MIXLIGHT: a flexible light transmission model for mixed-species forest stands

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Abstract

We describe the calibration and structure of a multi-species, two-scale light transmission model, and demonstrate its effectiveness for predicting instantaneous light availability at the stand scale across a wide range of forest stand compositions. The model, MIXLIGHT, calculates light transmission through the forest overstory at the stand or microsite scale using standard forest inventory data and two other parameters, foliage area density and foliage inclination. Since MIXLIGHT simulations on both scales are based on a list of individual tree characteristics, it allows for simple manipulation of stand structure to study the effects of silvicultural options on light availability. The two scales allow the input of various kinds of data, allowing predictions from data of different sources and completeness. A simple and rapid method of calibrating the foliage parameters for the species of interest is presented, using measurements of direct-beam light transmission measurements made in the shadows of newly isolated trees. In an independent validation, MIXLIGHT predicted light transmission at the stand level closely for 17 forest inventory plots with a wide range of density and species composition, during leaf-on and leaf-off seasons and under sunny and cloudy conditions. A sensitivity analysis indicated that the influence of the parameters in the model on stand level light transmission predictions was (highest to lowest): foliage area density, crown radius, crown length, and foliage inclination. Nonetheless, a test of the common assumption that the foliage is spherically inclined caused significant underestimation of light transmission. With this flexibility and demonstrated accuracy, we believe MIXLIGHT will provide an accessible and effective tool for forest stand management and regeneration modeling. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Boreal forest trees; Abies balsamea; Betula papyfera; Populus balsamifera; Populus tremuloides; Picea glauca; Pinus contorta; PAR; Light penetration; Leaf area density; Leaf angle distribution; Spatial scale

1. Introduction

The characterization of light in forests has been a key focus of work on forest community dynamics and regeneration. Numerous studies have established the dependence of tree and understory growth on available light in stands (Messier et al., 1989; Lieffers and Stadt, 1994) and microsites (Poulson and Platt, 1989; Alaback and Tappeiner, 1991; Klinka et al., 1992; McLure and Lee, 1993; Poage and Peart, 1993; Runkle et al., 1995). However, adequate measurement of light is time-consuming due to the heterogeneity in the arrangement and composition of the canopy, diurnal and seasonal changes in solar position and cloudiness.
Spatial approaches group foliage into identifiable objects such as individual rows, crowns, crown layers or shoots which are described geometrically as cylinders, ellipsoids, quarter ellipsoids or disks with measurements to locate these objects in three-dimensional space. Light penetration through these objects is modeled spatially and temporally, given the solar geometry for that latitude and time, and the position and size parameters of the geometric shapes. Pukkala et al. (1993) tested a crown model of this type in a pure Scots pine stand, Bartelink (1998) and Brunner (1998) tested somewhat similar models in pure Douglas-fir and beech stands, and Canham et al. (1994) and Ter-Mikaelian et al. (1997) developed models for mixed-species stands. These use size- or species-specific estimates of the projected leaf area density within the crown or similar extinction coefficients to compute through-crown light transmission. They do not account for light reflection, but are able to predict the spatial distribution of light at microsites throughout the stand with reasonable accuracy. Lastly, a number of models combine the spatial features of the geometric approach with a homogeneous layer model to deal with light reflection (Grace et al., 1987; Wang and Jarvis, 1990; Cescatti, 1997a). These have shown good spatial accuracy in pure stands, but require a suite of detailed input data.

