Modelling global radiation in complex terrain: comparing two statistical approaches

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Abstract

Two simple approaches for assessing global radiation in complex terrain are tested and compared. A parameterisation scheme for global radiation based on cloud cover observations was compared with interpolation of measured global radiation values from the Austrian climate observation network. Interpolation appears to be a useful method for a station density which has been available after 1992 in Austria (about 1000 km²/station). In that case interpolation is superior to parameterisation. The quality of interpolated data quickly drops with height and for elevations above 1500 m neither method delivers useful results. ©2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

The atmospheric environment constitutes a decisive factor controlling ecosystem processes. Most processes in the atmosphere and biosphere, such as evaporation, sensible heat flux, soil heat flux, photosynthesis or transpiration, are driven directly or indirectly by solar radiation. Many of them have been investigated and modelled in recent years by various disciplines not only in flat but also in complex terrain, where solar radiation constitutes a basic input.

In order to investigate and quantify possible relations between temporal and spatial distributions of ecosystem and atmospheric variables in complex terrain, the temporal and spatial resolution of both variable sets must match. Because of the high spatial variability of ecosystems in complex terrain, atmospheric information with a high spatial resolution is often seen as a prerequisite for ecological research there. Sometimes it is not easy to acquire the atmospheric data with the necessary spatial resolution in remote areas. Putting up and running a fully instrumented meteorological station can be costly. Points of investigation can be spread across a huge area, and not at each of them all atmospheric variables can be measured. If the measurement of only a selected set of atmospheric variables can be afforded, information about others which are also important, might be still lacking. In such situations one has to look for other sources of information:

1. Luckily most countries are running meteorological networks, which provide information about the atmospheric environment to be utilised. A straightforward solution would be to extrapolate and use...
the data from the nearest meteorological station. Because of the horizontal and vertical distance between the meteorological station and the place of interest, such a procedure can be dangerous and can deliver unsatisfying results. Unfortunately ecosystem research focuses often on such remote places where data are scarce. National climate observation networks have not been designed to supply data with a high spatial resolution in remote alpine areas. The more one moves from flat and inhabited areas to remote and complex terrain the more station density drops. Station density is also rapidly decreasing with increasing height.

2. In many cases a strong relationship between the desired atmospheric variable and topography can be found. Topographical information can be available more readily and with a much higher spatial resolution than for any atmospheric variable.

3. A third source of information can be found in the observation that atmospheric variables do not behave independently. One would have to select proper independent atmospheric variables which allow a tight relationship with the desired dependent variable. In case the independent atmospheric variable is measured with a higher spatial density than the dependent variable, additional spatial information for the dependent atmospheric variable can be deduced. Where radiation is concerned, sunshine duration, cloudiness, degree hours of temperature, relative humidity, precipitable water content, and composition, concentration and size distribution of aerosols are among the independent variables which have been used for parameterisation. The usefulness of a parameterisation scheme depends very much on the spatial behaviour of the chosen variables. Some authors, for instance, selected temperature and precipitation as independent variables which can cause problems, if applied in complex terrain. The parameterisation schemes might work at the few places where they were developed for but not necessarily at other sites. In order to apply a parameterisation scheme as it was developed by Bristow and Campbell (1984) or Lexer (1997), empirical coefficients would have to be interpolated in complex terrain. As the spatial behaviour of temperature and precipitation in complex terrain is as complex as the terrain, such an approach appears impractical.

4. As a fourth point, general knowledge about the physics of an atmospheric variable can add substantial information: laws describing the relative position of the sun on a tangential plane on the earth’s surface and shading through topographical features as a function of time help to calculate global radiation at a specific point in complex terrain. Or the idea of global radiation as a sum of direct radiation, diffuse radiation and radiation reflected from the earth’s surface greatly supports the reconstruction of global radiation from interpolated transmittance values.

This work aims at a straightforward and easily applicable procedure to produce radiation data in complex terrain based on the four above mentioned sources of information. Although the procedures which will be introduced show certain limitations, they can nevertheless prove their usefulness for a number of applications.

