Modeling wells in variably saturated soil with wellbore fluid gravity segregation

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Abstract

A method is presented for incorporating the dynamics of a well with finite wellbore volume into finite difference models of air and water flow in variably saturated porous media. The method is based on the assumption of fluid gravity segregation with water accumulation at the bottom of the well. The method accurately accounts for changing water levels in the well and the corresponding changes in air and water storage in the wellbore. The method was added to a three-dimensional finite difference model for multi-phase flow and transport and tested against three-dimensional pilot scale experiments of water injection into initially unsaturated soil. Well water levels and soil moisture contents monitored during the course of the experiments compared well with the model predictions. © 2000 Elsevier Science Ltd. All rights reserved.

1. Introduction

There is significant interest in the remediation of vadose zone contamination. Remediation schemes such as bioremediation and addition of chemical oxidants may potentially involve the addition of aqueous solutions through wells into unsaturated soils. In addition, techniques such as soil vapor extraction (SVE), dual phase extraction, and bioslurping may involve the removal of both gas and liquid from wells to which a vacuum is applied. Models capable of accurately simulating the dynamics of flow between wells and variably saturated porous media can be valuable tools for the design of field scale remediation programs involving these techniques. These models can also be used in the interpretation of slug tests conducted in unconfined aquifers in which conditions for analytical solution applicability are not met.

The most common method for incorporating wells into porous media flow models is through the use of prescribed flux or prescribed hydraulic head boundary conditions in finite difference or finite element cells [4]. Typically, the wellbore will be longer than the size of finite difference blocks used in simulating a field site, so that the wellbore will pass through several finite difference grid blocks. In most cases when the wellbore passes through several grid blocks, separate flux or head boundary conditions are applied to each of the blocks through which the wellbore passes. In many cases flux rates will vary over the length of the wellbore due to changing soil permeabilities, water saturations, or soil hydraulic conditions. Under these conditions it is very difficult to allocate the total flux to or from the well to the individual grid blocks since it is usually only the total well flow that is known. It is even possible that wellbore crossflow may occur where water flows into the well from one formation and flows out another section that has a lower hydraulic head [5]. In addition, the wellbore is of finite diameter and wellbore storage effects may be important, particularly in water table aquifers where the level of water in the wellbore may change significantly with time.

In petroleum reservoir engineering models it has been common to incorporate wells as individuals cells that are hydraulically connected to the grid blocks through which the wells pass. When the hydraulic head drop along the wellbore is small, the well can be incorporated into the model as one special cell with its own unique connection pattern. Hydraulic heads and fluid saturations are then calculated for the wellbore cell as they are...
for regular gridblocks in the model, and hydraulic head or flux boundary conditions can be applied to the wellbore cell as they would be applied to any grid block. It is assumed in this modeling approach that the fluids in the wellbore are completely mixed and no phase segregation occurs in the wellbore [1].

When the hydraulic head drops along the wellbore are significant, the wellbore can be discretized into several cells and the variations in hydraulic head and fluid saturation along the wellbore can be determined [2]. The equations used to determine hydraulic head drops along the wellbore are based on the assumption of completely mixed flow and no phase separation due to gravity segregation. Sudicky et al. [14] incorporated wells with significant wellbore hydraulic losses into a saturated water flow finite element model as a set of connected line elements. When wellbore sections are represented as line elements single phase water flow in the wellbore can be readily modeled when there is adequate wellbore resistance to flow to prevent matrix singularity problems. As with the method of Collins et al. [2] the line element method of Sudicky et al. [14] is not applicable to wells in variably saturated soils where gravity segregation of fluids occurs.

When gravity segregation occurs in a well, the lower portion of the well may be completely water saturated, due to accumulation of water at the bottom of the well, while the section of the wellbore above the air–water interface in the well will be completely unsaturated. When water is added directly to the well, it will likely be added to the bottom of the well, or will rapidly flow through the wellbore to the lower water-saturated section of the well. In the case where the water flows from the well into the soil the water will only be able to flow into the porous medium grid blocks that are connected to the cell below the air–water interface. When flow is from the soil into the well, the water can only flow into the cell from grid blocks that are at pressures greater than those in the adjacent wellbore segment. In the case of grid blocks connected to the air-filled segment of the wellbore, water from these blocks can only flow into the well if the grid block water pressure is greater than the air pressure in the wellbore (similar to a seepage face). Similarly, air can only flow from the wellbore to grid blocks above the air–water interface in the well.