Tools for modeling light are thus well-developed. Applications, however, have been limited by model inflexibility or the quantity and type of data required. Calibration has been particularly difficult. In MIXLIGHT, we aimed to build a flexible, accessible simulator which would allow light penetration theory to be applied to existing forest inventory data with minimal calibration and make light availability predictions for a wide range of forest stands. We designed our model to operate at either the microsite or stand scale to fit the data available and the spatial precision required. MIXLIGHT can generate light predictions from either a complete tree list or a minimal list of tree species and stem diameters. The complete list includes species, diameter, height, height to the base of the live crown, crown radius, and tree location coordinates. Where some of the crown size information is missing,
regressions are used to determine these parameters. A separate table of foliage area density, foliage inclination parameters and the crown shape for each species and/or each crown-class is required. These can be obtained following the calibration described here or from other sources. To account for exposure effects, stand slope and aspect are also required. Dates for deciduous leaf-on and leaf-off, and start/end-dates of the growing season can be added to simulate the seasonal light regime. This is critical for accounting for the leaf-off shoulder seasons in the spring and fall which are important for carbon fixation in wintergreen, evergreen and ephemeral species (Hutchison and Matt, 1977; Man and Lieffers, 1997). In large inventoried plots with stem location data available, light modeling can be carried out at the microsite scale. When the stem position data are missing, the inventoried stand is small and dominated by edge effects, or if only a stand-level estimate of light is required, MIXLIGHT will operate at the stand scale, using generalized canopy parameters calculated from the tree list. Since canopy parameters can also be obtained from remote-sensing or canopy analysis instruments, the capability to predict stand-level light directly from canopy leaf area index and inclination information is also possible.

In this paper, we present the structure of MIXLIGHT, describe a simple method of calibrating the foliage area density and inclination coefficients, and validate its stand-level predictions across a wide range of stand types in the boreal mixedwood forest of central Alberta, Canada. The importance of the key parameters of MIXLIGHT for stand-level estimation are evaluated in a sensitivity analysis.

2. Methods

2.1. Overview

An ideal forest light transmission model would integrate the light-attenuating effect of all foliage positioned between the sky and the measurement point. Since this is seldom possible, a common modeling approach is to sample the intervening foliage by tracing numerous rays from starting points scattered across the sky hemisphere to the measurement point \( (P_0) \) within the stand (Fig. 1A, Canham et al., 1994; Bartelink, 1998; Brunner, 1998). MIXLIGHT takes this approach, and uses the light penetration function (Eq. (1)) to calculate transmission along each of these rays based on the density of the foliage through which the ray passed, the degree to which this foliage area is oriented toward the light source, and the length of the ray’s path \((S)\) through individual tree crowns (Fig. 1B).
or the stand canopy (Fig. 1C). These ray transmission values are weighted by the strength of the light source and averaged to give an estimate of the integrated light transmission (the ratio of below to above canopy light) at the measurement point. The individual crown approach (Fig. 1B) accounts for horizontal variation in overstory foliage and produces microsite light predictions, while the canopy scale approach (Fig. 1C) distributes foliage randomly throughout the canopy and produces stand-level predictions. Calibration of the foliage area density and foliage projection parameters is accomplished by inverting the modeling process for the simple case of sunlight shining through the crown of an isolated tree (Fig. 2). This is repeated at several times of day at different sun angles, so that the foliage area density and foliage projection parameters can be determined. Simulations are then carried out using these species-specific foliage parameters with the tree lists from the sites of interest.

2.2. Theory

MIXLIGHT models the transmission of photosynthetically active radiation (PAR: 400–700 nm). We use ‘light’ as a synonym for PAR, though visible light is a slightly wider waveband. Foliage is taken to include leaves, branches and stem that are contained within the crown or canopy volume since these all attenuate light. Each fragment of foliage area is assumed to transmit and reflect no light so that a thin ray of light incident on the forest is either completely absorbed or completely transmitted, depending on whether it hits or misses foliage. If the foliage is randomly distributed within the crown or canopy region, the probability that the ray will not be intercepted \( P(0) \) is given by the Poisson distribution as shown in Eq. (1).

\[
P(0) = T = \frac{q[Z, \alpha]}{q_{0}[Z, \alpha]} = e^{- \sum_{j=1}^{n} G[Z, \chi_j] S_j F_j}
\]