As it was impossible to find a suitable procedure which could solve the stated problems, two approaches were developed and compared. One was the interpolation of measured global radiation sums (daily and monthly values) via atmospheric transmittance and the other, a parameterisation scheme with subsequent interpolation of the parameterised atmospheric transmittances. For parameterisation, cloud cover observations and the height of the cloud observing station were selected as independent parameters.

Comparing advantages and disadvantages of the parameterisation and interpolation approach, following factors have to be considered:
1. Station density for global radiation measurements (more than 1000 km\(^2\)/station in Austria).
2. Station density for cloud observations (less than 300 km\(^2\)/station in Austria).
3. Quality loss through parameterisation.
4. Quality loss through interpolation.

Parameterisation has a higher station density at its disposal, but suffers quality loss through a two-step process including parameterisation and interpolation. Interpolation of observed global radiation has to cope with a station density which is about a third of that for cloud observations, but saves the quality reducing step of parameterisation. The quality of both procedures will be compared quantitatively at each step.
2. Radiation in complex terrain

Various gaseous, liquid and solid components of the atmosphere, like ozone, aerosols, water droplets, cloud droplets, and the earth’s land and sea surfaces modify the atmospheric short wave radiation field by absorption and reflection. In complex terrain some specific effects add to these modifications. Multiple reflection between slopes at high altitudes, covered with fresh snow, can increase diffuse sky radiation. The intensity of diffuse sky radiation at a mountain observatory in 3106 m during winter time, for instance, was found to be 400% of that at a city station. Under varying cloud cover, short term radiation intensities can reach 140 to 150% of clear sky global radiation intensities or up to 118% of the extraterrestrial radiation intensities. Aerosol and cloud distribution causes a pronounced height dependence of radiation variables (Dirmhirn, 1951; Turner, 1958).

Apart from experimental work, models simulating radiation fluxes in the atmosphere have been developed. In case of a cloudless atmosphere, simple models are capable of reproducing solar irradiance within 5% of complex models and actual observations. This is not the case for overcast skies, where theoretical procedures are not yet well developed (Bristow and Campbell, 1984). An overview of simple models is provided by Goldberg et al. (1979) and a more recent and elaborate overview is given by Duguay (1993a). Multiple reflection between slopes in complex terrain is explicitly modelled, for instance, by Brühl and Zdunkowski (1983). State of the art radiation transfer models include information from remotely sensed data with high resolution digital elevation models (DEM) in order to calculate short and long wave radiation fluxes (Duguay, 1993b, 1995).

3. Procedures

3.1. Approach 1: interpolation of observed global radiation

As the number of semi-automatic stations recording global radiation has steadily been increasing in recent years, the question arises whether interpolation might have become a viable method to predict global radiation at a quality superior to any parameterisation scheme. Global radiation was interpolated applying the following procedure:
1. A sky view factor is calculated for all stations based on the station coordinates and a DEM.
2. For each station a seasonal cycle of daily extraterrestrial radiation sums is calculated taking into account the sky view factor at the station.
3. Daily transmittance values from daily sums of global radiation and extraterrestrial radiation are calculated.
4. Daily transmittance–height relationships are fitted with a second degree polynomial with data from all available stations in Austria.
5. The spatial interpolation procedure of Appendix B is applied to the daily transmittances.
6. Global radiation is then assessed at the stations or at the DEM grid points taking into account interpolated transmittance, slope, aspect and sky view factor according to the procedure in Appendix A.

The quality of the interpolation procedure was assessed by cross validation. Each station was successively skipped as data source for interpolation and instead daily values were interpolated for the skipped station from a specified number of neighbouring stations according to the above procedure. Finally the interpolated and measured time series were compared. As global radiation time series show a great degree of seasonal autocorrelation, the RMSE (Root Mean Square Error) was chosen as model quality variable.

The quality of interpolation is demonstrated by the height–RMSE distribution of Fig. 1 and the spatial distribution of the RMSE values in Fig. 2. The rapid drop in quality of the interpolated data could be explained by the drop in station density with height and a possible increase of variability with height.

