Network modeling of multiphase flow in fractures

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Abstract

We develop a pore-scale model of wetting in a single fracture. A variable fracture aperture distribution is represented as a square lattice of conceptual pores connected by throats. We borrow concepts from network modeling of flow in porous media to generate a fracture model that includes the effects of flow in wetting layers, snap-off, and cooperative piston-like advance. Cooperative filling accounts for both the curvature of the wetting front in the fracture plane and the effect of the fracture aperture itself. Viscous forces are accounted for using a perturbative method and simulations are performed for horizontal flow, as well as gravity stable and gravity unstable displacements. The wetting phase relative permeability and the trapped non-wetting phase saturation are computed for a fracture whose aperture distribution has been measured using CT scanning. © 2001 Elsevier Science Ltd. All rights reserved.

1. Introduction

Multiphase flow in fractures occurs in a variety of different situations, including the movement of pollutants, such as radioactive waste, in gas formed in an initially water-saturated system, the transport of dissolved contaminants through wetting layers in the unsaturated zone, and the migration of dense non-aqueous phase liquids through fractured bedrock. Many oil reservoirs are extensively fractured, and an understanding of how oil and water move through them is essential for the design of oil recovery projects.

Modeling multiphase flow through fractured systems on the field scale requires that we have estimates of average properties for the ability of the fractures to transmit each phase (the phase relative permeabilities) and for the ability of the surrounding matrix blocks to imbibe wetting phase and emit non-wetting phase (matrix/fracture transfer). These properties must be representative of the average behavior within a grid block that may have fractures that have different orientations and matrix block sizes [1].

In this paper we present a method for adapting a pore/throat network model to study multiphase flow in a single fracture. The method is similar to an approach used by Glass and co-workers [10,11] but accounts for wetting layer flow and uses an approximation to allow us to account for viscous effects. In another paper [13] we show how we can use the model to study the matrix/fracture interaction.

Traditionally, fracture relative permeabilities have been assumed to be unit slope, linear functions of saturation based on constant aperture, glass plate experiments by Romm [32]. Merrill [21] attempted to duplicate Romm’s experiments and found that the wetting phase saturations were scattered around a value of 0.72 for nearly the entire range of flow rates and fractional flows. Deviations from this behavior only occurred when both the total flow rate and the fractional flow were high. This behavior was also seen when Merrill performed experiments on Berea sandstone blocks that had been sealed with epoxy. The only difference was that the wetting phase saturation value where multiphase flow occurred was approximately 0.62.

Natural systems have variable aperture distributions [8,9,12,16,18,29]. Pyrak-Nolte et al. [30] computed drainage relative permeability curves using a percolation process, for a numerically generated, spatially correlated, log normal aperture distribution. They found that the relative permeabilities were non-linear functions of saturation and that the relative permeability curve
crossover point was essentially invariant of stress. Pyrak-Nolte et al. [28] modified the displacement model to be invasion percolation with trapping, which was then used for imbibition. Trapping introduced a strong shift in the crossover point towards lower relative permeability values and lower wetting phase saturation. Keller et al. [17] also used a percolation-type model to compute relative permeabilities and capillary pressures in a single fracture.

Pruess and Tsang [27] simulated percolation on a log normal aperture distribution. They found that for short-range correlations, very little multiphase flow occurred. This result concurs with that of Wilkinson and Willemsen [37] who have shown that for random percolation models, it is not topologically possible to have continuous pathways of two phases across a two-dimensional medium.

For correlations which were much stronger in the flow direction Pruess and Tsang [27] found that wetting phase relative permeability curves could be approximated as a power-law function of saturation (Corey-like). Non-wetting phase relative permeabilities started on a similar Corey-like path, but fell to zero once a critical grid block was filled.

Rossen and Kumar [33,34] used an effective medium approach on the Pruess and Tsang [27] short correlation length aperture distribution. They found relative permeabilities nearly identical to those found by Pruess and Tsang with less computational expense. Rossen and Kumar then used their effective medium approach to evaluate the effects of the variance of the fracture aperture distribution, gravity, and wetting layer flow on relative permeability.

Glass [10] used a modified invasion percolation model to simulate gravity-driven fingers in fractures. The modification to standard invasion percolation was to include the in-plane curvature in a manner similar to what Lenormand and Zarcone [19] called a series of I mechanisms, which depend on the number of adjacent areas filled with non-wetting phase. They called this modification capillary facilitation. A subsequent study [11] modified the facilitation term and the model was applied to horizontal fractures. Relative permeabilities were not presented and flow in wetting layers was not considered. When facilitation was not considered, the generic wetting behavior was found to be independent of the aperture distribution (invasion percolation). When facilitation was considered, the wetting behavior ranged from thin fingers for wide aperture distributions to flat frontal advance for narrow distributions.

