Estimation of leaf area and its vertical distribution during growth period

J. Ross a,*, V. Ross a, A. Koppel b

a Tartu Observatory, 61602 Tõravere, Estonia
b Estonian Agricultural University, 51014 Tartu, Estonia

Received 19 July 1999; received in revised form 15 December 1999; accepted 17 December 1999

Abstract

A statistical interpolation method is proposed which allows calculation of the leaf area index LAI(t), the downward cumulative leaf area index \( L(z, t) \) and the canopy leaf area density \( u(z, t) \) as the functions of height and time for any day of the growth period. The method is based on three functions: probability density of stem height, interrelationship between stem height and stem foliage area, and the shape function of stem foliage vertical distribution. The method was elaborated in detail using phytometrical measurements carried out on willow coppice (Salix viminalis L.) during the growth period of 1998 in Tõravere, Estonia. The estimated values of \( u(z, t) \) and those calculated by the method differ by about 10–15%.

The interpolation model can be used when phytometrical data are available for certain days throughout the growing season. It could be applied to pure stands: young forests, orchard plantations, and maize, sorghum and cotton crops, etc. ©2000 Elsevier Science B.V. All rights reserved.

Keywords: Statistical interpolation method; Leaf area index; Vertical distribution of leaf area; Willow coppice

1. Introduction

The spatial distribution of canopy elements is an important factor in canopy–atmosphere exchange processes, and the knowledge of the vertical profile of leaf area serves as a critical input in the models of these processes. Using the simulation model ‘MAESTRO’, Wang and Jarvis (1990b) demonstrated that tree crown structure influences strongly canopy PAR absorption, photosynthesis and transpiration. The vertical distribution of canopy leaf area is characterized by the leaf area density (LAD) function \( u(z) \) \( (m^2/m^3) \) which is related to another parameter of canopy foliage area — the downward cumulative leaf area index \( L(z) \).

The latter characteristic determines leaf area in \( m^2/m^2 \) ground area which is located in the upper canopy layer higher than the level \( z \). The leaf area index LAI is single-sided leaf area in \( m^2/m^2 \) ground area of the whole canopy. The total leaf area of the individual stem in \( m^2 \) is \( S_p \). The vertical distribution of the foliage area of the individual stem is expressed by the function \( S(z) \) which determines foliage area in \( m^2/m \) crown thickness at the height \( z \).

A great number of papers have been published in which the vertical profiles of LAD of many species and canopies have been expressed showing that \( u(z) \) and \( S(z) \) are highly variable and dynamic functions. To compare LAD of individual plants with different heights or of different species at different times during the growth period, Ross and Ross (1969, 1996a), and Ross (1981) introduced the so-called shape function of the vertical distribution of the stem foliage, \( S^*(z) \). This function is determined in the interval of the
relative height $z^*$ between $z_L^*$ and 1, where the relative lower level of the canopy foliage $z_L^*$ during the growth period increases monotonically.

The function $S^*(z^*)$ expresses the shape of $S(z)$ in relative units and allows comparision of plants with a different height and at different points of time. Different shapes of $S^*(z^*)$ of many species are given by Ross (1981).

Several attempts have been made to fit $S(z)$, $S^*(z^*)$ or $u(z)$ by some commonly used distribution functions (Table 1).

To simulate the asymmetric crown shape and LAD distribution inside the crown, Cescatti (1997a) elaborated the flexible model FOREST and used it for modelling radiative transfer in discontinuous Norway spruce stand (Cescatti, 1997b).

The empirical parameters of different distribution functions are not constant, they change during the foliated period and also from year to year during forest growth.

Long-term productivity, photosynthesis etc. models require the knowledge of the input functions of $L(z)$ or $u(z)$ for the whole growth season. For their estimation, direct phytometrical measurement or indirect determination methods are needed. Such measurements are very tedious and time-consuming and can be carried out in field conditions with 2–3-weekly intervals at best. However, the time step of comprehensive biophysical exchange and productivity models is usually much shorter, 1 h or day.

Hence the vertical distribution of LAD, $u(z, t)$, and the downward cumulative leaf area index, $L(z, t)$ as the functions of height $z$ and time $t$ should be considered, and methods for estimation of $u(z, t)$ or $L(z, t)$ for different canopy types should be elaborated.

