide and 3 m at neap tide. The horizontal distance between the mean high-tide line and the mean low-tide line ranges from 300 to 450 m. The lowest sea levels are around 0 m TAW (TAW indicates the Belgian datum level, 0 m TAW is 2.4 m below the mean sea level) and the highest levels are around +5.5 m TAW. With a frequency of about eleven cases in ten years the sea level overreaches +5.5 m TAW.

By the aid of twenty-nine borehole logs the lithological constitution of the phreatic aquifer on the shore of De Panne was studied. The described boreholes are situated in five different shore-normal cross sections. The distance between each of these cross sections is ca. 1 km (Fig. 1). These data were supplemented with a more detailed lithostratigraphical study made in the adjacent dune area. The substratum of the phreatic aquifer is formed by ca. 105 m thick clay layer of Eocene age of which the depth ranges between 33 and 28 m. The phreatic aquifer is formed by medium to fine-medium sands. At the French–Belgian border the phreatic aquifer is rather homogeneous, the lower part is only slightly coarser than the upper part. Towards the east the aquifer becomes more heterogeneous. In the middle of the aquifer a discontinuous sand layer may occur of which the silt and clay content ranges between 5% and 12% and of which the thickness ranges between 0 and 7 m.

In the adjacent dune area an extensive hydrogeological study was made. Based on a detailed lithostratigraphical study, geo-electrical well loggings, and pumping tests, the initial parameters of the mathematical model treating steady and unsteady state flow were estimated. The model was calibrated by the comparison of the observed tidal and seasonal fluctuations of the hydraulic head with the simulated fluctuations. The derivable hydraulic parameters were the horizontal conductivities of the pervious layers, the vertical conductivities of the semi-pervious layers, the specific elastic storage of the phreatic aquifer and the areal variation of the specific yield. One of the results of the calibrated mathematical model, which was important for this study, was the lateral variation of the freshwater flow from the dune area to the sea. At the French–Belgian border this flow is the largest and approximates very closely the flow when there was no pumping in the dune area. Towards the northeast the flow diminishes where it reaches a minimum before the water-collection area and the built-up area of De Panne (profiles K3 and K4). This lateral variation of the flow towards the sea can be deduced from the course of the groundwater divide line (Fig. 1).

2.2 Borehole resistivities

To deduce the distribution of the salt, brackish, and freshwater under the shore, resistivity measurements...
were made in twenty-nine boreholes located in five different shore-normal cross-section. The boreholes were made by the rotary drilling method with a rather small diameter, approximately 100 mm. The drilling mud was made of water and a biodegradable organic additive. The loss of drilling fluid was rather small because of the rather short drilling period (ca. 2 h), the large viscosity of the drilling mud and the formation of a less pervious mud cake on the borehole wall. So, one can expect that the invasion zone ranged between a few to maximum 10 cm. The resistivity was measured immediately after drilling with a long normal device. Here, a current and a potential ring electrode (diameter 40 mm, height 10 mm and interspace 1 m) were lowered in the borehole. The other current and potential electrodes were placed at the surface at a distance of 30 m, at opposite directions of the borehole, and parallel to the low tide line. It can be expected that the so measured resistivity approximates closely the average resistivity of the sediments within a sphere with a radius of one meter around the borehole electrodes center.

The results of the borehole resistivity logging are represented in five resistivity profiles (Fig. 2). In the profiles three resistivity regions are delineated. In the first region the resistivity is larger than 20 Ωm which consists mainly of sediments filled with freshwater. In the second region the resistivity ranges between 2.5 and 20 Ωm and in the third zone the resistivity is smaller than 2.5 Ωm. The sediments are mainly filled with brackish water in the second zone and with saltwater in the third zone. In the resistivity profile K0, at the French–Belgian border, a saltwater lens occurs above the freshwater which flow from the dunes towards the sea. This large inversion of pure saltwater above freshwater in a phreatic aquifer is not treated in the general hydrogeological literature. At the boundary between the dunes and the shore there is a nearly vertical brackish transition zone. Under the shore the transition zone is gently rising towards the sea. The resistivity of the sediments with freshwater decreases towards the sea. This indicates an increase of the salt content of the freshwater towards the sea. This deduction was confirmed by chemical analyses of water samples taken at different distances from the high tide line. This freshwater-saltwater inversion was found on the place with the largest flow of freshwater from the dunes to the sea.

The resistivity profiles K1 and K2, at 1 and 2 km from the French–Belgian border in the northeastern direction, show a distribution of fresh, brackish, and saltwater which deviates only slightly from the distribution found at the border. At profile K1 the brackish transition zone under the shore undulates around a

![Fig. 2. Shore normal resistivity profiles K0, K1, K2, K3, K4, and K5. The sediment resistivity \( \rho_i \) was measured with the long normal device.](image-url)
horizontal plane. In the lower part of the aquifer a larger downward resistivity gradient exists in the seaward direction than in the K0 profile. Consequently, the salt content of the freshwater shows a larger increase in the seaward direction so that under the low tide line the lower half of the aquifer is already completely filled with brackish water. In profile K2 the brackish transition zone rises less gently under the fore shore so that under the lower part of the fore shore the thickness of the upper saltwater lens is strongly reduced. Under the freshwater there is a foot of brackish water of which the salt content increases with depth. The rather small difference in salt-freshwater distribution in the profiles K0, K1, and K2 is due to the occurrence of discontinuous layers of finer sediments in the middle of the phreatic aquifer in the profiles K1 and K2.

There is remarkable difference between the resistivity profiles K3 and K4 and the resistivity profiles K0, K1 and K2. The profiles K3 and K4 are situated before the built-up area of De Panne and the water-collection area where the average flow towards the sea reaches a minimum and can even be slightly reversed. In these last two profiles there is a foot of freshwater underneath the saltwater on the back shore. On the fore shore there occurs a brackish water lens in the phreatic aquifer which is for the greater part filled with saltwater. In both profiles the brackish water lens occurs in the middle of the phreatic aquifer and close to the low-tide line.

2.3 Freshwater heads

The freshwater head is the head which would be measured in a well if the water column in this well is replaced over its full height by freshwater of density $d_f^{18}$. Depending of the observation site, two different kinds of freshwater head observation were made. In the dune area freshwater heads were observed monthly in eighty wells during a period of thirty months in the scope of a regional groundwater flow study of the dune area. The mean freshwater head of three of these wells were used to calibrate the model of salt and freshwater flow under the dunes, the shore, and the sea in a vertical cross section at the Belgian–French border (see below). These three wells are located close to the border and at three different distances to the high tide line. The screens of these wells have a length of 1 m and are situated around the 0 m TAW level.