Recently, there have been several experimental investigations in fractures with non-uniform aperture distributions. Fourar et al. [7] artificially roughened parallel glass plates by gluing 1 mm diameter glass beads to the plates with transparent epoxy. Simultaneous injection of water and air at relatively high flow velocities resulted in observed flow regimes which were similar to those observed in pipe flow (bubble flow, fingering bubble flow, complex flow, film flow, and droplet flow). The results were strong non-linear functions of saturation and flow rate when analyzed as a relative permeability.

Nicholl and Glass [23] measured end point (saturated) wetting phase relative permeabilities on an analog fracture. They found that the measured relative permeability was proportional to \( S_w^2 \), where \( S_w \) is the end point wetting phase saturation. In a subsequent study Nicholl et al. [25] found that the tortuosity of the flow path due to the residual non-wetting phase was the most significant factor that determined the satiated relative permeability.

Persoff and Pruess [26] performed experiments on natural rock fractures and on transparent epoxy replicas of natural fractures. Air and water were simultaneously injected into the fracture and attempts were made to reach steady state. Saturation measurements were not made. Plots of relative permeability vs. mass flow rate ratio and gas relative permeability vs. water relative permeability showed that significant phase interference occurred. Relative permeabilities were shown to be neither linear functions of saturation, nor were they Corey-like.

In this paper we propose an alternative method for studying the fundamentals of multiphase flow in fractured media, using pore network modeling to predict how the physics at the small scale affects macroscopic properties. The work relies on two major ideas.

First, we assume that the fracture can be modeled as a two-dimensional porous medium of pore spaces connected by throats. Pruess and Tsang [27] used this same conceptualization in their work and suggested that it is applicable to experimental images of fracture geometries as well as to their theoretical aperture distribution. This work will propose a method to represent a measured fracture aperture distribution as a regular lattice of conceptual pores and throats.

Second, we assume that the wetting phase remains connected throughout the system in wetting layers, even when the center of the fracture is occupied by non-wetting phase. All porous media contain grooves, roughness or crevices in which the wetting phase preferentially resides. The wetting phase that occupies these narrow regions is called a wetting layer in this paper.

The experimental evidence for this comes from the work of Tokunaga and Wan [35] which measured wetting layers in fracture experiments on Bishop Tuff. They found that the wetting layers were between 2 and 70 \( \mu \)m thick, were connected across the system and had flow velocities of 2–40 m/day. Wan et al. [36] also observed wetting layers in micromodel experiments.

We will model wetting layer flow as occurring in the corners of pores and throats. The non-wetting phase
flows through the central regions of the pore or throat while the wetting phase flows in wetting layers along the corners.

2. Model description

Full details of the model are provided in [14]. The description here follows [13]. We describe a model that is a network of pores and throats. At the end of the section we show how this type of model can be used to represent a fracture with a variable aperture.

Our model is a square or cubic lattice of pores connected by throats. For the results in this paper, the pores and throats have rectangular cross-sections. Wetting fluid resides in the corners of each element. Fenwick and Blunt [6] presented a similar three-phase model that assumed capillary forces dominate. Our model extends the two-phase portion of that model to include the effects of flow rate.

For primary imbibition (wetting phase invasion into an initially dry medium) we assume that wetting layers in the corners instantaneously form throughout the model. The conductivity for corner flow is a user-specified input parameter which is the same for every pore or throat throughout the displacement. The only exception is for very small throats for which this conductance would exceed the maximum permissible layer conductance even if the corners were entirely filled with wetting phase. In this case we set the corner flow to the maximum permissible value.

Secondary imbibition is simulated by filling the model with non-wetting phase (primary drainage) assuming that capillary forces dominate until a user-specified capillary pressure is reached. This leaves wetting phase in both pores and throats throughout the model. We again assume that the conductivity for corner flow is fixed throughout the network at the value set by the specified capillary pressure. Layer conductances do not need to be adjusted for small throats since these throats have not been invaded by non-wetting phase.

During imbibition a local capillary pressure, $P_c$, is computed for filling a pore or throat by either piston-like advance or snap-off. Combined with an approximate computation of the effects of viscous forces, this defines a filling sequence. We first describe how we compute the fluid conductances and saturation before giving details of how capillary pressures are found.