The aim of this paper is to develop a statistical interpolation method which allows determination of $u(z, t)$, $L(z, t)$ and LAI($t$) for any time throughout the whole growth period. This method is based on phytometrical measurements on sampling days $t_m$ with 2–3-weekly intervals, on estimation of $u(z, t_m)$, $L(z, t_m)$ and LAI($t_m$) for the days $t_m$ and on the calculation of $u(z, t)$, $L(z, t)$ and LAI($t$) for any day $t$ of the growth period. The method is applicable for pure stands (young forests, orchard plantations, and maize, sorghum, cotton crops, etc.) and it has been tested on the willow coppice.

2. Interpolation method

The method includes the following steps.

(a) Estimation of the stem height probability density function $f_z P(z_P)$ and fitting it by some known distribution. In many cases, the normal distribution, which is described by two parameters — mean $\langle z_P \rangle$ and standard deviation (STD) $\sigma_{z_P}$ — is applicable. For some species (e.g. willow coppice) the height distribution function in a certain growth period can be bimodal and $f_z P$ should be fitted as the superposition of two distribution functions. During the growth period, mean and STD change, can be regarded as the functions of time. Usually $\langle z_P(t) \rangle$ and $\sigma_{z_P}(t)$ are smooth functions and their fitting by means of a polynomial is possible. Thus the first step of the method allows estimation of stem height distribution function for any day of the growth period.
(b) Estimation of parameters for the correlation function between the stem leaf area \( S_P \) and stem height \( z_P \). For willow coppice the following power function can be used

\[
S_P(z_P) = b_0 z_P^b,
\]

where \( b_0 \) and \( b \) are empirical coefficients describing quite well correlation between leaf area and stem height. During the growth period, coefficients \( b_0 \) and \( b \) change and can be fitted by polynomials as the functions of time, i.e.

\[
b_0(t) = \sum_{i=0}^{n} b_{0i} t^i,
\]

and

\[
b(t) = \sum_{i=0}^{n} b_{1i} t^i,
\]

where \( b_{0i} \) and \( b_{1i} \) are polynomial coefficients.

Eqs. (1)–(3) allow calculation of the leaf area \( S_P(z_P, t) \) of the stem with the height \( z_P \) at any time \( t \) of the growth period, using data measured on sampling days.

(c) The most tedious step is estimation of the shape function of the stem leaf area vertical distribution

\[
S^*(z^*) = \frac{z_P}{S_P} S(z),
\]

where \( z_P \) is stem height and \( z^* = z/z_P \) is relative height.

The normalizing condition for \( S^*(z^*) \) is

\[
\int_{z_L}^{1} S^*(z^*) \, dz^* = 1,
\]

where \( z_L = z_L/z_P \) and \( z_L \) is the height of the canopy foliage at the bottom level. At the top of the plant, \( S(2z_P)=0 \) and \( S^*(1)=0 \).

On selected days \( t_m \), sample stems with different height (taking into account the height distribution function \( f_{zP} \)) are harvested. The stem is divided into 10 layers with the equal relative thickness \( \Delta z^* = 0.1 \). The leaf area \( S_L(z^*_L, t_m) \) in each layer is measured and the relative leaf area \( S^*(z^*_L, t_m) \) is calculated in accordance with Eq. (4). Using the data set \( S^*(z^*_L, t_m) \) of all sample stems, the shape function \( S^*(z^*, t_m) \) will be estimated fitting the \( S^*(z^*_L, t_m) \) by the polynomial

\[
S^*(z^*, t_m) = \sum_{i=0}^{1} a_i (t_m) \binom{z^*}{i},
\]

where \( a_i \) are the polynomial coefficients which change during the growth period. It should be noted that the fitting polynomial is valid in the interval \([z^*_L, 1]\) of the relative height \( z^* \).

To estimate the time dependence of \( S^* \) we used the following procedure.