On the shore and in the dunes close to the shore, the mean freshwater heads were deduced from two series of ‘continuous’ observations by means of pressure transducers connected to a data logger. A first series of freshwater head observation were made to determine the mean freshwater head just underneath the surface of the shore. Because the freshwater head fluctuates with the semi-diurnal tides, the pressures were measured during a period of six days with an interval of 10 min in six observation wells. These wells were installed in the same plane as the resistivity profile K0 and have their screens of 0.5 m length around the $-2$ m TAW level. The locations of the screens are indicated in Fig. 2. Four wells were placed on the shore at different distances of the high tide line. Well P1 is located at the lowest part of the shore. The wells P2, P3, and P4 are placed on the shore with an increasing distance to the low tide line. Well P5 is located between the mean high tide line and the high–high tide line whereas well P6 is situated in the dunes ca. 70 m from the high–high tide line.

In Fig. 3 the observed freshwater heads are plotted versus time. This measurement period started in rather exceptional circumstances during a northwestern storm where the sea level overreached $+5.5$ m TAW. These observations show the particular fluctuations of the freshwater head just underneath the surface of the shore. These fluctuations are characterized by two abrupt changes of the time derivative of the freshwater head during one tidal period. During the inundation of the shore the head fluctuates according to the tidal fluctuation of the sea level. From the moment that the sea retreats from the considered part of the shore then the head starts to lower very slowly. This slow lowering is

![Fig. 3.](image-url)
due to the relatively slow groundwater flow and the large specific yield of the phreatic aquifer. This lowering continues until the sea inundates again that part of the shore. From that moment on there is again infiltration of saltwater which causes a sudden strong rise of the head after which the head follows again the sea level fluctuations. With this knowledge it is now possible to calculate the mean freshwater head at each point of the shore based on the elevation of that point of the shore and on the tidal fluctuation of the sea level. These temporal means are used as estimates of the freshwater heads in the boundary condition at the shore surface for the simulation of the flow averaged over the time (Part 5). Finally, it should be mentioned that the runnels and the ridges on the shore can drastically change in location and on the tidal fluctuation of the sea level. During these measurements the pressure was made during a period of three days on and close to the shore. During these measurements the pressure was followed during the first observation series and in two parts of the phreatic aquifer (well P2') with those in the upper part (well P2). The mean freshwater head is there 0.7 m higher in the lower part of the aquifer than at the upper part (well P2). In Fig. 4 the freshwater head derived from these pressure measurements are represented versus time. These measurements allow us to compare the head fluctuations in the lower part of the phreatic aquifer (well P2') with those in the upper part (well P2). The average freshwater heads of the wells P2 and P4 are situated at the same distance of the low water line as the wells P2 and P4. In Fig. 4 the freshwater head derived from these pressure measurements are represented versus time. These measurements allow us to compare the head fluctuations in the lower part of the phreatic aquifer (well P2') with those in the upper part (well P2). The mean freshwater head is there 0.7 m higher in the lower part of the aquifer than at the upper part of the aquifer. In contrast with the fluctuations in the upper part of the aquifer, the time derivatives of the freshwater head in the lower part of the aquifer show a continuous course. These fluctuations show a typical asymmetric course with slow lowering of the head and a quicker rise. The difference between the maximum and the minimum is larger in well P2 than in well P2'. The average freshwater heads of the wells P2' and P4' are used in the calibration of salt and freshwater flow model.

3 THEORETICAL CONSIDERATIONS

3.1 Numerical model of solute transport

The numerical model of Konikow and Bredehoef11 was originally developed for the simulation of the flow and the solute transport in a twodimensional horizontal plane. The solute transport is simulated by the method of characteristics (MOC). This method uses a particle-tracking procedure to represent convective transport and a two-step explicit procedure to solve a finite-difference equation that describes the effect of hydrodynamic dispersion, fluid sources and sinks, and divergence of velocity. The result of the solute-transport equation is a matrix representing the areal distribution of the saltwater percentage. This saltwater percentage is now included in the groundwater flow equation so that this flow is density dependent14.

In this paper the model is applied in a vertical cross section. The horizontal and vertical components \( q_i \) of the Darcian velocity at a point \( i \) are then

\[
\begin{align*}
q_i &= k_i \frac{\partial h_i}{\partial x} + \frac{\partial z}{\partial t} \\
q_i &= k_i \frac{\partial h_i}{\partial z} + \frac{\partial x}{\partial t}
\end{align*}
\]

(1)

where \( k_i \) and \( \frac{\partial h_i}{\partial \phi} \) are the horizontal and vertical conductivity for freshwater, \( h_i \) and \( \frac{\partial h_i}{\partial \phi} \) are the densities of the freshwater and of the water in point \( i \) and \( \mu_i \) and \( \mu_i \) are their dynamic viscosities, \( \partial h_i/\partial x \) and \( \partial h_i/\partial z \) are the horizontal and vertical freshwater head gradients and \( (\delta_i - \delta_i)/\delta t \) is the buoyancy at point \( i \). Because the viscosity of saltwater is only about 1% larger than the viscosity of freshwater, the factor \( \mu_i/\mu_i \) is ignored in the adapted MOC model.

The buoyancy is deduced from the saltwater percentage which is derived from the solute transport equation. A simple linear relation is assumed between the salinity and the density at all the points. The higher degree terms and the influence of the temperature and pressure in this relation can be ignored within the limiting values of the salinity, temperature and pressure that can occur in the studied area. So the relation between the buoyancy and the saltwater percentage \( P_s \) can be written as follows

\[
\frac{\delta_i - \delta_i}{\delta_i} = \frac{P_s}{100} \frac{\delta_s - \delta_i}{\delta_i}
\]

(2)

The density of the freshwater \( (P_s = 0\%) \) with a total dissolved solid content (TDS) of 500 mg/l is taken equal to 1 g/cm³ and the density of the saltwater (TDS = 28.700 mg/l, \( P_s = 100\%) \) is taken as 1.022 g/cm³.

![Fig. 4. Freshwater head fluctuations observed at the French-Belgian border in wells located on the fore shore (P2, P2', P4'), on the back shore (P5), and in the dunes close to the sea (P6). Wells P2, P5, and P6 have screens of 0.5 m and are situated around the 2 m TAW level. Wells P2' and P4' have screens of 2 m and are situated around the 22 m TAW level.](Image)
The law of continuity for steady groundwater flow can be written as
\[ \frac{\partial q_h}{\partial x} + \frac{\partial q_v}{\partial z} = 0. \tag{3} \]
The finite-difference approximation of this continuity law was introduced in the model of Konikow and Bredehoeft\textsuperscript{11} and is solved with the alternation direction implicit method\textsuperscript{13}. The two-dimensional areal transport and dispersion of a non-reactive dissolved chemical species are calculated following the computer code of Konikow and Bredehoeft\textsuperscript{11} taking into account the density dependent dispersion of a non-reactive dissolved chemical species are calculated taking into account the newly obtained areal distribution of the saltwater percentage.