The cross-sectional area of an element (pore or throat) is $A_{tot} = bd$, where $b$ is the local fracture aperture and $d$ is the resolution of the aperture measurements (described in the Fracture Model section to follow). An element may be full of wetting phase, in which case the wetting phase area $A_w = A_{tot}$. If an element contains non-wetting phase, the wetting phase is present in layers and the area occupied by wetting layers in each element is given by Ma et al. [20]

$$A_w = 4r_{nw}^2 \left[ \cos \theta (\cos \theta - \sin \theta) - \frac{\pi}{4} + \frac{\theta}{2} \right],$$

where $\theta$ is the contact angle and $r_{nw} = \gamma/P_c$. $P_c$ is the maximum capillary pressure attained during primary drainage or the capillary pressure associated with the assumed wetting layer conductivity for primary imbibition. The area occupied by non-wetting phase is then

$$A_{nw} = A_{tot} - A_w. \quad (2)$$

The volume of each phase in each element $i$ is obtained as a ratio of the phase area to the total area

$$V_i^p = \frac{A_i^p}{A_{tot}} V_e^i,$$

where $V_e^i$ is the total volume of the element and the subscript $p$ indicates the phase.

Wetting phase saturation is then calculated from

$$S_w = \frac{\sum N_p V_i^w + \sum N_i V_i^m}{\sum N_p V_e^w + \sum N_i V_e^m},$$

where the sums are over the number of pores ($N_p$) and number of throats ($N_i$).

For elements full of wetting phase and for non-wetting phase flow in the centers of the elements, we define a hydraulic radius as

$$r_{eff} = \left( \frac{A_e}{\pi} + \frac{b}{2} \right)^\frac{1}{2}. \quad (5)$$

For elements full of wetting phase, $A_e = A_{tot}$. For non-wetting phase flow, $A_e = A_{nw}$ from Eq. (2).

For elements full of wetting phase, the conductivity is [4]

$$g^{w} = \frac{\pi r_{eff}^2}{8 \mu_w l},$$

and for the non-wetting phase,

$$g^{nw} = \frac{\pi r_{eff}^2}{8 \mu_{nw} l}, \quad (7)$$

where $\mu_w$ is the wetting phase viscosity, $\mu_{nw}$ the non-wetting phase viscosity, and $l$ is the length of the element.

Wetting phase conductivity in elements with two phases present is given by

$$g^{w} = \frac{4A_w r_{nw}^2}{\beta \mu_w l}. \quad (8)$$

The $\beta$ term is the crevice resistance factor for each corner proposed by Ransohoff and Radke [31]. We use an expression from Zhou et al. [38] to obtain $\beta$

$$\beta = \frac{12 \sin^2 \alpha (1 - B^2)^2 (\psi_1 - B \psi_2) [\psi_1 + f B \psi_2]^2}{(1 - \sin \alpha)^2 B^2 [\psi_1 - B \psi_2]^2}. \quad (9)$$
In this equation \( \psi_1, \psi_2, \psi_3, \) and \( B \) are all terms that depend on the corner half-angle \( (z) \) and the contact angle. For rectangular elements they are given by

\[
\psi_1 = \cos \theta (\cos \theta - \sin \theta),
\psi_2 = 1 - \frac{4\theta}{\pi},
\psi_3 = \cos \theta - \sin \theta,
B = \frac{\pi}{4}.
\]

The \( f \) term is a factor that ranges from 0 to 1 depending on the boundary condition at the fluid interface. We assume a free boundary and \( f = 0 \). In this paper we also use \( \theta = 0^\circ \) and hence from Eq. (9), \( \beta = 113.38^\circ \).

We calculate the phase pressure in each pore by applying volume conservation:

\[
\sum_{i-j} Q_{ij}^p = 0, \tag{14}
\]

where the sum is over all pores \( j \) connected to pore \( i \). \( Q_{ij}^p \) is obtained from a Darcy-type law in each pore–throat element as

\[
Q_{ij}^p = g_{i,j}^p (P_i^p - P_j^p), \tag{15}
\]

where the conductance \( g_{i,j}^p \) is the harmonic mean conductance between pores \( i \) and \( j \) obtained from

\[
\frac{1}{g_{i,j}^p} = \frac{1}{g_i^p} + \frac{1}{g_j^p} - \frac{1}{2} \left( \frac{1}{g_i^v} + \frac{1}{g_j^v} \right), \tag{16}
\]

where \( g_i^v \) is the conductivity of the throat connecting pore \( i \) to pore \( j \). We solve Eq. (14) for \( P_w \) for a fixed flow rate across the network and constant pressure across the inlet and outlet.

3. Displacement modeling

For each element in the network we calculate the wetting phase pressure drop between the inlet and the element \( (\Delta P^w) \). We then calculate a sorting pressure \( (P_{\text{sort}}) \) which is defined by

\[
P_{\text{sort}} = \Delta P^w - P_c - \Delta \rho gh \sin \varphi, \tag{17}
\]

where \( P_c \) is displacement capillary pressure that is appropriate for the mechanism that applies to the element (snap-off, piston-like advance or pore-filling), \( \Delta \rho \) the density difference between the phases, \( \varphi \) the inclination angle from horizontal of the fracture, \( g \) the gravitational constant, and \( z \) is the distance from the inlet to the element. In a later section (Fracture Model) we describe how \( P_c \) is found for simulating flow in a fracture.