(1)

\[
q[Z, \alpha] = q_{0}[Z, \alpha] e^{- \sum_{j=1}^{n} G[Z, \chi_j] S_j F_j}
\]

(2)

Eq. (2) is rearranged to yield light flux density \( q[Z, \alpha] \) rather than transmission \( T \), which is a more convenient form for modeling the diffuse and seasonal light flux (see Eq. (4)). In Eqs. (1) and (2), \( q[Z, \alpha] \) is the flux density \( (\mu\text{mol m}^{-2} \text{s}^{-1}) \) of a ray of light through a surface perpendicular to the ray direction, originating from a distant point source defined by its zenith \( Z \) (angle from vertical) and azimuth \( \alpha \) (compass orientation), \( q_{0}[Z, \alpha] \) is the light flux density \( (\mu\text{mol m}^{-2} \text{s}^{-1}) \) incident on the canopy from the same source, \( G[Z, \chi_j] \) (unit-less) is the projection of one unit of foliage area perpendicular to the direction of the light source, \( \chi_j \) is a parameter describing the foliage area inclination (see below), \( S_j \) is the path length (m) through the light-absorbing region, and \( F_j \) is the foliage area density \( (\text{m}^2 \text{m}^{-3}) \). By analogy with...
one ray is traced). In Eq. (4),

\[
\frac{T_{\text{direct}}}{Q_{0,\text{direct}}} = \frac{\cos Z q[Z, \alpha_s]}{\cos Z q_0[Z, \alpha_s]} \tag{3}
\]

\[
T_{\text{diffuse or seasonal}} = \frac{Q_{\text{diffuse or seasonal}}}{Q_{0,\text{diffuse or seasonal}}}
= \int_0^{\cos Z=1} \cos Z \, q[Z, \alpha] \, d\cos Z
= \int_0^{\cos Z=1} \cos Z \, q_0[Z, \alpha] \, d\cos Z
\approx \sum_{c=1}^{nc} \sum_{a=1}^{na} \left[ \cos Z_c \, q[Z_c, \alpha_a] \right]
= \sum_{c=1}^{nc} \sum_{a=1}^{na} \left[ \cos Z_c \, q_0[Z_c, \alpha_a] \right] \tag{4}
\]

For instantaneous estimates of light transmission, Eq. (2) is applied once for direct-beam radiation transmission \(T_{\text{direct}}\) using the sun’s position angles \((Z_c, \alpha_s)\); Eq. (3)), and multiple times using angles identifying the centers of equal-area sectors of the sky (i.e. sky-elements of equal solid angle) for integrating the transmission of diffuse or seasonal skylight (Eq. (4)). Note that Eq. (3) simplifies to Eq. (1) since only one ray is traced). In Eq. (4), \(c=1, \ldots, nc\) is the index for the zenith angles \((Z_c)\) which are divided into \(nc\) equal increments of the cosine of the zenith from \(1 (Z_c=0^\circ)\) to \(0 (Z_c=90^\circ)\) to achieve equal-area sampling of the sky, and \(a=1, \ldots, na\) is the index for the azimuth angles \((\alpha_a)\) which are divided into \(na\) equal increments from 0 to 360°.

For the Beer–Lambert law (Swinehart, 1962), we consider the product \(G(Z, \alpha)S_jF_j = -(\ln T)\) as the absorbance for the region \(j\). Since absorbances are additive, the summation in Eqs. (1) and (2) deals with any number of light absorbing regions \((n)\) that intersect the light path, from a single canopy \((n=1, \text{Fig. 1C})\) to an array of individual crowns \((n=\text{number of trees, Fig. 1B})\).

**2.3. Study area**

Calibration and validation of MIXLIGHT were performed in a number of stands across west-central Alberta (53°30’–54°10’N, 114°50’–118°W). Mean annual temperature in this region is 0.9–2.6°C, and precipitation is 530–560 mm per annum, approximately 60% of it falling during the growing season (May–August) (Anonymous, 1982a, b). All stands were upland sites with submesotrophic to mesotrophic nutrient status and mesic to hygic ecological moisture regime (Anonymous, 1994).