3.2 Resistivity–salinity relation

The model of the parallel electrical conductivities\textsuperscript{19} poses that the electrical conductivity of sediments filled with pore water is equal to the sum of the electrical conductivity of the matrix and the electrical conductivity due to the pore water. This last electrical conductivity is equal to the reciprocal value of the product of the true formation factor \[ \frac{1}{\rho_t} = \frac{1}{\rho_{mat}} + \frac{1}{F \rho_w}, \tag{4} \]
where \( \rho_t \) is the resistivity of the sediment saturated with pore water and \( \rho_{mat} \) the resistivity of the matrix or of the solids.

The resistivity of the pore water \( \rho_w \) can be expressed as a function of the saltwater percentage \( P_s \), the resistivity of the pure freshwater \( \rho_f \) and the resistivity of the saltwater \( \rho_s \)
\[ \frac{1}{\rho_w} = \frac{P_s}{100 \rho_s} + \frac{100 - P_s}{100 \rho_f}. \tag{5} \]
By substitution of eqn (5) in eqn (4) one obtains a relation between the resistivity of the sediment \( \rho_t \) and the saltwater percentage \( P_s \) or vice versa
\[ \frac{1}{\rho_t} = \frac{1}{\rho_{mat}} + \frac{1}{F} \left( \frac{P_s}{100 \rho_s} + \frac{100 - P_s}{100 \rho_f} \right), \tag{6} \]
\[ P_s = 100 \left( \frac{F}{\rho_f} - \frac{F}{\rho_{mat}} - \frac{1}{\rho_t} \right) \left( \frac{1}{\rho_s} - \frac{1}{\rho_f} \right). \]
By means of the adapted MOC model and eqn (6) the sediment resistivity can be calculated. On the other hand the saltwater percentages of the pore water in the sediments are derived from the observed sediment resistivity by the second equation in eqn (6).

3.3 Residuals

Four types of residuals are considered: the first concern the electrical resistivities, the second the saltwater percentage, the third the freshwater head and the fourth the hydraulic parameters. These residuals are called respectively the resistivity residuals, the saltwater percentage residuals, the freshwater head residuals and the parameter residuals. They are defined as
\[ r_p = \ln \rho_i^* - \ln \hat{\rho}_i, \]
\[ r_{sr} = \ln P_i^* - \ln \hat{P}_i, \]
\[ r_{nh} = h_i^* - h_i, \]
\[ r_p = \ln \rho^* - \ln \hat{\rho}. \]
where \( r_p \) is the vector of the resistivity residuals, \( \rho_i^* \) the vector of the observed sediment resistivities, \( \hat{\rho}_i \) the vector of the calculated resistivities, \( P_i^* \) is the vector of the saltwater percentage residuals, \( \hat{P}_i \) the vector of the saltwater percentages derived from the measured sediment resistivities and the second relation in eqn (6), \( \hat{P}_i \) the calculated saltwater percentage with the adapted MOC model, \( \rho_w \) the vector of the observed sediment resistivities, \( \hat{\rho}_i \) the vector of the calculated resistivities, \( \rho_{mat} \) the resistivity of the matrix or of the solids.

3.4 Sensitivities

Several types of sensitivities are calculated. The first two types concern the sensitivities of the calculated resistivities: the first with respect to the hydraulic parameters HP such as the effective porosity \( n \), the longitudinal dispersivity \( \alpha_L \), the transverse dispersivity \( \alpha_T \), the horizontal conductivity \( k_h \) and the vertical conductivity \( k_v \), the second type with respect to the electrical parameters EP such as the formation factor \( F \), the resistivities of the pure saltwater \( \rho_s \), the resistivities of the pure freshwater \( \rho_f \) and the resistivity of the matrix \( \rho_{mat} \)
\[ J_{1,i,ph} = \frac{\ln \hat{\rho}_i(HP_{ph} \cdot sf) - \ln \hat{\rho}_i}{\ln sf}, \]
\[ J_{2,i,pe} = \frac{\ln \hat{\rho}_i(EP_{pe} \cdot sf) - \ln \hat{\rho}_i}{\ln sf}, \tag{8} \]
where \( sf \) is the sensitivity factor, \( \hat{\rho}_i \) the calculated sediment resistivity corresponding to the \( x \) and \( z \) coordinates of the \( i \)th observation with the initial estimates of the hydraulic parameters and the electrical parameters (or the estimates of the former iteration), \( \hat{\rho}_i(HP_{ph} \cdot sf) \) is the same as \( \hat{\rho}_i \) with the only difference that the estimate of the \( j \)th hydraulic parameter is multiplied by the sensitivity factor and \( \hat{\rho}_i(EP_{pe} \cdot sf) \) is the same as \( \hat{\rho}_i \) with the only difference that the estimate of the \( j \)th electrical parameter is multiplied by the sensitivity factor.

The second two types concern the sensitivities of the calculated saltwater percentage: the first with respect to
the hydraulic parameters HP, the second type with respect to the electrical parameters EP

\[
\begin{align*}
J_{3, k} &= \frac{\ln \tilde{P}_a(HP_{ph} \cdot sf) - \ln \tilde{P}_a}{\ln sf}, \\
J_{4, k} &= \frac{\ln \tilde{P}_a(EP_{ph} \cdot sf) - \ln \tilde{P}_a}{\ln sf},
\end{align*}
\]

where \( \tilde{P}_a \) is the calculated saltwater percentage corresponding to the \( x \) and \( z \) coordinates of the \( k \)th observation with the initial estimates of the hydraulic parameters and the electrical parameters (or the estimates of the former iteration), \( \tilde{P}_a(HP_{ph} \cdot sf) \) and \( \tilde{P}_a(EP_{ph} \cdot sf) \) are similarly defined as \( \tilde{P}_h(HP_{ph} \cdot sf) \) and \( \tilde{P}_h(EP_{ph} \cdot sf) \). The fifth type concerns the sensitivities of the freshwater head to the hydraulic parameters

\[
J_{5, k} = \frac{\tilde{h}_h(HP_{ph} \cdot sf) - \tilde{h}_h}{\ln sf}, \quad J_{6, k} = 0,
\]

where \( \tilde{h}_h \) is the calculated freshwater head corresponding to the \( x \) and \( z \) coordinates of the \( k \)th observation with the initial estimates of the hydraulic parameters (or the estimates of the former iteration), \( \tilde{h}_h(HP_{ph} \cdot sf) \) is the same as \( \tilde{h}_h \) with the only difference that the estimate of the \( j \)th hydraulic parameter is multiplied by the sensitivity factor.

The freshwater heads are independent of the electrical parameters. Consequently, all elements of the sensitivity matrix \( J_6 \) of the freshwater heads with respect to the electrical parameters are zero. The last considered sensitivity matrix \( J_7 \) of the hydraulic and electrical parameters with respect to their own is an identity matrix.