We assume that the non-wetting phase pressure gradient is negligible, and so \( P_{\text{sort}} \) can be viewed as an inlet pressure necessary to fill each element. We fill the element with the smallest value for \( P_{\text{sort}} \). This procedure is repeated until all the non-wetting phase has been produced or are trapped.

Because the method is rule based, we have found that we only need to update the pressure field after filling approximately 25 elements rather than after each fill [14]. Fully dynamic models, that may better capture the complex dynamics of the flow, are more computationally expensive and require at least two solves per displaced element [22].

4. Relative permeability

Using the pressure solution for a single phase across the network we can determine the permeability of the network. This definition for the permeability leads to one possible method for defining the relative permeabilities for the model. If the entire model is considered a representative elemental volume (REV), then the inlet and outlet pressures obtained from the solution for the pressure field at intermediate saturations can be used to calculate the relative permeability since the flow rate is kept constant. We call this the pseudo method, and the relative permeabilities are defined as

\[
k_{wp} = \frac{P_{\text{In}}^\phi - P_{\text{Out}}^\phi}{P_{\text{In}}^w - P_{\text{Out}}^w}, \tag{18}
\]

where \( P_{\text{In}}^\phi \) and \( P_{\text{Out}}^\phi \) are the inlet and outlet pressures from the single-phase pressure field solution, and \( P_{\text{In}}^w \) and \( P_{\text{Out}}^w \) are the inlet and outlet pressures from the pressure field solution for phase \( p \) at intermediate saturations. For the wetting phase this definition is general, since we assume that the wetting phase is continuous across the model and can therefore obtain a pressure solution for the network. This definition is problematic when there is no continuous path of non-wetting phase from the inlet to the outlet, and \( k_w = 0 \).

5. Model approximations

Our model makes several approximations [13,14].

First, we assign a fixed conductance to all the wetting layers throughout the displacement. We do this so that we can obtain a solution for the wetting phase pressure across the model. If we do not fix the conductance, wetting layers become very small towards the outlet end of the model and we cannot obtain a solution for the pressure field. In reality, the wetting layers swell near an advancing imbibition front. Second, when we find the pressure field, we assume a fixed flow rate in the wetting phase throughout the medium. In reality, at the advancing front there is displacement, meaning that the wetting phase flow rate decreases towards the end of the
model and non-wetting phase flow increases. Third, we neglect the pressure gradient in the non-wetting phase in our calculation of $P_{\text{sort}}$. This makes our approach suitable for water/gas flows or water into light NAPLs, but will be inappropriate if the displaced fluid has a high viscosity (where viscous fingering could occur). For viscous stable flows, all these approximations would lead to more snap-off and a wetting front that has more roughness.

Both this model and that of Blunt and Scher [2] use a fixed wetting layer conductance. In this model we explicitly compute the pressure field and account for pressure drops both in wetting layers and in filled regions. Blunt and Scher [2] only approximately accounted for pressure drops in the wetting layers and ignored pressure drops in the filled regions.

We have shown [14] that our model successfully reproduces the generic features of wetting invasion observed in micromodels. Nevertheless, some phenomena seen in steady-state and high flow rate experiments, namely the movement of trapped ganglia or bubble flow and cooperative transport, are not modeled.

We have also used this model [14] in two and three dimensions to study the effects of flow rate, contact angle and initial wetting phase saturation on imbibition relative permeability and flow patterns. We identified five generic flow regimes, which we categorized in terms of typical trapped non-wetting phase saturation and relative permeabilities.

We showed that a key parameter in determining the flow behavior was

$$\alpha = \frac{\text{Average conductivity of the throats}}{\text{Wetting layer conductivity}},$$

which is the ratio of the conductance of a fully saturated throat, to the wetting layer conductance. For primary imbibition, $\alpha$ is a user-specified parameter. For secondary imbibition it is derived from $P_c$.

In [14] we used the model to study the effects of flow rate in porous media. In [13] we used the model to study flow in fractures and matrix/fracture transfer. In this paper we extend this work to study fracture flow and the combined effects of flow rate and gravity.