On sampling days \( t_m \), using the Eq. (4), we calculate for the relative heights \( z^*_n = 0.9, 0.8, \ldots, z_L \) the values of \( S^*(z^*_n, t_m) \) and use them for constructing the function \( S^*(z^*, t) \). To determine the dependence of \( S^* \) on time at the fixed relative height \( z^*_n \) we employ approximation by the third order polynomial

\[
S^*(z^*_n, t) = \sum_{k=0}^{3} a_{k} (z^*_n)^{k} t^k.
\]

Polynomial coefficients \( a_{k} (z^*_n) \) are regarded as the functions of the relative height \( z^* \) and will be approximated by the fifth order polynomial

\[
a_{k} (z^*_n) = \sum_{i=1}^{5} a_{ik} (z^*)^i.
\]

Finally, from Eqs. (6), (6a) and (7) we obtain

\[
S^*(z^*, t) = \sum_{k=0}^{3} \sum_{i=1}^{5} a_{ik} (t)^k (z^*)^i.
\]

The last equation allows calculation of the shape function of the stem leaf area at any time of the growth period.

(d) The final step of the method is calculation of the canopy \( u(z, t), L(z, t) \) and \( LAI(t) \). Consider the canopy elemental layer with the thickness \( \Delta z \) at the height \( z \). Total leaf area in this layer consists of leaves from individual stems whose height \( z_P > z \). One stem with the height \( z_P > z \) contributes to the layer \( \Delta z \) whose leaf area, in accordance with Eq. (4), is

\[
\frac{S_P(z_P)}{z_P} S^*(z^*) \Delta z.
\]

At the height \( z \) the number of stems with the height \( z_P \) is \( N_p(t) f_{zP}(z_P, t) \). The contribution of all stems with \( z_P > z \) to canopy leaf area in the layer \( \Delta z \) is

\[
N_p f_{zP}(z_P) \frac{S_P(z_P)}{z_P} S^*(z^*) \Delta z.
\]

The last expression yields the foliage area of all stems with the height \( z_P \) in the layer \( \Delta z \).
To obtain total leaf area in the layer $\Delta z$ one must summarize all stems with a different height. Finally, the canopy leaf area density $u(z, t)$ at the height $z$ and time $t$ is

$$
u(z, t) = N_{p}(t) \int_{z_{U}}^{z_{L}} f_{z_{p}}(z_{p}, t) S_{p}(z_{p}, t) dz_{p}$$

where $z_{U}$ is the upper level of the canopy foliage and $S_{p}(z_{p}, t)$ is given by Eqs. (1)–(3), and $S^{*}(z_{p}, t)$, by Eq. (6b).

The downward cumulative leaf area index $L(z, t)$ and the leaf area index LAI(t) can be calculated by summarizing $u(z, t)$ over $z$.

In conclusion, when one has a set of required phytometrical data for sampling days $t_{m}$ within the growth period, interpolation method allows calculation of the canopy leaf area characteristics $u(z, t)$, $L(z, t)$ and LAI(t) for any day of the growth period.

3. Study area and methods

Phytometrical measurements were carried out during the vegetation period of 1998 in a willow coppice of Salix viminalis (L.), clone 78021, located close to Tartu Observatory, Tõravere (58°16’ N, 26°28’ E, 70 m asl), Estonia. For more details see (Koppel et al., 1996; Ross and Ross, 1998).

The plantation (0.2 ha) was established in May 1993 on the flat top of a small hill on light pseudopodzolic soil (Planosoil). Cuttings were planted in double rows, the distance between the rows being 0.75 and 1.25 m and the distance between the plants in the rows 0.5 m (planting density, 20,000 cuttings per ha). The azimuth angle of the rows was 75°E. After 4 years of growth the plants were harvested in winter 1997–1998, and a new rotation started in 1998. The stem density $N_{p}$ (stems per m²) decreased from 30 stems in June to 21 stems in October. After midsummer, some shoots became suppressed by competition of dominant shoots, the lowest leaves started to die, while the height of the lower boundary $z_{L}$ of the foliage tended to rise.

During the growth period phytometrical measurements were carried out on sampling days $t_{m}$ with 2–3-weekly intervals. As the first task, the height $z_{p}$ of 250–350 stems was measured, and a histogram of $z_{p}$ was constructed. Then 15–20 model stems of different height classes were selected and harvested so that the number of stems in the $z_{p}$-class corresponded to the shape of the histogram of $z_{p}$. Each sample stem was divided into 10 equal layers with the relative thickness $\Delta z^{*}=0.1$, leaf area in each layer $S(z^{*})\Delta z^{*}$ was estimated, and the respective quantities $S^{*}(z^{*})$ and $S_{p}(z_{p})$ were calculated. To estimate leaf area, a gravimetric method was used (Ross and Ross, 1996b). The mean value of specific leaf area, averaged over all canopy layers, was used for a sampling day. The values of specific leaf area varied between 53 and 62 (cm²/g fresh weight) during the growth period.