**2.4. Model calibration**

**2.4.1. Foliage area density**

The foliage area density of various tree species was determined on newly-isolated residual trees on clearcut edges and in partial-cut stands. We visited 10 cutblocks from late June to August in 1996 and 1997 for leaf-on calibration, and in April 1997 for leaf-off measurements. Before cutting, the sites were dominated by either aspen (Populus tremuloides Michx.), white spruce (Picea glauca (Moench) Voss) or mixtures of these, balsam fir (Abies balsamea (L.) Mill.), balsam poplar (Populus balsamifera L.), and white birch (Betula papyrifera Marsh.). Trees were measured as soon as possible after logging (1 week–3 months) so they still retained most of the leaf and shape characteristics they had while growing in the stand.

Light measurements were taken with an 80 sensor linear radiometer (Model SF-80, Decagon Devices, Pullman, WA) which measures PAR in quantum units (\(\mu\text{mol m}^{-2} \text{s}^{-1}\)). Measurements were made during periods of direct sun when the trees cast a clearly delineated shadow on the ground. At each sample tree, crown width, total shadow length \(h_i^t\) (stem base to shadow apex), length to the base of the shadow’s live-crown \(h_i^l\) and time of measurement were recorded first for crown size determination (Fig. 2A). The light sampling strategy depended on the crown shape. Parallel transects at 1–2 m spacing (wider spacing for larger crowns) perpendicular to the central axis of the crown (see Fig. 2A) were used for longer crowns. Radial transects at 45° intervals originating from the center of the crown shadow.
were used to sample ellipsoidal crowns (Fig. 2B). To compensate for the horizontal movement of the sun (0.25° min⁻¹), the origin of each transect was aligned with the central axis of the shadow before commencing each transect. Vertical changes in the sun’s position (<0.1° min⁻¹) were small enough to be ignored during the sampling period. Where necessary, ground slope and aspect were measured to later transform the transect positions to horizontal coordinates. Every 50 cm from the transect origin, the radiometer wand has held at right angles to the transect (Fig. 2C), six light measurements taken over 6 s and the average recorded. Sampling extended 100–150 cm past the edge of the shadow to provide at least one unshaded light measure \((Q_0)\) for that transect. Length from the transect origin to the shadow edge \((d, \text{Fig. } 2\text{C})\) was recorded for crown size determination. Total \((Q_0)\) and diffuse light \((Q_{\text{diffuse}})\) were measured in unshaded sunlight adjacent to the crown shadow immediately before and after sampling each tree by exposing then shading a single sensor of the radiometer with a small disk at 2 m distance, and recording the output from that sensor. Sampling time depended on tree size, but was usually less than 20 min per tree.

An appropriate geometric shape (ellipsoid, paraboloid, cone, cylinder, or rocket (a cone of 25% of the crown length perched on a cylinder 75% of the crown length)) was chosen for the crown of each species (Table 1). To obtain the dimensions of these shapes (crown length (tree height minus live-crown height) and radius), actual height and live-crown height measurements were adjusted for the solar zenith angle at the time of measurement (Eq. (5)).

\[
\text{actual length} = \frac{\text{shadow length}}{\tan Z_s} \quad (5)
\]

We assumed the crowns were circular in cross-section so that their crown radius was not affected by solar angle. The solar zenith and azimuth at the time, day, latitude and longitude of measurement were obtained using the formulae of Iqbal (1983).