### 6. Aperture measurements

The method we use to simulate flow in a single fracture requires knowledge of the aperture distribution on a grid. Several methods for obtaining these aperture distributions have been presented in the literature: epoxy or liquid metal injection [9,29]; profilometry [3]; light diffraction [24,26]; CT scanning [16]; NMR imaging [18]. Some measurement resolution is implicit in the results of each type of measurement. The advantage to the non-destructive methods (light diffraction, CT scanning or

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**Fig. 1.** Measurement of fracture apertures. The fractured core (a) has a variable aperture size throughout. Measurements of the aperture are made at some resolution, $d$. The measurements are typically made on a grid in two dimensions as shown in (b). The reported individual apertures, $b$, are actually averaged values within the resolution of the measurement device as shown in (c). NMR imaging is that flow experiments can be conducted on the measured aperture field either as part of the measurement process or immediately following the measurements. Fig. 1 shows a schematic of the measurement of a fractured core in a laboratory by non-destructive means. The fracture apertures are labeled $b$ and the resolution of the measurement, $d$. Fig. 1(a) shows a cartoon of a core with a variable aperture fracture. The apertures will be measured on a grid system that has some resolution in the different directions. Fig. 1(b) shows one of the directional resolutions, $d$. The fracture apertures are obtained for each of the grid locations and are usually averaged values over the distance, $d$, and depend on the resolution as shown in Fig. 1(c). For the discussion that follows, it is assumed that the resolution is the same in the different directions.

### 7. Fracture model

For our model, we assume that the fracture plane is a square lattice of pores and throats. The distance, $d$, between pores is the resolution of the data.

When modeling low capillary number flows, the physical law that controls the displacement is the Young–Laplace equation which provides the local capillary pressure ($P_c$) for a set of principal radii of curvature ($R_1$ and $R_2$)

$$P_c = \gamma \left( \frac{1}{R_1} + \frac{1}{R_2} \right),$$

where $R_1$ is the radius of curvature perpendicular to the fracture plane and effectively measures the local aperture. The $R_2$ term is the radius of curvature in the plane of the fracture and accounts for the local distribution of fluid. The different $I_n$ mechanisms for pore filling in

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imbibition presented by Lenormand and Zarcone [19] and the capillary facilitation terms presented by Glass [10] and Glass et al. [11] are effectively a way to account for $R_2$ in a network model. We use this approach to study wetting in a fracture. In this case the pores and throats are entirely conceptual, and the lattice spacing, $d$, depends on the resolution of the measurements.

Fig. 2 shows how the model uses a number of adjacent filled elements to determine the capillary pressure for filling a pore. First, if there is frontal advance and three of the adjacent elements are filled with non-wetting phase (Fig. 2(a)), we take the in-plane curvature to be infinite. Therefore, only the fracture aperture is involved in the Young-Laplace equation and the capillary pressure is given by

$$P_c = \gamma \left( \cos \frac{\theta}{b/2} \right),$$

where $b$ is the fracture aperture.

If two adjacent elements are filled (Fig. 2(b)), we assume that the in-plane curvature will no longer be infinite. The capillary pressure for this $I_2$ mechanism is a modified form of that given by Lenormand and Zarcone [19]

$$P_c = \gamma \left( \cos \frac{\theta}{b/2} + \frac{1}{d} - \frac{1}{\sqrt{2d}} \right).$$

The modification that we have made is that the middle term has $\cos \theta$ in the numerator in the Lenormand and Zarcone [19] paper. We assume that the contact angle is $0^\circ$ for the in-plane curvature. This allows our model to satisfy the requirement that the capillary pressures for the $I$ mechanisms be monotonic ($P_{I_1} > P_{I_2} > P_{I_3}$). A drawback of our model is that if the wetting phase occupies throats that are on opposite sides of the pore, we still assume that we have an $I_2$ mechanism.

If three adjacent elements are filled with wetting phase (Fig. 2(c)), the $I_1$ capillary pressure is given by [19]

$$P_c = \gamma \left( \cos \frac{\theta}{b/2} + \frac{\sqrt{2}}{d} \right).$$

We assume that the central portion of the grid block has the measured aperture value and is represented by a pore whose diameter is the local aperture. The remaining portion of the grid block is assigned to throats. A throat is assigned a diameter that is the smaller of its adjacent pores. The pore length is $\frac{1}{2}d$ and the pore volume is $\frac{1}{2}d^2b$. There are four throats connected to each pore and they connect adjacent grid blocks. Each throat has a length of $\frac{1}{2}d$ (or $d$ in each grid block) and a volume of $\frac{1}{2}d^2(b_i + b_j)$, where $b_i$ and $b_j$ are the apertures of the two grid blocks connected by the throat. In this way the total volume of the fracture is independent of the resolution.

Snap-off events are modeled as they are in the granular media simulations [19]. Here a throat fills in advance of the connected wetting front by swelling of the wetting layer. We assume that the curvature of the fluid interface is infinite in the aperture plane and so the snap-off capillary pressure is given by

$$P_c = \gamma \left( \cos \frac{\theta - \sin \theta}{b/2} \right).$$

Fig. 2. Fracture $I$ mechanisms: (a) shows $I_3$; (b) shows $I_2$; (c) shows $I_1$. Gray blocks are filled with wetting phase while white blocks are filled with non-wetting phase.
It is assumed that once a conceptual pore is filled in a fracture, the adjacent conceptual throats fill immediately. This re-emphasizes the fact that for a fracture, a pore or a throat is entirely conceptual and if a grid location has been invaded by wetting phase, the entire location should be filled. Our model is similar to that of Glass and co-workers [10,11]. However, there are several differences in the way that the conceptual pores are filled.