In this paper a method for incorporating discrete wellbore dynamics into a finite difference model for variably saturated air and water flow is presented. The method is incorporated into the three-dimensional, multiphase, multicomponent finite difference model of Sleep and Sykes [13] and McClure and Sleep [7]. The model is applied to pilot scale tests of water injection from wells into initially unsaturated soil. Predicted well water levels and soil moisture contents around the wells are compared to measured values to test the model.

2. Modeling discrete wellbore dynamics

In developing the method for representing wells in finite difference models for variably saturated air and water flow, it is assumed that any water entering the well accumulates at the bottom of the well and that capillary effects are negligible. Hydraulic head gradients in the air and water phases in the wellbore are assumed to be negligible so that each well can be represented by one segmented finite difference cell as shown in the example grid in Fig. 1. This grid contains 9 regular grid blocks, so the well cell is computational block 10. The boundaries of the segments (Seg 1, Seg 2, and Seg 3 in Fig. 1) of the well cell correspond to the boundaries of the grid blocks (4, 5, and 6) through which the wellbore passes. The well, cell 10, is thus connected to the grid blocks 4, 5, and 6 and flows between cell 10 and grid blocks 4, 5, and 6 must be calculated.

To model flow of air and water between the well segments and grid blocks air and water pressures in the well segments must be determined. If the overall water and air saturations in the well cell are known, then the position of the air–water interface can be calculated from the wellbore length and elevation of the bottom of the wellbore. With the assumption that the wellbore fluids are in hydrostatic equilibrium, and knowing the position of the air–water interface and the phase densities, the fluid pressures anywhere in the well can be calculated from the heights of air and water above that point and their respective densities, and the gas pressure at the top of the wellbore (or any fluid pressure at any location in the wellbore).

Once the pressure in the wellbore segment is determined at the same elevation as the elevation of the grid block center, the flow of each of the water and gas phases between wellbore segment and grid block can be calculated from the pressure difference between the

![Fig. 1. Example grid with one well.](attachment:image.png)
wellbore segment and the grid block, and the phase transmissivity for wellbore–grid block flow. The equation for wellbore segment–grid block flow is given by

\[ q_p = T_p(h_p - h_w^b), \] (1)

where \( \beta \) represents the phase (water or air), \( q_p \) is the wellbore–grid block Darcy flow rate, \( h_p \) is the grid block hydraulic head, \( h_w^b \) is the wellbore hydraulic head, and \( T_p \) is the transmissivity for the \( \beta \) phase for wellbore–grid block flow. For the well in Fig. 1 there are connections to grid blocks 4, 5, and 6, giving the following molar balance equation for the water phase for the well (cell 10):

\[
\frac{V_{10}}{\Delta t} \left[ (S_w \rho_w)^{n+1}_{10} - (S_w \rho_w)^{n}_{10} \right] =
\left[ (T_w \rho_w)^{n+1}_{4-10}(h_w^{b} - h_w^{e}) + (T_w \rho_w)^{n+1}_{5-10}(h_w^{b} - h_w^{e}) + (T_w \rho_w)^{n+1}_{6-10}(h_w^{b} - h_w^{e}) \right],
\] (2)

where \( V_{10} \) is the total wellbore volume (m\(^3\)), \( \Delta t \) the time step size (s), \( S_w \) the water saturation, \( \rho_w \) the water phase molar density (g-mol/m\(^3\)), and \( h_w \) is the water hydraulic head. The \( n \) and \( n+1 \) superscripts represent time step numbers, the single subscript numbers represent cell numbers, and the pairs of subscript numbers (4–10 for example) represent properties (transmissivities and densities) evaluated at the interface between the cells corresponding to the subscript numbers. A similar equation can be written for the air phase.

Each grid block connected to the well will contain a term for flow between the wellbore cell and that grid block. For example, for grid block 4, the water molar balance equation is

\[
\frac{V_4}{\Delta t} \left[ (nS_w \rho_w)^{n+1}_{4} - (nS_w \rho_w)^{n}_{4} \right] =
\left[ (T_w \rho_w)^{n+1}_{4-10}(h_w^{b} - h_w^{e}) + (T_w \rho_w)^{n+1}_{5-10}(h_w^{b} - h_w^{e}) + (T_w \rho_w)^{n+1}_{6-10}(h_w^{b} - h_w^{e}) \right],
\] (3)

where \( n \) is the porous medium porosity. A similar equation can be written for the air phase.