### 3.5 Simulation-regression procedure

The simulation-regression procedure minimizes the sum of squared resistivity residuals and the weighted sums of the squared saltwater percentage residuals, of the freshwater head residuals and of the parameter residuals in order to estimate the values of the hydraulic parameters as well as parameters of the resistivity–salinity equations (eqn (6)). The object function which must be minimized is expressed as

\[
\text{SSR} = \sum_{i=1}^{n} \lambda_{P_i} \left( \tilde{P}_i - P_i \right)^2 + \lambda_{hf} \sum_{j=1}^{m} \left( \tilde{h}_f - \hat{h}_f \right)^2 + \lambda_{sf} \sum_{j=1}^{m} \left( \tilde{s}_f - \hat{s}_f \right)^2 + \lambda_{p} \sum_{k=1}^{p} \left( \tilde{P}_a - \bar{P}_a \right)^2,
\]

where \( n \) is the number of resistivity observations, \( m \) the number of freshwater head observations and \( p \) the number of considered parameters. The weighting factors \( \lambda_{P_i}, \lambda_{hf} \) and \( \lambda_{sf} \) are calculated by an iterative process. These values are first put equal to one. The object function (11) is minimized with the linearization method after which the new \( \lambda \) values are calculated so that the variances of the resistivity, the saltwater percentage, the freshwater head and the parameter residual populations are the same. With this new \( \lambda \) values the objective function is again minimized. The iterative process is continued until the changes in \( \lambda \) values are small.

The regression analyses are performed following the general guidelines given in Cooley and Naiff. During the successive iterations of the regression analysis the parameter correction vector can be multiplied by a factor smaller than one to prevent overshoot. The new parameter values are obtained taking into account that the parameters are considered into their logarithmic space.

### 3.6 Joint confidence region of parameters

When the weighted residuals approximate a normal distribution and the freshwater heads and the logarithms of the resistivities and saltwater percentages can be approximated as a linear function versus the parameters within this joint confidence region, then the joint confidence region can be described by the optimal values and the covariance matrix \( \text{cov}_p \)

\[
\text{cov}_p = \sigma^2_p (J^TWJ)^{-1},
\]

where \( \sigma^2_p \) can be estimated as \( \text{SSR}/(n(1 + \lambda_{P_i}^2) + m\lambda_{hf}^2 + p(\lambda_{sf}^2 - 1)) \). The marginal standard deviation \( s_m \) of the \( j \)th parameter is the square root of the \( j \)th diagonal term of the covariance matrix. The optimum value increased and decreased by the marginal standard deviations multiplied by a factor result respectively in the maximum and the minimum value of the parameter on the bounds of a joint confidence region which can be approximated by a \( p \)-dimensional ellipsoid. Because these marginal standard deviations give only a partial measure of the covariance matrix, they can be supplemented by other statistical parameters, the conditional standard deviations. These deviations can be calculated by means of the eigenvalues and the eigenvectors of the covariance matrix as shown by Lebbin.

\[
S_{ij} = \left( \sum_{k=1}^{p} \beta_{jk}/x_k \right)^{-1/2},
\]

where \( x_k \) is the \( k \)th eigenvalue of the covariance matrix and \( \beta_{jk} \) is the eigenvalue of the covariance matrix. The conditional standard deviations help to locate the intersections of the bounds of the joint confidence region with the parameter axes that go through the optimal values. The joint confidence region is here approximated by the \( p \)-dimensional ellipsoid which can also be described by the eigenvalues and the eigenvectors of the covariance matrix. In the case where the conditional standard deviation is only slightly smaller than the marginal standard deviation of the same parameter, then this hydraulic parameter has no large correlation with other parameters. If, however, a large difference exists between the conditional and the marginal standard deviation of a considered parameter, then a correlation exists between this parameter and one or several other parameters. The correlation matrix can also be deduced from the
covariance matrix. The partial correlation coefficient between the parameters $p_j$ and $p_{j+1}$ is $\text{cov}_{\text{pj}}(p_j, p_{j+1})/\text{cov}_{\text{pj+1}}(p_{j+1}, p_{j+1})^\dagger$.

The comparisons of the marginal and the conditional standard deviations along with the correlation matrix are helpful tools in the regression diagnostic.

4 SYNTHETIC PROBLEM

With this synthetic problem the regression modelling procedure is examined and demonstrated on a fresh-saltwater flow problem which is similar to the field problem. In the synthetic problem the input data of the inverse problem are first calculated by the adapted MOC model. The hydraulic parameters used during these calculations are considered to be the “real” values. The generated input data are borehole resistivity logs and freshwater heads. At the start of the regression the initial estimates of the parameters are different from the “real” values. By the representation of the evolution of the parameter values during the successive iterations of the inverse model, it is demonstrated how the initial estimates converge towards the “real” values. The simulated time span is smaller in the synthetic problem than in the field problem so that the regression modelling procedure requires less computer time.

4.1 Generation of data

With the adapted MOC model the flow is simulated in a vertical section. The section is situated under the dunes, shore and the sea (see Fig. 5). The vertical boundary under the dunes coincides with the groundwater divide of the phreatic aquifer. In the finite-difference model the aquifer is subdivided in nineteen layers with a thickness of 2 m and thirty-eight columns, each with a width of 30 m. The total thickness of the phreatic aquifer in the synthetic problem is 38 m. This is larger than the real thickness of the phreatic aquifer at De Panne. Eighteen columns are located under the dunes, fourteen under the shore and six under the sea.

At the initial time the phreatic aquifer is completely filled with saltwater. As explained in Part 5 (Regression modelling of the field problem) this initial condition can be supported by sedimentological data. During the simulation period the boundary conditions stay unaltered. The lower boundary of the aquifer and the vertical boundary that coincides with the water divide line are impervious. The upper boundary under the dunes is a constant vertical flow boundary where freshwater infiltrates with a rate of $7.39 \times 10^{-4}$ m day$^{-1}$. The upper boundary under the shore and the sea is a constant freshwater head boundary. The values of the freshwater heads under the sea correspond with the mean sea level. Under the shore the freshwater heads increase from the sea in the landward direction. In the synthetic problem, it is, however, assumed that the gradient of the freshwater head under the shore is half the gradient observed on the shore of De Panne. This last gradient is derived from the observed fluctuation of the freshwater heads (Fig. 3). The infiltrated water on the shore has a saltwater percentage of 100%. The seaward vertical boundary is also a constant hydraulic head boundary. The freshwater head values correspond there with the mean sea level and it is assumed that there is no vertical flow of pure saltwater. When water flows through this vertical
boundary into the modelled area this water has a saltwater percentage of 100%.