First, we take the in-plane curvature to be infinite for a flat front \( I_3 \) mechanism and therefore neglect in-plane curvature. Glass [10] and Glass et al. [11] take \( R_2 = \frac{2}{d} \).

For an \( I_2 \) mechanism, our model uses \( \frac{1}{d} - \frac{\sqrt{2}}{d} \) for \( 1/R_2 \). The first Glass model [10] assumes no contribution from an \( I_2 \) configuration \( (R_2 = \infty) \), while the second [11] would set \( R_2 \) to \( -2.356d \) if only the three closest blocks are used [11].

For the \( I_1 \) mechanism, this work uses \( \sqrt{2}/d \) for \( 1/R_2 \). Again, Glass [10] would ignore the second term in this configuration \( (R_2 = \infty) \), and Glass et al. [11] would take \( R_2 \) to be \( \infty \).

For all the mechanisms, our model determines the capillary pressure for displacement based on the local configuration of fluids before a grid block fills, while Glass and associates [10,11] generally use the configuration of fluids when a grid block is half-filled to determine their displacement capillary pressures. Glass and co-workers concentrate on finding \( P_c \) for configurations where the wetting phase bulges out ahead of a flat front, whereas we principally consider \( P_c \) for dips in the wetting front.

Additionally, while we have presented comparisons with Glass et al. [11] on the basis of nearest neighbor filling, the Glass et al. [11] work actually uses a method to compute \( R_2 \) based on the correlation length and all of the grid blocks that are filled with wetting phase that are within a one correlation length radius away from the block being filled.

The other primary difference between this model and those of Glass and co-workers [10,11] is that this model includes wetting layers and hence allows snap-off in narrow regions. This allows filling away from a connected front which increases trapping. We also account for viscous forces. Glass and co-workers [10,11] were interested in modeling primary imbibition where wetting layers are not present initially. We are also interested in cases where wetting phase is already present. Wetting layers in these systems are important and, depending on the flow rate, can have a large impact on the wetting behavior.

8. Resolution sensitivity

To simulate accurately multiphase flow in fractures, the model should be independent of the resolution of the measurement system. The model we have proposed suggests that the resolution of the data is a primary parameter in the calculations. To test the sensitivity of the model to the resolution of the data, a sequence of simulations was conducted on a measured fracture aperture distribution.

The data set used in this section was core C from Keller [16] and consisted of a 160 × 160 matrix of aperture measurements. The core was 166 mm in length and 52.5 mm in diameter. The first and last 3 mm of length were not included in the data set; which means that the resolution of the data received was 1 mm in the \( x \)-direction and 0.33 mm in the \( y \)-direction. Fig. 3 shows the original data. Note that there are two regions where the aperture values are very large. Small apertures are located along the sides of the fracture (the top and bottom of the figure) and near the outlet end. Excluding points of contact (zero aperture sizes) the aperture distribution was approximately log normal with a mean aperture of 0.825 mm, a standard deviation of 0.683 mm and a 6 mm correlation length [16].

From this, four additional data sets were created for use in the simulations by averaging and interpolating values from the original data set. The approximate resolution for each of the data sets were 4, 2, 1, and 0.5 mm, respectively.

Using the data at various resolutions, we simulated a primary imbibition experiment at a flow rate of \( 1 \times 10^{-8} \text{m}^3/\text{s} \) (0.6 cm³/min). This rate was chosen to approximate Keller’s [15] primary imbibition.
experiment which was conducted at a flow rate that was stated to be between 0.5 and 1 cm³/min. Keller’s experimental results did not show a significant amount of fluid accumulation at great distances from an advancing front. Keller et al. [17] in fact used a percolation model to simulate flow on this same distribution. The percolation model was able to capture the general flow behavior but will be inappropriate for investigation of flows that are not capillary-dominated. The value for $\alpha$ was set for each resolution so that the reported pressure drop ($\sim$6800 Pa) was obtained. Thus $\alpha$ varied between 5002 and 9813.

We used constant flow rate inlet and constant pressure inlet and outlet boundary conditions along with no flow boundary conditions for the sides of the fracture. Additional parameters for the simulations are: $\mu_w = 1 \times 10^{-3}$ Pa s; $\gamma = 72$ mN/m; $\theta = 0^\circ$.

We define a capillary number for this model as

$$N_{Ca} = \frac{Q h_w}{b d N_y},$$

where $Q$ is the volumetric flow rate, $b$ the mean aperture size, $d$ the resolution, and $N_y$ is the number of conceptual pores in the $y$-direction. This definition yields a capillary number of $4.2 \times 10^{-6}$.