Eqs. (2) and (3) illustrate that the finite difference equations for a system containing a discrete stratified well are of the same general form as in the standard finite difference method, but with specially computed wellbore–gridblock transmissivities and irregular wellbore–gridblock connections. For example, in the system in Fig. 1, the well (cell 10) is connected to grid blocks 4, 5, and 6. The grid block 4 is connected to the well (cell 10) as well as the regular grid blocks 1, 5, and 7.

The transmissivities, \( T_w \) and \( T_p \) for the air phase, between regular grid blocks in Eq. (3) may be calculated using standard methods [13], but a different approach is required for transmissivities between the wells and grid blocks. Peaceman [9–11] developed formulae for flow between wells and grid blocks. For a vertical well the transmissivity for horizontal flow to the well is given by:

\[ T_p = \left[ \frac{2\pi k \mu_L}{\rho_p} \right] \left[ \ln \left( \frac{r_0}{r_w} \right) \right]^{-1}, \] (4)

where \( k \) is the intrinsic isotropic soil permeability, \( k_\beta \) the relative permeability, \( L \) the wellbore segment length, \( \rho_p \) the viscosity, \( r_w \) the wellbore segment radius, and \( r_0 \) is the effective radius of influence of the well. One can calculate \( r_0 \) from

\[ r_0 = 0.28 \frac{\left( \frac{k_x}{k_w} \right)^{1/2} \Delta x^2 + \left( \frac{k_y}{k_w} \right)^{1/2} \Delta y^2}{\left( \frac{k_x}{k_w} \right)^{1/4} + \left( \frac{k_y}{k_w} \right)^{1/4}}, \] (5)

where \( k_x \) and \( k_y \) are intrinsic permeabilities in the \( x \) and \( y \) directions, respectively, and \( \Delta x \) and \( \Delta y \) are the grid block dimensions in the \( x \) and \( y \) directions, respectively.

Phase relative permeabilities for each wellbore segment can be calculated as the fraction of the length of the wellbore section that is occupied by the particular phase. These fractions can be calculated from the positions of the phase interfaces. When a segment is completely below the air–water interface the phase relative permeability will be unity and the air phase relative permeability will be zero. Conversely, when the segment is completely above the air–water interface the air phase relative permeability will be unity and the water phase relative permeability is zero. When the air–water interface is within a well segment the relative permeability for a particular phase can be set equal to the fraction of the wellbore segment occupied by that phase.

When calculating transmissivities for well segment–grid block flows using Eq. (4), it is necessary to calculate a weighted relative permeability from the well segment relative permeability and the grid block relative permeability (calculated from the grid block saturation). It is convenient to use upstream weighting for the relative permeability in Eq. (4). The upstream weighted relative permeability will be that of the appropriate wellbore segment when the calculated wellbore pressure is greater than the corresponding porous medium grid block pressure. If either water or air is absent from the wellbore segment under consideration then, for that section, the wellbore relative permeability for that phase will be zero. The calculated flow of the particular phase into the porous medium grid block under consideration will be zero, even if the overall wellbore saturation of that phase is nonzero, since the phase relative permeability is zero for the segment of the wellbore in that grid block. When the pressure for a particular phase is higher in the porous medium grid block than in the wellbore segment then the upstream weighted relative permeability for that phase will be that of the grid block.
The wellbore formulation described has been incorporated into the three-phase, three-dimensional compositional simulator described in McClure and Sleep [7] and Sleep and Sykes [13]. As the equations of air–water flow in porous media are nonlinear, they are solved using the Newton Raphson technique with primary variable substitution [13]. Just as with regular grid blocks, primary variables for well cells are chosen from the set of water and gas phase pressures and water and gas phase saturations (one set for the complete wellbore, from which the pressures, saturations, and relative permeabilities of each of the wellbore sections can be determined). The particular choice of primary variables from this set will be the same as those used for grid blocks, except that gas phase pressure will be used for all cases where the wellbore contains a non-zero gas phase saturation. This choice is motivated by the fact that when the well is open the gas phase pressure is fixed to be atmospheric pressure. If the well is operated as a vacuum extraction well then the gas phase pressure may be a control variable. For each stratified well added to a simulation one extra set of equations (each set has a number of equations equal to the number of primary variables) must be solved for each of the individual wells added.