4. Result

4.1. Estimation of fitting parameters

4.1.1. Distribution function of stem height

As an example, Fig. 1 expresses the dynamics of the probability density of the stem height $z_{p}$ and its fitting by the normal distribution $f_{z_{p}}(z_{p})$. At the beginning of the vegetation period, data are sufficiently well described by the normal distribution. In the second half of summer, however, stems can be classified into two hierarchical categories — fast growing and suppressed stems. The height distribution function becomes bimodal. During the following years of growth (Ross and Ross, 1998) this tendency becomes even more pronounced.

Table 2 presents the values of the parameters of the normal distribution during the growth period of 1998. They increase monotonically with time and are fitted by the third order polynomials as a function of time:

$$\langle z_{p}(t) \rangle = 0.61 + 0.55t + 2.49 \times 10^{-2}t^{2}$$

$$-1.62 \times 10^{-3}t^{3}, \quad R^2 = 0.99,$$

$$\sigma_{z_{p}}(t) = -4.62 + 1.22t - 0.99 \times 10^{-2}t^{2} + 0.38 \times 10^{-2}t^{3}, \quad R^2 = 0.99.$$  

(9)

Time count in days ($N$) starts from 1 June.

On 23 November the distribution function $f_{z_{p}}(z_{p})$ was fitted in the unimodal way (by one normal distribution) and in the bimodal way (by superposition of two normal distributions). In Table 2 the bimodal situation on 23 November corresponds to the first (I) and the second (II) parts of the bimodal distribution curve.
Fig. 1. Probability histograms of the willow stem height $z_p$ ($\Delta z_p=0.1\,\text{m}$) on 4 days through the growing season in 1998. $f_1$ — height distribution of suppressed stems, $f_2$ — height distribution of dominating stems, $f_1+f_2$ — superposition, $f$ — all stems together.

Table 2
Dynamics of the parameters of the normal distribution — mean $\langle z_p \rangle$ and standard deviation $\sigma_{z_p}$ — used for fitting the crown height distribution function $f_{z_p}(z_p)$ during the growing season in 1998$^a$

<table>
<thead>
<tr>
<th>Date</th>
<th>$N$</th>
<th>No obs.</th>
<th>$\langle z_p \rangle$ (m)</th>
<th>$\sigma_{z_p}$ (m)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>19 June</td>
<td>19</td>
<td>335</td>
<td>0.81</td>
<td>0.16</td>
<td>0.960</td>
</tr>
<tr>
<td>1 July</td>
<td>31</td>
<td>336</td>
<td>0.94</td>
<td>0.23</td>
<td>0.948</td>
</tr>
<tr>
<td>21 July</td>
<td>51</td>
<td>318</td>
<td>1.35</td>
<td>0.37</td>
<td>0.912</td>
</tr>
<tr>
<td>11 August</td>
<td>72</td>
<td>313</td>
<td>1.68</td>
<td>0.48</td>
<td>0.757</td>
</tr>
<tr>
<td>26 August</td>
<td>87</td>
<td>261</td>
<td>1.92</td>
<td>0.51</td>
<td>0.713</td>
</tr>
<tr>
<td>9 September</td>
<td>101</td>
<td>261</td>
<td>2.01</td>
<td>0.54</td>
<td>0.642</td>
</tr>
<tr>
<td>16 September</td>
<td>108</td>
<td>347</td>
<td>2.08</td>
<td>0.61</td>
<td>0.514</td>
</tr>
<tr>
<td>23 November</td>
<td>176</td>
<td>336</td>
<td>1.78</td>
<td>0.87</td>
<td>0.380</td>
</tr>
<tr>
<td>Suppressed I</td>
<td>–</td>
<td>–</td>
<td>1.33</td>
<td>0.47</td>
<td>–</td>
</tr>
<tr>
<td>Fast growth II</td>
<td>–</td>
<td>–</td>
<td>2.53</td>
<td>0.31</td>
<td>–</td>
</tr>
</tbody>
</table>

$^a N$ — number of days counted from 1 June.