Fig. 4 shows the simulation results at the four resolutions at the end of the displacement. Black indicates a region filled with wetting phase, while white indicates a region filled with non-wetting phase. The results for the last three resolutions are broadly similar with the 80 and the 160 results being the most alike. The results for the lowest resolution case show patterns that deviate from the higher resolution cases and also deviate from both the simulation and experimental results reported by Keller and co-workers [15,17]. Glass et al. [11] suggest that the aperture field resolution must be smaller than the correlation length. Our simulations seem to confirm this result since the 40 × 20 solution is approximately equivalent to the correlation length. To account properly for both $R_1$ and $R_2$ in the Young–LaPlace equation (Eq. (20)), we suggest that the ideal resolution is the one similar to the mean aperture as long as the mean aperture is smaller than the correlation length.

Residual saturation values for the four simulations were 0.43 for the 4 mm case, 0.60 for the 2 mm resolution, 0.63 for the 1 mm resolution and 0.58 for the 0.5 mm resolution. End point wetting phase relative permeability values were 0.061 for the 4 mm resolution, 0.052 for the 2 mm resolution, 0.034 for the 1 mm resolution and 0.046 for the 0.5 mm resolution.

9. Flow rate effects

We conducted simulations where we varied the flow rate in a secondary imbibition process. Primary drainage was conducted to a capillary pressure of 800 Pa with an initial wetting phase saturation of 2%. This left the wetting phase in the narrow regions of the fracture which tended to be along its outside edges and near the outlet. The $\alpha$ parameter (Eq. (19)) was 6486.

The effect of changing the flow rate is seen in Fig. 5(a)–(e). These figures are the wetting patterns at the end of the displacement when all elements have either been filled or are trapped. At very high flow rates (Fig. 5(a)), the wetting phase advances as a flat front with very little bypassing of the non-wetting phase. As flow rate decreases, more bypassing of the wide regions occurs, and the residual non-wetting phase saturation increases. At the lowest flow rate (Fig. 5(e)), only areas in front of the first wide region and near the outlet are filled. This behavior is due to wetting phase bypassing the wider regions through flow through the filled regions along the
edges of the model and flow through wetting layers. Only the smallest aperture areas are filled by snap-off and cooperative pore filling. Viscous forces are too small to force the wetting phase into the wider regions.

Fig. 6 shows the residual non-wetting phase saturation and end point wetting phase relative permeability ($k_{\text{max}}^{\text{rw}}$) as functions of the capillary number, $N_{\text{Ca}}$. The plot of residual saturation is a moderately sloped function for capillary numbers between $10^{-4}$ and $10^{-3}$. The plot of end point relative permeability is also a moderately sloped function for this range of capillary numbers. At capillary numbers greater than $10^{-4}$, viscous forces allow filling of the wider regions of the fracture, and consequently, a marked increase in the end point relative permeability. Fig. 7 shows the results for simulations at $N_{\text{Ca}} = 10^{-4}$ and $N_{\text{Ca}} = 10^{-3}$. The residual non-wetting phase saturation for the higher rate is 0.27 and it is 0.49 for the lower rate case. The inlet and outlet ends show

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Fig. 5. Fracture flow simulations: (a) $N_{\text{Ca}} = 10^{-1}$; (b) $N_{\text{Ca}} = 10^{-3}$; (c) $N_{\text{Ca}} = 10^{-5}$; (d) $N_{\text{Ca}} = 10^{-7}$; (e) $N_{\text{Ca}} = 10^{-9}$. Note the increased amount of trapping and the changes near the inlet and outlet ends as the capillary number gets smaller. The pictures show the fracture when all the non-wetting phase is trapped. $P_0 = 800$ Pa, $z = 6486$, and $S_{\text{wi}} = 0.02$.

Fig. 6. The effects of capillary number, $N_{\text{Ca}}$, on: (a) residual non-wetting phase saturation, $S_{\text{nr}}$; (b) the end point wetting phase relative permeability, $k_{\text{max}}^{\text{rw}}$.

Fig. 7. Changes in the distribution of fluid dramatically affects flow parameters: (a) is for $N_{\text{Ca}} = 10^{-4}$ and has $k_{\text{max}}^{\text{rw}} = 0.27$; (b) is for $N_{\text{Ca}} = 10^{-3}$ and has $k_{\text{max}}^{\text{rw}} = 0.79$. 
more filling and there is more filling in the areas around the two wide regions for the high rate case. The maximum wetting phase relative permeability is 0.79 for the high rate case, while it is 0.29 for the lower rate case.