In the code of McClure and Sleep [7] the same pointer and index matrices are used for specifying connections in assembly of the mass balance equations and for solving the matrix equations. Incorporation of the wellbore formulation required a straightforward modification of the subroutine for construction of the connection pointer and index matrices. In addition, new subroutines to determine air–water interface levels in the wells, and to determine pressures and relative permeabilities in each wellbore segment were required. The subroutine for calculation of transmissivities for flow between grid blocks was also modified to allow calculation of transmissivities between wellbore segments and the neighboring grid blocks.

3. Experimental methods

3.1. Three-dimensional pilot scale aquifer

A pilot scale aquifer was constructed for the purposes of conducting a variety of experiments for testing computer models of multiphase flow and transport in porous media. The tank containing the pilot scale aquifer had dimensions that were nominally 3.5 m by 3.5 m by 1.7 m high. The walls and bottom of the tank were constructed from 3.2 mm 304 stainless steel sheets. The tank was equipped with time domain reflectometry probes to measure moisture contents, and probes that could be used to measure water, gas, and organic phase pressures, and to take water, gas, and organic samples. There were 150 sample points, 25 per horizontal plane, on a total of 6 horizontal planes (see Fig. 2(a) and (b)). To avoid problems packing sand around bundles of coaxial cables and tubing, 5 cm diameter, 2 m foot long vertical pipes were located at each of the sampling locations (M-2, M-3, etc.), where there was not a vertical well. The coaxial cables and tubing were run inside of these 5 cm pipes. At the appropriate levels the tubing or coaxial connection was passed through the side of the pipe wall (see Fig. 2 (b)). The 5 cm pipes were closed at the bottom, and open at the top.

Five vertical wells were installed in the tank, at positions shown in Fig. 2(a) (W-1, W-5, W-12, W-21, and W-25). The well outer casings were constructed from 10 cm i.d. aluminum Sch. 40 pipe. The outer casings had five 2.5 cm wide, 90 cm long, slots cut vertically into their lower halves, starting at 2.5 cm from the bottom of the tank. The slots were equally spaced radially, and were covered by 100 mesh stainless steel screen, to prevent migration of soil fines into the

Fig. 2. (a) Plan view of tank showing monitoring locations and wells; (b) vertical locations of sampling instruments.
wellbores. The outer casings were fitted with TDR probes, thermocouples, and pressure/sampling points, installed in the same manner as in the 5 cm pipes. 5 cm i.d. and 2 m long, aluminum pipes were placed inside the 10 cm well casings to act as an inner conduit. These pipes also had slots cut into them to allow fluid to flow freely from the outer wellbore annulus into the interior of these 5 cm pipes. All tubing and coaxial cable was confined to the annulus between the 10 cm casing and the 5 cm inner pipe. The purpose of these inner pipes was to provide a passage clear from tubing and coaxial cable, in which to measure water and organic levels. The vertical wells were open at the bottom. The top of each well was closed with a machined aluminum cap, containing a 1.9 cm hole through which the tubing and coaxial cables passed. The hole was sealed around the cables and tubing with silicone. The caps also contained a 5 cm hole to access the inner 5 cm pipes, and a 1.9 cm NPT hole through which gas, water, or organic could be pumped.

3.2. Soil properties

Two types of soil were placed in the box. The finer material, placed at the bottom of the box to a depth of 83.8 cm was an unscreened soil that was primarily sand, but contained a fraction of clay-sized particles. The coarser material, placed to a thickness of 86.4 cm on top of the fine sand was a natural sand that was screened. The sands were placed in the box in lifts of about 10 cm, and compacted with a 30 cm by 30 cm steel plate. The porosities, measured volumetrically, were 0.3 and 0.33 for the coarse sand and the fine sand, respectively. The permeabilities of the soils, measured using a standard falling head test, were measured to be $1.2 \times 10^{-11}$ and $3.4 \times 10^{-12}$ m$^2$ for the coarse sand and the fine sand, respectively. To provide data for estimation of in situ effective permeabilities, a slug test was performed at Well 12, in which 8 l of water was added over 20 s. To determine capillary pressure saturation parameters of the soils, dynamic drainage and imbibition experiments were performed in a small cell similar to that described by She and Sleep [12]. The hysteretic capillary pressure saturation parameters determined are given in Table 1. In determining these parameters it was assumed that the same $n$ values applied to both imbibition and drainage as is commonly assumed [6].