It should be noted that since the role of suppressed shoots in the canopy’s radiative transfer processes as well as in photosynthesis and other processes is rather small, then instead of bimodal height distribution, the averaged distribution as a rougher approximation can be used.

The top level of the canopy foliage $z_U(t)$ can be considered as a random quantity. We will determine it by

$$z_U(t) = \langle z_p(t) \rangle + 2\sigma_{z_p}(t).$$

The lower level of the canopy foliage, $z_L$, increasing monotonically with time, was fitted by

$$z_L(t) = 3.37 \times 10^{-1} - 1.29 \times 10^{-2}t$$

$$+ 1.42 \times 10^{-4}t^2, \quad R^2 = 0.97$$

Fig. 2 expresses the dynamics of some willow coppice parameters during the growth period of 1998.

Thus when one has fitting polynomials for the parameters of the normal distribution, $\langle z_p(t) \rangle$ and $\sigma_{z_p}(t)$ (Eq. (9)), the distribution of the stem height
4.1.2. Correlation between stem foliage height and foliage area

The correlation was estimated seven times during the growth period of 1998, \( R^2 \) being between 0.89 and 0.98. In Fig. 3, as an example, the curves \( S_P(z_P) \) express the correlation for June, August and October.

During the growth period, \( b_0 \) and \( b \) change (Fig. 4) and can be fitted by the polynomials

\[
b_0(t) = 7.07 \times 10^{-2} - 4.34 \times 10^{-4} t, \\
R^2 = 0.94;
\]

\[
b(t) = 3.44 - 2.48 \times 10^{-2} t + 1.53 \times 10^{-4} t^2, \\
R^2 = 0.63.
\]

4.1.3. Estimation of the shape function of stem foliage vertical distribution

On sampling days \( t_{mk} \), the leaf area in different layers of sample stems was measured, and \( S^s(z^s_{1k}, t_k) \) was calculated using Eq. (6). As an example the values of \( S^s(z^s_{1k}, t_k) \) for four sampling days in different months are presented in Fig. 5. Scattering of points characterizes the error made in determination of the shape function. Typically, in midsummer months, a more rapid increase is seen in the shape function \( S^s(z^s_{1k}, t_k) \) with \( z^s \), starting from \( z^s = 0.6 \). This may be partly caused by the sprouting of small lateral branches on the stem and partly by fact that the largest leaves in the foliage layer are located within the relative height \( \Delta z^s = 0.8 - 0.9 \). In accordance with Eq. (6b) \( S^s(z^s_{1k}, t_k) \) as the function of the relative height \( z^s_{1k} \) was approximated by the fifth order polynomials whose coefficients \( a_{ijk} \) are given in Table 3.
Fig. 5. Shape function $S^*(z^*)$ of stem foliage vertical distribution throughout the growth period in 1998. Points correspond to measured values, curves — to fitted polynomials.

The fitting polynomial corresponds to some ‘mean’ stem foliage vertical distribution averaged over the whole shoot height range. During the growth period, maximum leaf area density is located at $z^* = 0.8$, and the maximum value of $S^*$ fluctuates between 1.5 and 2.5.

Having a set of coefficients $a_{ik}$ and applying Eq. (4) to each day and each relative height $z^*$ it is possible to calculate the values of the shape function $S^*(z^*, t)$.

4.1.4. Calculation of leaf area density and leaf area index

Finally, one can calculate the canopy leaf area density, $u(z, t)$ as the function of height and time. In accordance with Eq. (8) the leaf area density $u(z, t)$ is determined by three functions: distribution of stem height, interrelationship between stem height and stem total leaf area, and the shape function of stem foliage vertical distribution.

The downward cumulative leaf area index $L(z)$, estimated for different days of the growth period, is given in Fig. 6.

5. Discussion

The presented interpolation method has several constraints. In the first half of the growing season, stem height distribution was approximated by the normal distribution, sample size being 250–350 stems. However, agreement was not the best (see Table 2). Conjecturing that the sample size was too small, we doubled the number of samples during a special experiment, which still did not improve agreement essentially.