For the generation of the ‘observed’ data, the groundwater reservoir was considered homogeneous and anisotropic. The horizontal and vertical conductivity are respectively put equal to 10 and 0.2 m day$^{-1}$. The used values of the effective porosity, the longitudinal and transverse dispersivity are respectively equal to 0.38, 0.1524 and 0.0381 m. The formation factor in the resistivity–salinity relation is put equal to 3.979 whereas the used values for the resistivity of the saltwater, of the freshwater, and of the matrix resistivity are 0.4362, 20.14, and 319.8 $\Omega$m. For the synthetic problem the fresh-saltwater distribution was simulated after 100 years of infiltration of freshwater in the dune area in a phreatic aquifer which was initially completely filled with saltwater. The generated observations are borehole resistivity logs and freshwater head measurements. Ten resistivity logs are ‘made’ at different distances from the high water line (Fig. 5). In each log 35 resistivity observations are made with a depth interval of 1 m. At the same location of the resistivity logs four freshwater heads are generated at four different levels with a depth interval of 10 m (for location see Fig. 5). Consequently, in the synthetic problem 350 resistivities and 40 freshwater heads are used as input data of the inverse model. In Fig. 5 the generated resistivities are represented by continuous lines. For the simulation of the solute transport nine particles per grid cell are used along with a maximum cell distance of 0.85 per move of the particles. After each time step of one year the buoyancy was adjusted following the obtained saltwater percentage after which the groundwater velocity field was recalculated.

Fig. 6 shows the results of the adapted MOC model corresponding with the “real” values. The vertical axis is 10 times exaggerated with respect to the horizontal axis. The freshwater heads and the saltwater percentages are represented by contour lines. These lines are obtained by bilinear interpolation between the values of the grid-centered nodal points. The groundwater flow velocities are plotted at each nodal point.

### 4.2 Initial parameter estimates

The initial estimates of the hydraulic and electrical parameters are estimated in a similar way as in a field problem. The estimated horizontal conductivity is one half of the “real” value. Normally, the horizontal conductivity of a homogeneous phreatic aquifer can be determined with a higher precision by the execution of a simple pumping test. In most pumping tests, however, no observations are made to deduce the vertical conductivity of such an aquifer. Therefore, the initial value estimated for the vertical conductivity deviates more from the “real” value. An initial estimate of 0.05 m/d was taken. This is four times smaller than the “real” value. Better estimates of such a vertical conductivity can for example been obtained by the execution of a pumping test where only the lower part of the aquifer is pumped. The drawdowns are then not only observed in the directly pumped part of the aquifer but also at different level in the upper undirectly pumped part of the aquifer at relatively small distances of the pumped well where a relatively large vertical gradient in the hydraulic head can be expected in the beginning of the pumping test.

The initial estimates of the longitudinal and transverse dispersivity are 0.3048 and 0.1524 m. This is respectively two and four times the “real” values. In general, these two parameters are less well known. Usually they are identified by the calibration of a solute transport model to an observed evolution of a concentration in one or a limited number of wells. Here, these values are identified.

![Fig. 6. Calculated freshwater head distribution (m TAW), contour lines of saltwater percentage and velocity field corresponding with “real” values of hydraulic parameters (x- and y-axes are marked in m).](image-url)
by means of concentration gradients in the salt-freshwater transition zone. The initial estimate of the effective porosity is equal to 0.36. This is relatively close to the “real” value 0.38. In contrast to hydraulic conductivities, the effective porosity of a single formation has normal rather than log-normal distribution. Because the flow is considered to be in a homogeneous sediment, it is supposed that the effective porosity can be closely estimated. So, the initial value of the effective porosity is estimated very close to the “real” value.

The initial estimates of the electrical parameters is based on the comparison of borehole resistivities with resistivities of water samples taken at comparable depths. The ratio between these resistivities results in an apparent formation factor. The initial estimate of the formation factor is taken equal to the average value of the apparent formation factor found in the dune area of De Panne and in its adjacent areas. The resistivity of the freshwater (20 Ωm) is the average resistivity of the freshwater samples collected in the dune area. The resistivity of the saltwater was taken equal to 0.4 Ωm and could be deduced from frequent resistivity measurements of the sea water. Until now, no systematic study was made to derive the matrix resistivity of the medium to fine medium sand which forms the studied phreatic parameters that govern the groundwater flow are involved in the regression process: the vertical and horizontal conductivity residuals is faster than the decrease of the sum of the squared freshwater head residuals. In the third and fourth iteration these hydraulic parameters that govern the groundwater flow are alternated with the parameters that govern the hydrodynamic dispersion. Also, in the fifth and sixth iteration these parameters are separately considered. In the seventh iteration these four hydraulic parameters are simultaneously introduced in the regression process.

During the first seven iterations a sensitivity factor \( sf \) of 1.122 is used. This corresponds with \( \log_{10}sf = 0.05 \). During the following iteration the sensitivity factor was reduced to 1.0471 or \( \log_{10}sf = 0.02 \). In the eighth iteration the effective porosity is added to these four hydraulic parameters. In the ninth iteration the resistivity–salinity

### 4.3 Development of regression modelling

The evolution of the simulation regression process is represented in Table 1. The initial values are given in the first line; the “real” values in the last line of the table. During the first iteration only the two hydraulic parameters that govern the groundwater flow are involved in the regression process: the vertical and horizontal conductivity. Due to the adjustments of these parameters at the end of the first iteration of the regression process the sum of the squared freshwater head residuals becomes 39 times smaller whereas the sum of the squared resistivity residuals becomes only 2.3 times smaller. In the second iteration only the longitudinal and traverse dispersivities are considered. Due to the adjustments of these parameters at the end of the second iteration the decrease of the sum of the squared resistivity residuals is faster than the decrease of the sum of the squared freshwater head residuals. In the third and fourth iteration the hydraulic parameters that govern the groundwater flow are alternated with the parameters that govern the hydrodynamic dispersion. Also, in the fifth and sixth iteration these parameters are separately considered. In the seventh iteration these four hydraulic parameters are simultaneously introduced in the regression process.