3.3. Measurement of moisture contents

Moisture contents were measured with time domain reflectometry. Three-wire TDR probes based on the design of Zegelin et al. [15] were used in this study. The probes were installed in the outer walls of the monitoring access pipes (Fig. 2(b)). A coaxial connector was installed into a hole drilled through the side of the pipe.

The connector was epoxied into place, so that no water could leak into the interior of the pipe. A coaxial cable was run down the interior of the pipe to the coaxial connector. A wire, consisting of a 15 cm piece of stainless steel TIG welding rod (diameter = 1.6 mm) was fit into the center of the coaxial connector making an electrical connection with the center conductor of a coaxial cable. Two outer wires were fit into holes countersunk in the pipe walls, at distances of 1.5 cm from the central wire. These wires were in electrical connection with the outer conductor of the coaxial cable, through mutual contact with the pipe wall through the coaxial connector.

Electromagnetic signals were generated by a Tektronix 1502C cable tester. As the tank contained 150 TDR probes, it was necessary to construct an automated data acquisition system to measure moisture contents. Five TDR switchcards (Soil Moisture Corporation) were assembled in a chassis, and connected to a computer. The cable tester and the probes were connected to the switchcard system. A Windows-based computer program was written (C programming language) to control the switchcard system, to control the cable tester, and to acquire waves from the cable tester, and to analyze these waveforms. The program could perform automatic analysis of the waveforms as they were acquired to determine moisture contents, or it could be used in manual mode to analyze stored waveforms. This analysis consisted principally of identifying the points on the waveform corresponding to the start and end of the TDR probes. It was found that the automatic analysis program, based on the method used by Heimovaara and Bouten [3] was reliable for low moisture contents, but was not very consistent at higher moisture contents, necessitating manual analysis of waveforms. The time required to acquire one waveform consisting of 251 points was about 30 s, so that the set of 75 probes simultaneously connected to the switchcard system could be read every 37 min.

<table>
<thead>
<tr>
<th>Property</th>
<th>Screened sand</th>
<th>Fine sand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Van Genuchten $n$</td>
<td>5.0</td>
<td>4.3</td>
</tr>
<tr>
<td>Van Genuchten $\alpha$ (m$^{-1}$) for imbibition</td>
<td>8.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Van Genuchten $\alpha$ (m$^{-1}$) for drainage</td>
<td>4.0</td>
<td>2.5</td>
</tr>
<tr>
<td>Irreducible water saturation ($S_{wr}$)</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>Maximum residual air saturation ($S_{rw}$)</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>Horizontal permeability (m$^2$) (from slug test)</td>
<td>$4.0 \times 10^{-11}$</td>
<td>$6.0 \times 10^{-12}$</td>
</tr>
<tr>
<td>Vertical permeability (m$^2$) (from slug test)</td>
<td>$2.0 \times 10^{-11}$</td>
<td>$3.0 \times 10^{-12}$</td>
</tr>
</tbody>
</table>
3.4. Water pressure and water level measurement

To allow measurement of pressures, and to allow withdrawal of water, gas, or organic samples, 150 pairs of probes were placed in the tank, one pair at each of the 150 sampling locations. The probes consisted of Swagelok fittings with ceramic disks or porous Teflon disks sealed into the fittings. The ceramic disks were preferentially water-wet compared to air and organics, and could be used to measure water pressures, and take water samples. The porous Teflon disks were preferentially nonwetting to water in air–water systems. They could be used to measure gas pressures and take gas samples in water–air systems. The Swagelok fittings holding the ceramic and Teflon disks were connected to 1/8” Teflon tubing. The tubing passed into the access pipes, or the vertical wells, via bored – through 1/8” Swagelok fittings seated in the pipe walls. The tubing was brought up through the pipe to the surface of the box. At the ground surface the tubing ends were connected to electronic pressure transducers (Sensotec, Model WD-JE,+/-60 kPa differential,+/-0.1% accuracy), or were capped with Swagelok fittings that were sealed with rubber septa. The electronic pressure transducers were connected to a data acquisition system (Sciemetric) interfaced with a computer. Water levels in wells W-5, W-12, and W-21 were measured with water level transducers located at the bottoms of the wells. The water level transducers were connected to the data acquisition system.