3.2. Approach 2: parameterisation of global radiation

In order to estimate global radiation at an individual point with the parameterisation procedure, two steps have to be performed. At first, transmittance has to be parameterised at each station with cloud observations and afterwards the parameterised transmittance has to be interpolated for the desired point with the method described in Approach 1.
The relationship between transmittance and cloudiness has been investigated by a number of authors (Kasten and Czeplak, 1980; Neuwirth, 1982). To illustrate this relationship, a scatter plot of transmittance and cloud cover is shown from daily data at Innsbruck in Fig. 3. The maximum of the transmittance does not occur at 0 cloudiness, but at low cloudiness. This is not unique and not necessarily an artefact of the regression. Neuwirth (1982) compared mean values of the transmittance at different levels of cloudiness, corroborating the earlier mentioned observation.

The transmittance–height relationship displays a pronounced seasonal cycle in Austria which is obviously governed by height, density and frequency of cloudiness (Fig. 4). High frequency of fog in low lying areas and of clear days at higher altitudes create a steep slope of the transmittance–height regression in Figs. 4 and 5. In spring, transmittance is still high at high altitudes, but the influence of foggy weather quickly disappears in Austria’s lowlands, and transmittance gains there a maximum in spring (Figs. 4 and 6). In May, the dullest places appear to be the medium altitudes at the northern and southern fringe of the Alps. Lifting of moist unstable air masses impinging on the Alpine chain promotes free convection, condensation, cloud cover and precipitation. During August spatial differentiation is at its minimum (not shown).

For the empirical model, cloudiness and height of the cloud observing station were chosen as independent variables and in a two-step process, regression coefficients were fitted for a second degree polynomial with monthly resolution. 105 stations with global radiation measurements and 303 sites with cloud observations were available with varying observational periods (up to 15 years).
Cloudiness $C$ calculated from three daily observations at 07:00, 14:00 and 19:00 hours local mean time with $C = (C_{07} + 2C_{14} + C_{19})/4$ turned out to deliver much better results than just a single daily observation. $\bar{H}_0$ represents a calculated reference radiation, for instance, extraterrestrial radiation or direct radiation, and $\bar{H}$ the measured global radiation. The coefficients $a, b, c$ from (1) were modelled as a function of station elevation $z$ as second independent variable:

$$a = a_0 + a_1 z + a_2 z^2$$  \hspace{1cm} (2)

$$b = b_0 + b_1 z + b_2 z^2$$  \hspace{1cm} (3)

$$c = c_0 + c_1 z + c_2 z^2$$  \hspace{1cm} (4)

At this point one has to look at the spatial stability of the regression, whether the variation with height represents the only spatial variability or there exists an additional horizontal variability. If the latter is true, Austria would have to be divided into subregions of similar height regressions, if not, only one regression could be applied to the whole country. In order to test the spatial stability of the coefficients, the regression was once deduced and validated at a set of individual stations and another time the regression coefficients were deduced from data of all available stations and validated at same set of previously used individu-

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**Fig. 3.** The atmospheric transmittance–cloudiness relationship at Innsbruck. Transmittance was calculated from 3653 daily ratios of global/extraterrestrial radiation.

**Fig. 4.** Transmittance vs height for 105 stations in Austria for January (left) and May (right).
It turned out that there is no significant difference between both validation results. One must conclude that there is no significant horizontal variation in the regressed relationships. This is a major advantage compared with other parameterisation schemes using spatially highly variable atmospheric variables. The spatial distribution of the RMSE of the transmittances parameterised using Eqs. (1)–(4) and cross validated afterwards appears fairly homogeneous with only a slight deterioration at higher altitudes (Figs. 7 and 8).