10. Gravity effects

We next conducted simulations where the flow was either in a gravity stable or a gravity unstable direction. We used the same initial conditions as those in the previous section. The capillary number was $1 \times 10^{-7}$. For these simulations $\Delta \rho$ was 1000 kg/m$^3$ and $\phi = -90^\circ$ for gravity stable cases while $\phi = 90^\circ$ for gravity unstable cases.

The effect of changing the flow direction is seen in Fig. 8(a)–(c). For gravity stable flows (Fig. 8(a)), the wetting phase advances in a manner similar to the $N_{Ca} = 1 \times 10^{-5}$ case of the previous section. When flow is in a gravitationally unstable direction, more bypassing takes place and the residual non-wetting phase saturation increases significantly (Fig. 8(b)). Due to this particular aperture distribution, most of the bypassing occurs along the edges of the fracture in a manner similar to that seen in the lowest capillary number flow case. Fig. 8(c) is a gravity neutral flow shown for comparison and is a repeat of Fig. 5(d).

We next used the same histogram of the aperture distribution from Keller [16] that we used before, but distributed the apertures with a correlation length of 5 mm in each direction using the geostatistical software GSLIB [5]. Fig. 9 shows the aperture map that we used and the initial saturation distribution for $P_c = 1000$ Pa ($S_{wi} = 1.7\%$). We simulated flow at a capillary number of $1 \times 10^{-7}$ on a model with 128 $\times$ 128 grid blocks where $d$ was 1 mm. We simulated gravity stable, gravity unstable and gravity neutral conditions. Fig. 10 shows the results at breakthrough for these simulations. Breakthrough is defined when there is a connected path of elements completely full of wetting fluid across the system. When flow is in a gravity stable direction, the local capillary pressure and pressure drop across the model cause a rough frontal advance. When gravity is acting in the same direction as flow, gravity and the local capillary pressure act to channelize the flow in the narrow regions. For the gravity neutral simulation, we see a pattern more similar to the gravity stable case than to the unstable case, but with a higher degree of roughness.

Fig. 8. Fracture flow simulations for: (a) gravity stable; (b) gravity unstable; (c) gravity neutral conditions. $N_{Ca} = 10^{-7}$. The pictures show the fracture when all the non-wetting phases are trapped. $P_c = 800$ Pa, $\alpha = 6486$, and $S_{wi} = 0.02$.

Fig. 9. Fracture aperture map (a) and initial saturation configuration for $P_c = 1000$ Pa (b). The $\alpha$ parameter was 6363.
Our simulations are a bit less “blocky” than those of Glass [10] and have more tortuosity in the wetting structure for the gravity unstable case. For the gravity stable case, our simulation shows significantly more trapping than those shown by Glass [10]. It is difficult to determine whether these differences are due to differences in the topology of the fractures, differences in the numerical scheme or differences in the initial wetting saturation.

Glass et al. [11] show that the dimensionless interfacial curvature, $C$, which is defined as the ratio of the mean aperture to the correlation length times absolute value of the cosine of the contact angle, has a strong influence on the structure of the wetting front. Our mean aperture, correlation length, and contact angle combination yields a value of 0.165 for $C$. Our gravity neutral case can then be compared to Plate 1, Fig. (a) or Fig. (c) from [11]. Our simulation compares reasonably well with these figures. The end point wetting phase saturation reported by Glass et al. [11] was approximately 0.4 for their distribution (see Fig. 4 of [11]), while we obtain an end point wetting phase saturation of 0.33 for our distribution. Our simulations are also for fractures with an initial wetting phase present.

### 11. Conclusions

We have described a pore-scale network model of wetting in a fracture. A variable aperture distribution is represented as a lattice of conceptual pores and throats. We simulated wetting taking into account the effects of viscous forces, gravity, cooperative piston-like advance, and snap-off. We computed the wetting phase relative permeabilities and trapped non-wetting phase saturation.

As a test case we studied the effects of flow rate and gravity in a fracture whose aperture distribution had been measured by CT scanning. We also simulated flow for a numerically generated aperture distribution with an isotropic correlation length. For the measured distribution, wetting phase relative permeability was controlled by the amount of filling that occurred in the wider regions of the fracture. Reasonably high viscous forces were required to cause this additional filling. Flow in a gravity stable direction was similar to flow in a gravity neutral direction at a much higher capillary number. Flow in a gravity unstable direction was similar to gravity neutral flows at a much lower capillary number. For the numerically generated aperture distribution, gravity stable flows result in rough frontal advance, while flow in a gravity unstable direction results in highly channelized flow.

Fig. 10. Gravity effects on a model with a 5 cm correlation length. In all the figures flow is from left to right: (a) is the simulation for a gravity stable condition – gravity is acting from right to left in the figure; (b) is the simulation for an unstable condition – gravity is acting from left to right; (c) shows the simulation for a gravity neutral condition. $N_{Ca} = 10^{-7}$. 
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