In the second half of the growing period, stem height distribution function has a tendency to transform from unimodal to bimodal. This is caused by competition between stems and by development of stem hierarchy (e.g. suppressed and dominant stems). As we have demonstrated earlier for the second year of growth, height distribution becomes bimodal and can be fitted by superposition of two normal distributions (Ross and Ross, 1998). Actually, the share of short stems in total canopy leaf area is not very large, and more careful fitting is essential in the range of long stems.

Correlation between stem height and stem foliage area is good during the whole growth period. The empirical formula $S_P = b_0 z^*_P$ used is a good approx-
Table 3
Empirical coefficients $a_{ik}$ in Eq. (6b) for calculation of the shape function of the stem foliage vertical distribution $S^*(z^*, t)$ at different relative heights $z^*_n$

<table>
<thead>
<tr>
<th>$z^*_n$</th>
<th>$a_{i0}$</th>
<th>$a_{i1}$</th>
<th>$a_{i2}$</th>
<th>$a_{i3}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>$-6.35 \times 10^{-1}$</td>
<td>$8.06 \times 10^{1}$</td>
<td>$-1.1 \times 10^{-3}$</td>
<td>$3.95 \times 10^{-6}$</td>
<td>0.894</td>
</tr>
<tr>
<td>0.8</td>
<td>$6.55 \times 10^{-1}$</td>
<td>$4.09 \times 10^{-2}$</td>
<td>$-4.76 \times 10^{-4}$</td>
<td>$2.11 \times 10^{-6}$</td>
<td>0.975</td>
</tr>
<tr>
<td>0.7</td>
<td>1.45</td>
<td>$-4.05 \times 10^{-1}$</td>
<td>$3.86 \times 10^{-5}$</td>
<td>$1.43 \times 10^{-6}$</td>
<td>0.994</td>
</tr>
<tr>
<td>0.6</td>
<td>1.58</td>
<td>$-1.66 \times 10^{-2}$</td>
<td>$3.00 \times 10^{-4}$</td>
<td>$-1.26 \times 10^{-6}$</td>
<td>0.997</td>
</tr>
<tr>
<td>0.5</td>
<td>1.53</td>
<td>$-2.23 \times 10^{-2}$</td>
<td>$4.05 \times 10^{-4}$</td>
<td>$-2.50 \times 10^{-6}$</td>
<td>0.984</td>
</tr>
<tr>
<td>0.4</td>
<td>1.72</td>
<td>$-3.33 \times 10^{-2}$</td>
<td>$5.71 \times 10^{-4}$</td>
<td>$-3.44 \times 10^{-6}$</td>
<td>0.992</td>
</tr>
<tr>
<td>0.3</td>
<td>$6.56 \times 10^{-1}$</td>
<td>$3.24 \times 10^{-2}$</td>
<td>$-7.82 \times 10^{-4}$</td>
<td>$3.88 \times 10^{-6}$</td>
<td>0.981</td>
</tr>
<tr>
<td>0.2</td>
<td>2.85</td>
<td>$-9.14 \times 10^{-2}$</td>
<td>$9.63 \times 10^{-4}$</td>
<td>$-3.32 \times 10^{-6}$</td>
<td>0.999</td>
</tr>
</tbody>
</table>

Estimation ($R^2=0.86/0.98$) and the seasonal change of the empirical parameters $b_0$ and $b$ can be successfully fitted by polynomials, $R^2$ being 0.94 and 0.89, respectively.

Estimation of the shape function of the vertical distribution of the stem foliage is less exact. When defining the shape function we assume that the shape function of the vertical distribution of the stem foliage is similar for all stems. However, this assumption is not exact for the second half of the growth period. Short branches with a length of 3–8 cm develop in the upper part of long stems causing additional increase in leaf area density at the relative height of about $z^*=0.7–0.9$.

This may increase dispersion of the experimental points in Fig. 4. One source of errors is the limited number of sampled stems. Practically, we were able to measure leaf area distribution for 15–30 stems on sampling days, which should represent all height classes of $z_P$. However, in radiative transfer and photosynthesis, leaf area distribution in upper canopy layers plays a more important role and hence the share of dominant trees is larger.