3.5. Water injection schedule

The initial saturations of both the coarse and fine soils were approximately 25% when placed in the tank. As a result of evaporation of water from the top of the tank, and some downward movement of water in the tank, the saturations at the soil surface were about 3%, while they were 25–35% at the bottom of the tank.

Tap water was added to the tank through W-21 first for approximately 170 min at the rates and times given in Table 1. The water was added using a pump, connected to a 200 l container. The tap water was placed into the container before being added to the tank to allow it to reach room temperature, and to avoid air degassing into the water in the tank. Water flow rates were metered with a rotameter. 193 h after water was first added to W-21 water addition to W-5 was started (rates and times measured from commencement of water addition to W-5 are given in Table 1). During water addition to W-21 and W-5, water levels were monitored in W-5, W-12, and W-21 using water level transducers placed at the bottoms of the wells, and connected to the data acquisition system. Water saturations and pressures were also monitored at selected locations in the tank during water addition.

3.6. Modeling discretization and boundary conditions

A finite difference grid with 18, 19 and 20 blocks in the x, y and z directions, respectively was used for all simulations in this study. In the x and y (horizontal) directions grid dimensions ranged from 0.05 cm in the center of the model domain to 0.26 m at the boundaries. Blocks containing wells had horizontal dimensions of 0.2 m by 0.2 m. In the vertical direction the grid block thicknesses from top to bottom were: two of 0.2 m, four of 0.08 m, six of 0.04 m, and eight of 0.1 m.

All grid boundaries were set to no flow for the water phase. The side and bottom grid boundaries were set to no flow for the air phase, while at the top boundary gas pressures were prescribed to be atmospheric pressure. The well nodes had prescribed gas pressures set to atmospheric pressure, as these wells were opened frequently during the study. Water additions were simulated as flux boundary conditions for the wells to which water was added. Time steps were dynamically adjusted by the model as a function of the rate of convergence of Newton iterations.

In modeling the water addition to W-21 the initial water pressures at the bottom of the tank were set to −0.57 m H2O. Hydrostatic equilibrium for both water and gas phases was assumed to exist throughout the tank. Initial water saturations were calculated from initial water pressures, assuming primary drainage conditions at the start of the simulation. The initial conditions for simulation of water addition to W-5 were based on the final conditions (193 h after starting water addition) resulting from simulation of water addition to W-21. For simulation of the slug test the initial water table was at an elevation of 0.84 m above the bottom of the tank. Hydrostatic equilibrium was assumed to exist and it was assumed that water saturations could be calculated from primary drainage curves. The hysteretic capillary pressure saturation relationships proposed by Parker and Lenhard [8] and the relative permeability relationships of Lenhard and Parker [6] were used in modeling. A maximum of six scanning curves were allowed.

4. Results and discussion

The model was first applied to the analysis of the slug test performed on W-12. The horizontal and vertical intrinsic permeabilities of the two soils in the model were adjusted manually to improve the visual match between measured and predicted water levels in the W-12 during the slug test. The predicted water levels for the calibrated permeability values are plotted in Fig. 3. The permeabilities that produced the best match (see Table 1) were greater than the values measured with small soil samples in the falling head permeameter by a factor of
three or less. The calibrated permeabilities also had horizontal to vertical anisotropies of 2.0. As an exhaustive calibration was not performed it is possible that other combinations of permeabilities might provide an equivalent or better match to the slug test data. However, changes in any of the intrinsic permeabilities given in Table 1 of more than 20% produced visibly poorer fits to the slug test data. The impact of grid refinement was not examined, and it is possible that further refinement may have had a small impact on the permeabilities determined to best match the slug test data.