### Table 1. Evolution of the parameter values, the sums of the squared resistivity and fresh water head residuals during the different iteration of the inverse synthetic problem

<table>
<thead>
<tr>
<th>Iteration</th>
<th>( k_h ) (m day(^{-1}))</th>
<th>( k_v ) (m day(^{-1}))</th>
<th>( \alpha_L ) (m)</th>
<th>( \alpha_T ) (m)</th>
<th>( n ) (dim.less)</th>
<th>( F ) (dim.less)</th>
<th>( \rho_s ) (Ωm)</th>
<th>( \rho_l ) (Ωm)</th>
<th>( \rho_{mat} ) (Ωm)</th>
<th>( \sum_{i=1}^{10} r_{hf,i}^2 )</th>
<th>( \sum_{i=1}^{40} r_{sf,i}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.000</td>
<td>0.5000</td>
<td>0.3048</td>
<td>0.15240</td>
<td>0.3600</td>
<td>4.0000</td>
<td>0.4000</td>
<td>20.00</td>
<td>500.0</td>
<td>27.65</td>
<td>0.7798</td>
</tr>
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<td>0.1587</td>
<td>0.3048</td>
<td>0.15240</td>
<td>0.3600</td>
<td>4.0000</td>
<td>0.4000</td>
<td>20.00</td>
<td>500.0</td>
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</tr>
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<td>4.0000</td>
<td>0.4000</td>
<td>20.00</td>
<td>500.0</td>
<td>27.65</td>
<td>0.7798</td>
</tr>
<tr>
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<td>0.2055</td>
<td>0.09952</td>
<td>0.3600</td>
<td>4.0000</td>
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<td>20.00</td>
<td>500.0</td>
<td>27.65</td>
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</tr>
<tr>
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<td>0.1886</td>
<td>0.08372</td>
<td>0.3600</td>
<td>4.0000</td>
<td>0.4000</td>
<td>20.00</td>
<td>500.0</td>
<td>27.65</td>
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</tr>
<tr>
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<td>0.1690</td>
<td>0.1886</td>
<td>0.08372</td>
<td>0.3600</td>
<td>4.0000</td>
<td>0.4000</td>
<td>20.00</td>
<td>500.0</td>
<td>27.65</td>
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</tr>
<tr>
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<td>500.0</td>
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</tr>
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<td>0.7798</td>
</tr>
<tr>
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<td>0.1554</td>
<td>0.05009</td>
<td>0.3785</td>
<td>4.0000</td>
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<td>3.9871</td>
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</tr>
<tr>
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<td>0.1581</td>
<td>0.04215</td>
<td>0.3785</td>
<td>3.9871</td>
<td>0.4350</td>
<td>20.00</td>
<td>500.0</td>
<td>27.65</td>
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<tr>
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<td>0.1945</td>
<td>0.1579</td>
<td>0.04217</td>
<td>0.3785</td>
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<td>500.0</td>
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<td>500.0</td>
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<td>0.03810</td>
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<td>3.9791</td>
<td>0.4362</td>
<td>20.135</td>
<td>319.8</td>
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<td>–</td>
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</tbody>
</table>

Parameter values written in bold are included in the simulation regression process during the considered iteration. Parameter values given in last line of this table indicated by an * are “real” values.
relation is adjusted by the introduction of the two
electrical parameters that determine the most this rela-
tion: the formation factor and the saltwater resistivity.
This results in a further decrease of the sum of the
squared resistivity residuals. The inclusion of these pa-
rameters in the simulation regression process along with
the hydraulic parameters that determine the ground-
water flow and solute transport is, however, not possi-
ble. This causes large absolute values of non diagonal
elements of the correlation matrix. During the four
following iterations the five hydraulic parameters are
simultaneously included in the regression process so that
the sum of the squared residuals becomes four times
smaller as well as the sum of the squared freshwater
head residuals. During the fourteenth iteration the sali-
nity resistivity relation is further adjusted by the sepa-
rate consideration of the formation factor and saltwater
resistivity. In the last three iterations the five hydraulic
parameters are again simultaneously included in the
regression process. This results in very small change of
the hydraulic parameters values and a slight oscillation
of the sums of the squared residuals.

5 REGRESSION MODELLING OF THE FIELD
PROBLEM

5.1 Input data

With the two-dimensional model the flow is simulated in
the vertical plane at the French-Belgian border where
the flow of freshwater from the dune towards the sea is
the least influenced by the pumping in the dune area.
Therefore, it is assumed that the flow in the considered
area is parallel to the modelled cross-section. At this
cross-section the phreatic aquifer shows the most ho-

geneous constitution of the studied profiles. The
aquifer is subdivided in nineteen layers each with a
thickness of 1.6 m and thirty-eight columns each with a
width of 30 m. Eighteen columns are located under the
dunes, fourteen under the shore and six under the sea.
Here, the groundwater divide of the phreatic aquifer in
the dunes is estimated to be at 540 m from the high
water line.

The phreatic aquifer is also here completely filled
with saltwater at the initial time. Before the formation of
the dune ridge the sea inundates periodically the area.
This can be deduced from the sedimentological data. All
sediments under the eolian sands are tidal flat deposits.
Hence, one can suppose that before the rise of the dune
ridge the tidal flat deposits, which form the major part
of the phreatic aquifer are completely filled with salt-
water.

During the simulation period the boundary condi-
tions do not change. The boundary conditions for the
groundwater flow and the solute transport are the same
as in the synthetic problem except for the freshwater
head values. Under the shore and the sea these values
are derived from the observed fluctuations (Fig. 3). At
the seaward vertical boundary the freshwater head val-
ues correspond with the mean sea level whereas no
vertical flow of pure saltwater is assumed.

The groundwater reservoir was considered homoge-
neous and anisotropic. The prior estimates of the
horizontal and vertical conductivities are respectively
10 and 0.4 m day$^{-1}$. For the effective porosity and the
longitudinal and transverse dispersivity the prior esti-
mates are respectively 0.38, 0.01 and 0.002 m. The prior
estimate of the formation factor is equal to 4.0. The
parameters in the resistivity–salinity relation are de-
duced from field observations. The resistivity of salt-
water ($P_s = 100\%$) and freshwater ($P_s = 0\%$) correspond
with 0.35 and 20 $\Omega$m. The resistivity of the matrix was
put equal to 500 $\Omega$m. These last mentioned values of
the resistivities are also considered as prior estimates.

The initial values of the hydraulic parameters to the
regression modelling were the prior estimates of the
hydraulic parameters.

During each simulation the evolution of the fresh-
water lens under the dunes and the shore are calculated
for a period of eight hundred years. Then the saltwater
percentage distribution approximates dynamic equilib-
rium so that it can be compared with the observation of
the $K_0$ resistivity profile. For each simulation the fol-

dowing model parameters were used: nine particles per
grid cell, 0.85 as maximum cell distance per move of the
particles, after each time step of eight years the buoy-
ancy was deduced from the obtained saltwater percent-
apages and the groundwater flow velocity field was
recalculated. The sensitivity analyses were performed for
a sensitivity factor $sf$ of 0.944 or $\log_{10}sf = -0.025$.

5.2 Results

By this regression the hydraulic parameters that define
the groundwater flow and the solute transport are first
identified. Their sensitivities $J_1$, $J_3$ and $J_5$ with respect to
the hydraulic parameters are considerably larger than the
sensitivities $J_2$, $J_4$ and $J_6$ with respect to the elec-
trical parameters of the model of the parallel electrical
conductivities. Therefore, the first mentioned param-
ters can better be identified than the electrical parame-
ters. The following optimal values for the hydraulic
parameters are found: 10.12 and 0.5616 m day$^{-1}$ for the
horizontal and vertical conductivity, 7.010 and 1.591
mm for the longitudinal and transverse dispersivity, and
0.363 for the effective porosity. The values found for the
average formation factor is 3.98, 0.434 and 20.1 $\Omega$m
for the resistivity of pure saltwater and pure freshwater, and
320 $\Omega$m for the resistivity of the sediment matrix.