The success of the method depends also on the fitting accuracy of the temporal course of model parameters during the growing period. Increasing the number of sampling days improves the situation. Abnormal weather conditions (drought, late frosts, etc.) may affect leaf and shoot growth and cause a non-smooth change of model parameters. Under such conditions input parameters should be estimated at shorter intervals.

It is difficult to analyse the accuracy of the proposed interpolation method. We made an attempt to test its accuracy by using the data of phytometric measurements for 24 August, which were not included in the calculation of the coefficients of the method. For this day the functions $u(z)$, $L(z)$ and the value of LAI were estimated on the basis of phytometrical measurements as well as by using the interpolation method. The estimated and calculated values of LAI were 3.6 and 4.1, respectively (Fig. 7). In the upper part of the canopy, which is more important considering light attenuation, the results obtained by interpolation method are in good agreement with the direct measurements of LAI. Differences in the leaf area density $u(z,t)$ did not exceed 10–15%.
Fig. 7. Vertical distribution of (a) the leaf area density $u(z)$ and (b) the downward cumulative leaf area index $L(z)$ on 24 August 1998. 1 — experimental data, 2 — calculated using interpolation method.

6. Concluding remarks

Long-term calculation of canopy photosynthesis, transpiration, growth and other ecophysiological processes need a daily input parameter of the vertical distribution of canopy leaf area. Its direct measurement is very tedious and time consuming and therefore corresponding phytometrical measurements are usually carried out after 15–20 days.

This paper presents a statistical interpolation method which allows calculation of the function $u(z)$ as well as the downward cumulative leaf area index $L(z)$ and the leaf area index LAI for every day during the growth period when necessary phytometrical measurements are performed on sampling days. The method is based on the estimation of three statistical phytometrical functions: (i) probability density function of stem height, (ii) interrelationship between stem height and stem foliage area, and (iii) shape function of stem foliage vertical distribution. The method was applied for willow coppice in Estonia in the growth period of 1998.

Due to the great variability of the canopy leaf area density $u(z)$, the error of our method has been estimated at about 10–15%. Among different input functions the greatest error is caused by the shape function of stem foliage vertical distribution. Increasing the number of sampling days during the growth period the error of method decreases. The method uses time fitting polynomials up to the fifth order, which does not present a problem for a PC owner.

To our knowledge, the present study is the first attempt to elaborate a statistical interpolation method to be used for calculation of plant canopy foliage area and its vertical distribution for any day of the growing season.

The results of the study can be employed as the input parameters of comprehensive models of canopy photosynthesis and productivity. The applicability of the method for other plant canopy types needs further research.

7. List of symbols

- $b_0(t)$: empirical constant (m)
- $b(t)$: empirical constant (–)
- $f_{zp}(z_p, t)$: stem height probability density distribution function
- $L(z, t)$: downward cumulative leaf area index ($m^2 m^{-2}$)
- $LAI(t)$: leaf area index ($m^2 m^{-2}$)
- $N$: number of days counted from 1 June (day)
- $N_p$: stem density ($stem m^{-2}$)
- $S(z)$: vertical distribution of foliage
area of individual stem (m² m⁻¹ per stem)

\( S_P(z_P) \)
total leaf area of individual stem at height \( z_P \) in m² (m² per stem)

\( S^*(z^*) = \frac{z^*}{z_P} S(z) \)
shape function of the vertical distribution of the stem foliage (–)

\( u(z,t) \)
single side area density (m² m⁻³)

\( z_L \)
bottom level of canopy foliage (m)

\( z_P \)
stem height (m)

\( \langle z_P(t) \rangle \)
top level of canopy foliage (m)

\( z_U \)
mean stem height (m)

\( z^* = z/z_P \)
relative height (–)

\( z^*_L = z_L/z_P \)
relative bottom level of canopy foliage (–)

\( \sigma^*_L \)
standard deviation of stem height (–)

Acknowledgements

The research was supported by the ESF Grant No. 2668. The authors thank Mrs. Eneken-Mall Ross for assistance in fieldwork and data processing, Mrs. Viivi Randmets for technical aid, Mrs. Ester Jaigma for revision of the English text and Dr. Madis Sulev and Matti Mõttus for useful comments. The regional editor of the journal Dr. J.B. Stewart and the anonymous reviewers are highly acknowledged for their comments and suggestions in improving the manuscript.

References