If interpolation is done after parameterisation, a significant change happens to the RMSE–height scatter. The RMSE curve steepens and cuts the original RMSE curve at about 1000 m. The interpolation procedure improves parameterised global radiation data unexpectedly at elevations below 1000 m, but leads to a pronounced deterioration above (Fig. 7). The drop in quality with height approaches the shape of the quality curve of interpolation of observed data.
Fig. 7. Solid curve and open squares: RMSE (Root Mean Square Error) vs height of first parameterised, then interpolated and cross validated global radiation. Dashed curve and crosses: RMSE vs height of parameterised and cross validated global radiation. Daily data from 1994 to 1996.

4. Comparison between Approach 1 and 2

It was expected that the density of cloud observing stations would be higher at higher elevations relative to global radiation observing stations. As a consequence the parameterisation approach should be able to compete with the interpolation of observed data (Fig. 9).

But differences in station density above 1500 m turned out to be insignificant.

The height–RMSE relationships of interpolated observed and interpolated plus parameterised global radiation data appear as parallel shifted curves (Fig. 1). Although station density is about threefold in the case of parameterised radiation data, interpolation quality remains below that of observed data. Station density decreases and scattering increases with increasing
elevation so that the power of explanation of the curves above 1000 m is strongly reduced. Interpolation of observed transmittances before 1992 produce unrealistic spatial distributions. For the time period previous to 1992 parameterisation could be more reliable for certain areas with a low density of stations with global radiation observations. Data resources have not been exhausted for this project and adding observations from non-automatic stations could extend the application of interpolation as a method of choice to the time before 1992.

5. Conclusions

1. The parameterisation scheme for atmospheric transmittance appears spatially robust. There is practically no loss of quality through a generalisation of the scheme for the whole region of Austria (84 000 km²), which includes complex terrain.
2. There is no significant loss of quality for lowlands due to interpolation of parameterised atmospheric transmittance but a significant reduction of quality for areas above 1000 m.
3. The number of global radiation measuring sites has been increasing steadily during recent years so that interpolation has become a viable alternative to acquire radiation data either for a point or for an area of interest.
4. After 1992 interpolation is superior to parameterisation over most of Austria.
5. Areas at elevations above 1500 m still pose a problem and require supplementary radiation information, for instance, provided by satellite data.

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Appendix A. The calculation of daily (monthly) average insolation on inclined surfaces

The approach used here for daily radiation sums follows a suggestion by Klein (1977) for daily averages of a month. The daily radiation \( \hat{H}_S \) on an inclined surface is

\[
\hat{H}_S = \mathcal{R} \mathcal{T} = \hat{R} \hat{K}_T \hat{H}_0 \tag{A.1}
\]

where \( \hat{H} \) is the daily radiation sum on a horizontal surface (MJ m\(^{-2}\) day\(^{-1}\)); \( \hat{H}_0 \) is the daily sum of the extraterrestrial radiation (MJ m\(^{-2}\) day\(^{-1}\)); \( \hat{R} \) is the ratio of the daily radiation on an inclined surface to that on a horizontal surface; and \( \hat{K}_T \) is the atmospheric transmittance represented by the ratio \( \hat{H} / \hat{H}_0 \).

\( \hat{R} \) can be estimated for the direct and diffuse radiation and for the fraction which is reflected from the earth’s surface. Diffuse radiation is assumed to be isotropic and the earth’s albedo is kept constant for the area considered:

\[
\hat{R} = \left( 1 - \frac{\tilde{H}_\delta}{\hat{H}} \right) \tilde{R}_b + \frac{\tilde{H}_\delta (1 + \cos s)}{\hat{H}} \tag{A.2}
\]

where \( \tilde{H}_\delta \) is the daily sum of the diffuse radiation (MJ m\(^{-2}\) day\(^{-1}\)); \( \tilde{R}_b \) is the ratio of the average direct or extraterrestrial radiation on the inclined surface to that on a horizontal surface; \( s \) is the tilt of the surface from horizontal (degrees); and \( \rho \) is the surface albedo.

In order to apply this approach in complex terrain, atmospheric transmittance data have to be parameterised or interpolated for the point or the area of interest. \( \tilde{R}_b \) was set equal to the ratio of direct radiation on the slope to direct radiation on a horizontal surface. Extraterrestrial radiation could also be used for this ratio.