The results of the simulation are represented in
Figs. 7 and 8. Fig. 7 shows the calculated and observed
resistivities in the modelled cross sections. At
each borehole the variation of the logarithm of the
resistivities is represented versus depth for the optimal solution. Because the thickness of the modelled and observed transition zones is similar, it is concluded that the longitudinal and transverse dispersivity are rather well defined. The obtained values for these parameters are very small compared with values obtained from calibrated two-dimensional models in the horizontal plane with concentration data collected from observation wells in the same aquifer but at different depths and locations (e.g., Iribar et al. in the magnitude of 100 m).

These small values are due to the fact that the groundwater flow and the solute transport take place in an aquifer which is composed of well-sorted medium to fine-medium sands with a slight trend to a less coarse sediment toward the base of the aquifer (see part 2.1). Such an aquifer shows all geological characteristics which limits the non-idealities on the microscopic, macroscopic and megascopic scale in the aquifer and so the mechanical dispersion which is caused by the variability of the direction and rate of transport. Xu and Eckstein show that the uniformity coefficient, which is a characteristic on the microscopic scale, can explain 71% of the dispersivity deduced from column tests. The uniformity coefficients is $d_{60}$ divided by $d_{10}$. The diameters $d_{60}$ and $d_{10}$ correspond respectively to 60% and 10% finer by weight on the grain-size distribution curve. The uniformity coefficient of the sediments in which the transition zone between the fresh and the saltwater is situated, ranges between 1.544 and 1.659.

According to the best fitted relation of Xu and Eckstein ($x_L = -3.51 + 4.41d_{60}/d_{10}$), the dispersivity on the scale of the column test (diameter 63 mm, length 310 mm) of these sediments should range between 3.3 and 3.8 mm. The larger value found in this study can be ascribed to non-idealities on the larger scales and to the groundwater flow fluctuations. These flow fluctuations are principally due to changes of the freshwater head configuration on the shore (neap tide, spring tides and irregular high water levels due to northwestern storms) and due to seasonal fluctuation of the infiltration rate of freshwater in the dunes.

In three boreholes the depths of the simulated transition zone are quite different from the observed depths. The locations of these places are probably not only defined by the hydraulic parameters but also by the average heads on the shore and the sea floor which are
is slightly larger than in a normal distribution. This results from the fact that in regions with pure freshwater or pure saltwater the resistivity and the saltwater percentage residuals are zero or very small.

The covariance matrix is studied to obtain a notion about the extension of the joint confidence region in the parameter space. This covariance matrix is indirectly given by the marginal standard deviations (Table 2) and the partial correlation matrix (Table 3). In Table 2 the marginal and the conditional standard deviation are given of the different deduced hydraulic parameters and of the electrical parameters. The marginal and conditional standard deviations of the hydraulic parameters are smaller than those of the electrical parameters. Within the hydraulic parameters there is a correlation between the transverse dispersion and the vertical conductivity. This can also be inferred from the large differences between their conditional and marginal standard deviations. The most important electrical parameter is the formation factor. The formation factor shows, however, a small correlation with the freshwater resistivity and the saltwater resistivity. Because of these correlations, their marginal and conditional standard deviations are quite different. The matrix resistivity is the electrical parameter that influences the least the regression process. Its conditional and marginal standard deviations are both large and the correlation with the other electrical parameters is small. The correlation between the electrical and the hydraulic parameters is very small.

### Table 2. Results of the modelling regression of the field problem, the optimal value \( v_o \), the marginal standard deviation \( s_m \), and the conditional standard deviation \( s_c \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>( v_o )</th>
<th>( s_m )</th>
<th>( s_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>dim.less</td>
<td>0.363</td>
<td>0.0047</td>
<td>0.0033</td>
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<tr>
<td>( \sigma_T )</td>
<td>mm</td>
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<td>0.0154</td>
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</tr>
<tr>
<td>( \sigma_L )</td>
<td>mm</td>
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<tr>
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<tr>
<td>( k_h )</td>
<td>m day(^{-1} )</td>
<td>10.1</td>
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</tr>
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<td>( F )</td>
<td>dim.less</td>
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<tr>
<td>( \rho_T )</td>
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<td>( \Omega ) m</td>
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### Table 3. Partial correlation matrix of the modelling regression of the field problem

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<th>( \sigma_T )</th>
<th>( \sigma_L )</th>
<th>( k_q )</th>
<th>( k_h )</th>
<th>( F )</th>
<th>( \rho_T )</th>
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<td>0.1965</td>
<td>-0.0163</td>
<td>-0.0349</td>
<td>0.0100</td>
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</tr>
<tr>
<td>( \sigma_L )</td>
<td>-0.2644</td>
<td>-0.5305</td>
<td>1.0000</td>
<td>0.4946</td>
<td>0.0797</td>
<td>0.0315</td>
<td>-0.0062</td>
<td>0.0200</td>
<td>-0.0150</td>
</tr>
<tr>
<td>( k_q )</td>
<td>-0.2502</td>
<td>0.9188</td>
<td>0.4946</td>
<td>1.0000</td>
<td>-0.3625</td>
<td>0.0093</td>
<td>0.0155</td>
<td>-0.0167</td>
<td>-0.0078</td>
</tr>
<tr>
<td>( k_h )</td>
<td>0.0811</td>
<td>0.1965</td>
<td>0.0797</td>
<td>-0.3625</td>
<td>1.0000</td>
<td>-0.0232</td>
<td>-0.1154</td>
<td>0.1065</td>
<td>-0.0125</td>
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<tr>
<td>( F )</td>
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<td>-0.0163</td>
<td>0.0315</td>
<td>0.0993</td>
<td>-0.0232</td>
<td>1.0000</td>
<td>-0.7883</td>
<td>-0.7772</td>
<td>-0.1105</td>
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<tr>
<td>( \rho_T )</td>
<td>0.0460</td>
<td>-0.0349</td>
<td>-0.0662</td>
<td>0.0155</td>
<td>-0.1154</td>
<td>-0.7883</td>
<td>1.0000</td>
<td>0.5114</td>
<td>0.2118</td>
</tr>
<tr>
<td>( \rho_S )</td>
<td>0.0157</td>
<td>-0.0100</td>
<td>0.0200</td>
<td>-0.0167</td>
<td>0.1065</td>
<td>-0.7772</td>
<td>0.5114</td>
<td>1.0000</td>
<td>0.0616</td>
</tr>
<tr>
<td>( \rho_{max} )</td>
<td>0.0646</td>
<td>-0.0143</td>
<td>-0.0150</td>
<td>-0.0078</td>
<td>-0.0125</td>
<td>-0.1105</td>
<td>-0.2118</td>
<td>0.0616</td>
<td>1.0000</td>
</tr>
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</table>
follow the saltwater encroachment of the freshwater lens under the dunes due to overexploitation. When the pumping started the infiltration rate of saltwater on the back shore increased. Owing to this pumping the saltwater lens under the shore enlarged and an isolated fresh-brackish lens is formed under the lower part of the fore shore as was observed by the K4 and K5 resistivity profiles (Fig. 2). When the overexploitation continues this isolated fresh-brackish lens moves slowly in the seaward direction while its salinity increased. Under the dunes the saltwater encroachment advances in the lower part of the aquifer towards the water-collection area. Under the water-collection area, first the water with a larger salt content at the base of the phreatic aquifer rises towards the lower cells where water is withdrawn. Subsequently, the salt content of the water under the water-collection area increases steadily due to the saltwater encroachment on the shore.