After calibrating the model to the slug tests the initial water injection tests were simulated. The water levels during water addition to W-21 are plotted in Fig. 4(a), while those measured during water addition to W-5 are plotted in Fig. 4(b)–(d). The agreement between model predictions and experimental measurements is excellent. The peak water levels and the rates of water level increase and decrease predicted in W-21 and W-5 during water addition to those wells (Fig. 4(a) and (b)) are very close to those measured. This indicates that the coupling between the wellbore nodes and the soil gridblocks incorporated into the model was able to accurately simulate the dynamics of the flow between the wellbore and the soil. The increases in water levels at W-12 and W-21 (Fig. 4(c) and (d)) as a result of water addition to W-5 also agree reasonably well with the measured data, indicating that the movement of water across the tank away from W-5 and the flow into wells W-12 and W-21 was accurately simulated.

There were small peaks in the measured water levels in W-12 during water addition to W-5 at approximately 6000 and 17 500 min when the water from W-5 reached

![Fig. 3. Slug test in W-12.](image)

![Fig. 4. Water levels in wells: (a) W-21 during water addition to W-21; (b) W-5 during water addition to W-5; (c) W-12 during water addition to W-5; (d) W-21 during water addition to W-5.](images)
W-12. Subsequent to the observation of these peaks, the water level in W-12 decreased as water redistribution throughout the tank continued. Peaks were not observed at W-21 at corresponding times as it was further from W-5. The small peaks at W-12 at 6000 and 17 500 min were not predicted by the model. This is perhaps due to the level of discretization used in the model and associated inability to resolve sharp moisture fronts. Variations in measured water levels during periods when water was not being added (see Table 2) are due to drift in the water level transducers and variations in atmospheric pressure.

The measured and predicted water saturations at a few locations during the experiments are shown in Fig. 5(a)–(c). The location labels represent horizontal and vertical locations in the tank (for example, 5-6 represents level 6 at W-5, 4-6 represents level 6 at M-4). The times at which water saturations began to increase or decrease at various locations as a result of starting or stopping water addition into the wells were very close to those predicted by the model, even at the probes right beside the water addition wells (21-5 in Fig. 5(a), 5-3 in Fig. 5(b)). The peak water saturations measured are higher than those predicted in many cases. This may have been due to inaccuracies in TDR measurements, as it was difficult to analyze TDR waveforms at high moisture contents. It is also possible that soil porosities around the probes were larger than the bulk porosities due to soil settlement during filling of the tank. The peak water saturations predicted are also affected by the residual air saturations used, which may not have been uniform throughout the tank. Finer grid discretization may have also increased the ability to resolve moisture fronts in modeling.

The predicted porewater pressures during water addition to W-5 were in good agreement with those measured (Fig. 6), although the water pressure measured at 5-6 after stopping water addition to W-5 decreased more slowly than that predicted by the model. The different rates of response may have been partially due to the

<table>
<thead>
<tr>
<th>Time for well 21 (min)</th>
<th>Rate for well 21 (m³/s)</th>
<th>Time for well 5 (min)</th>
<th>Rate for well 5 (m³/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–34</td>
<td>1.26E–05</td>
<td>0–120</td>
<td>6.31E–05</td>
</tr>
<tr>
<td>34–71</td>
<td>2.52E–05</td>
<td>4341–4400</td>
<td>6.31E–05</td>
</tr>
<tr>
<td>71–136</td>
<td>3.79E–05</td>
<td>5226–5525</td>
<td>6.31E–05</td>
</tr>
<tr>
<td>136–156</td>
<td>5.05E–05</td>
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<td>156–170</td>
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<td>5.68E–05</td>
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<td>17306–17321</td>
<td>5.36E–05</td>
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Fig. 5. Water saturations during (a) water addition to W-21, (b) water addition to W-5 and (c) water addition to W-5.
slow response of the pressure transducers as a result of slow water flow through the ceramic cup tensiometers used.

5. Summary

A method was presented for the incorporation of discrete wellbores into finite difference models of air and water flow in porous media. The wellbore formulation was based on the assumption of fluid gravity segregation, with water accumulation at the bottom of the wellbore. The model is capable of simulation of water addition through wells, water accumulation in monitoring wells, and changing well water levels. These processes that cannot be simulated through the use of distributed sources and sinks to represent wells. Although not tested, the model is also capable of simulation of soil vapor extraction and processes such as water table rise as a result of the application of a vacuum to a well.

The model predictions, with permeabilities calibrated from a slug test, matched the experimental results for water injection into variably saturated soil very well. The discrete wellbore formulation incorporating fluid segregation is an effective method for representing wells in variably saturated soils within finite difference models of air and water flow.

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References