When Eq. (A.2) is put into the second term of Eq. (A.1) three terms are created. The first term

\[
\hat{H}_c = (\hat{H} - \tilde{H}_\delta) \tilde{R}_b \tag{A.3}
\]

represents the direct radiation \( \hat{H}_c \) (global radiation \( \hat{H} \) minus diffuse radiation \( \tilde{H}_\delta \)), which is modified by slope, aspect and sky view of the receiving surface. This modification is taken into account by \( \tilde{R}_b \). The second term
\[ \tilde{H}_d \frac{(1 + \cos s)}{2} \]  \hspace{1cm} (A.4)\\
describes the diffuse radiation taking into account the reduction of sky view by the slope. In case of a vertical wall, for instance, the cosine reduces to zero and only half the diffuse radiation is received from half of the hemisphere. The diffuse radiation is reduced not only by the slope but also by the surrounding terrain above the horizon, which results in the total reduction of the sky view expressed by \( h \), the percentage of the sky hemisphere covered by terrain. Thus the expression \( \frac{1}{2}(1 - \cos s) \) can be replaced by \( 1 - h \). The third term
\[ \tilde{H}_d \rho \frac{(1 - \cos s)}{2} \]  \hspace{1cm} (A.5)\\
describes the radiation, which is received from surfaces projecting above the horizon. This term represents in some sense the complement of Term 2 and therefore shows also the inverted sign. As mentioned earlier, the total sky view reduction \( h \) replaces the expression \((1 - \cos s)/2\).

As measurements of \( \tilde{H}_d \) are usually not available on a routine base, \( \tilde{H}_d \) is assessed from a few stations, where diffuse radiation is measured. Neuwirth (1979) used the expression
\[ \tilde{N}_H \frac{1}{2}(1 - \cos s) \]


\[ T_j = a + b z_j + cz_j^2 + \cdots \]  \hspace{1cm} (B.1)\\
where \( T_j \) is the transmittance at station \( j \); \( j \) has an index from 1 to \( n \), the total number of available stations; \( a, b, c, \ldots \) are the coefficients of the polynomial; and \( z_j \) is the elevation (m) of station \( j \).

In this work a second degree polynomial was applied. The coefficients \( a, b, c, \ldots \) were fitted by a generalisation of the least squares method for linear combinations of any functions (Press et al., 1992).

Based on the above fitted transmittance–height relationships, daily residuals \( R_j \) at all \( n \) stations can be calculated:
\[ R_j = (a + b z_j + cz_j^2 + \cdots) - T_j \]  \hspace{1cm} (B.2)\\
In case \( R_j > 0 (R_j < 0) \), transmittance at station \( j \) will be overestimated (underestimated) by Eq. (B.1). The field distribution of the residuals can be looked upon to be an approximation of the field which has its transmittance–height relationship removed.

The residuals of \( m \) selected neighbouring stations are multiplied by the weight of the inverse square of their distance to the point of interest. Therefore the inverse squared distances \( d_{ij} \) between the point of interest \( i \) and station \( j \) are summed up
\[ D_i = \sum_{j=1}^{m} \frac{1}{d_{ij}^2} \]  \hspace{1cm} (B.3)\\
The residual \( R_i \) at point \( i \) will be interpolated by weighing and summing the residuals \( R_j \) of all selected \( m \) neighbouring stations
\[ R_i = \sum_{j=1}^{m} \left[ \frac{1/d_{ij}^2}{D_i} R_j \right] \]  \hspace{1cm} (B.4)\\
via the transmittance–height relationship and the interpolated residual \( R_i \) at point \( i \) the transmittance \( T_i \) can be calculated:
\[ T_i = R_i + (a_j + b z_j + cz_j^2 + \cdots) \]  \hspace{1cm} (B.5)\\
The optimum number of neighbouring stations was found to be about 30 and the optimum power for the IDW (Inverse Distance Weighting) procedure was two.
References