This simulation demonstrates clearly that the water-collection areas are, here, not jeopardized by the mean sea level at the mean water line as can be deduced from the general saltwater intrusion literature\textsuperscript{1,7,9,23} but by the high water levels at the high water lines. This phenomenon occurs also on beaches with rather limited tidal fluctuation but with significant wave heights as was pointed out by a recent study in a narrow coastal aquifer at New South Wales, Australia\textsuperscript{24}. The appearance of a semi-pervious layer at a limited depth will reduce the infiltration of saltwater on the shore and consequently the extend of the upper saltwater lens under the shore as was observed in the unconfined aquifer at the eastern Belgian coast\textsuperscript{17}. In thick coastal aquifers (e.g., South and North-Holland, The Netherlands), it is expected that the freshwater which flow from the dunes to the sea is squeezed through the upper saltwater lens on the shore and the generally known lower saltwater feet. The depth of the lower transition zone around the dune shore line depends of the saltwater infiltration which took place on the back shore. Ignoring this saltwater infiltration will result in an underestimation of the depth.
of the transition zone and of the hydraulic head in the
dunes close to shore.

The significance of the observed and simulated flow
phenomenon is not only limited to shores with a
pervious layer just underneath the surface and with
rather high tidal fluctuations or rather high waves but
can also be extended to shores with small tidal fluctua-
tion and wave heights as for example the Mediterrane-
ian sea. In these coastal areas the freshwater
resources can be jeopardized by exceptional high water
with a rather low frequency of once per decade or even
once per several decades. This phenomenon is also of
interest for the water resources of small isles in oceans
or seas either with rather high tidal fluctuations, or
with significant wave heights, or with exceptional high
waters. Finally, the significance of this phenomenon
can further be enlarged to the banks of streams or
rivers with tidal fluctuations or exceptional high waters.
Here, the fresh groundwater is not only jeopardized by
infiltrating saltwater but also by polluted stream or
river water.

7 CONCLUSIONS

A particular salt-freshwater inversion was found by
means of a series of borehole resistivity measurements
on a sandy shore with semi-diurnal tides and used for
the identification of the hydraulic parameter of the sol-
ute transport. At the French–Belgian border, where a
large flow of freshwater occur from the dunes towards
the sea, a saltwater lens exists above the freshwater. This
inversion can be simulated by means of a coupled den-
sity dependent groundwater flow and solute transport
model (the adapted MOC model). In this model the
freshwater heads observed on the slightly sloping sandy
shore were taken into account. Due to tidal fluctua-
tions of the sea-level the average freshwater head increases
from the sea in the landward direction. The saltwater
lens in the upper part of the phreatic aquifer is caused by
the infiltration of saltwater on the back shore during the
high tides and by the seepage of saltwater on the fore
shore during the low tides.

A regression modelling procedure was proposed in
which the hydraulic parameters of the groundwater flow
and the solute transport can be simultaneously consid-
ered with the parameters of the resistivity–salinity rela-
tion. During this procedure an object function was
minimized which comprises resistivity residuals, salt-
water percentage residuals, freshwater head residuals
and parameter residuals. First the freshwater heads and
the saltwater percentage were calculated with the nu-
merical model after which the calculated saltwater per-
centage was converted in sediment resistivities by means
of the resistivity–salinity relation. The inverse of this
relation helps to convert the observed sediment resist-
ivities into saltwater percentages.

The regression modelling procedure was first applied
on a synthetic problem. At the end of this procedure the
hydraulic parameters which govern the groundwater flow
and the solute transport are simultaneously de-
duced. However, in the beginning of this procedure the
hydraulic parameters that govern the groundwater flow
were separately considered in one iteration. They were
alternated with the hydraulic parameters that govern the
solute transport in the other iterations. In a number of
iterations the two most important electrical parameters
of the resistivity–salinity relation were separately ad-
justed. By the application of the modelling regression
procedure on the field problem of fresh-saltwater flow
under the shore with semi-diurnal tides it is shown that
it was possible to identify the longitudinal and trans-
verse dispersivities along with the effective porosity and
the horizontal and vertical conductivity from a detailed
picture of the saltwater distribution derived from bore-
hole resistivities and from some freshwater head data.

The found optimal values of the dispersivities were
very small. These small values are due to the fact that
the groundwater flow and the solute transport take place
in an aquifer which is composed of well-sorted medium
to fine-medium sands with a slight trend to slightly
coarser sediment toward the base of the aquifer (see part
2.1). Such an aquifer shows all geological characteristics
which limits the non-idealities on the microscopic,
macroscopic and megascopic scales in the aquifer and so
the mechanical dispersion which is caused by the vari-
bility of the direction and rate of transport\(^1\).

Finally, it was shown that the high waters on the back
shore form the main threat of saltwater encroachment
from the sea side of the dunes and that the isolated
fresh-brackish lens found under the lower part of the
shore before the build up area of De Panne was due to
the overexploitation of freshwater lens in the dunes. The
significance of the observed and simulated flow phe-
nomenon is not only limited for similar shores with
similar tidal movements but can be extended to all
shores and stream or river banks with a rather small
slope, with a pervious subsoil, and with tidal fluctua-
tions, or with significant wave heights, or even with
exceptional high water with a rather low frequency, e.g.,
one per decade or even once per several decades.

ACKNOWLEDGEMENTS

The investigation was supported by the National
Fund for Scientific Research (Belgium) and by the
AVICENNA-project of the Commission of the Euro-
pian Community AVI-73. The author wishes also to
express his gratitude to Prof. Dr. W. De Breuck, Head
of the Laboratory of Applied Geology and Hydrogeol-
ogy, for the help in obtaining the field data. The review
comments of the anonymous reviewers are gratefully
acknowledged.
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