The transmission of direct beam light \((T_{\text{direct}})\) was calculated by subtracting the diffuse component \((Q_{\text{diffuse}})\) from the light measured \((Q)\) at the point and from the light measured outside the crown shadow \((Q_0)\), then taking the ratio of these two (Eq. (6)).

\[
T_{\text{direct}} = \frac{Q - Q_{\text{diffuse}}}{Q_0 - Q_{\text{diffuse}}} \quad (6)
\]

The diffuse light actually decreases for measurements nearer the tree since its stem and crown capture more of the diffuse light. The effect on direct-beam transmission and the resulting increase in the foliage density estimate that results from this is small, however, because the direct-beam flux is so much greater (>4 times) than the diffuse. The path length of the sun through the crown \((S, \text{Fig. } 2\text{B})\) was estimated by solving for the intersection of the ray vector described by the measurement point and the sun angle with the geometric crown. Equations for determining the two intersection points \((P_1(x_1, y_1, z_1)\) and \(P_2(x_2, y_2, z_2)\)) for a ray passing through the various crown shapes are presented in Appendix A. The distance formula (Eq. (7)) was then applied to the intersection points to obtain the path length.

\[
S = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad (7)
\]

The projection of a unit of foliage area in the sun’s direction \((G[Z_s, \chi_j])\) was calculated using Eq. (9), as described below. Lastly, the foliage area density \((F_j)\) was calculated from these variables by inverting Eq. (1) (see Eq. (8)).

\[
F_j = \frac{-\ln T_{\text{direct}}}{G[Z_s, \chi_j]} S \quad (8)
\]

Foliage densities were averaged over all measurement points within each tree’s shadow. Trees were then averaged within a species to yield that species’ average foliage area density. Estimates calculated from radial transects were weighted by the length from the transect origin to the measurement point to even out the area sampling rate.

### 2.4.2. Foliage inclination and projection

Since the foliage area is not perpendicular to every angle it is viewed from, nor to each light source, it is necessary to correct the apparent projected area to actual (one-sided) surface area. Campbell and Norman (1989) demonstrated that a simple and effective method of modeling the inclination of many leaves is to assume the foliage is oriented as if it was spread over the surface of an ellipsoid. If the orientation with regard to azimuth is assumed to be random, the ellipsoidal inclination distribution can be modeled with one
<table>
<thead>
<tr>
<th>Season</th>
<th>Species</th>
<th>No. of trees</th>
<th>DBHb (cm)</th>
<th>Height (m)</th>
<th>Crown characteristics</th>
<th>Foliage area density (m² m⁻²)</th>
<th>Foliage inclination (ϕf)³</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Length (m)</td>
<td>Radius (m)</td>
<td>Shape</td>
</tr>
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<td>Leaf-on</td>
<td>Balsam fir</td>
<td>3</td>
<td>12–23 (17)</td>
<td>8.7–15.5 (11.6)</td>
<td>3.6–7.8 (5.6)</td>
<td>1.5–2.1 (1.8)</td>
<td>Paraboloid</td>
</tr>
<tr>
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<td>Overstory white spruce</td>
<td>5</td>
<td>39–67 (48)</td>
<td>20.1–28.9 (26.0)</td>
<td>5.9–12.1 (9.7)</td>
<td>1.2–2.2 (1.7)</td>
<td>Rocket</td>
</tr>
<tr>
<td></td>
<td>Understory white spruce</td>
<td>5</td>
<td>13–20 (16)</td>
<td>6.7–11.8 (9.4)</td>
<td>3.4–5.9 (4.6)</td>
<td>1.4–2.1 (1.7)</td>
<td>Rocket</td>
</tr>
<tr>
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<td>Lodgepole pine</td>
<td>6</td>
<td>18–28 (23)</td>
<td>18.2–23.7 (20.9)</td>
<td>2.1–4.5 (3.3)</td>
<td>0.7–1.5 (1.2)</td>
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</tr>
<tr>
<td></td>
<td>Aspen</td>
<td>11</td>
<td>25–62 (43)</td>
<td>21.3–28.5 (24.0)</td>
<td>3.4–8.9 (6.5)</td>
<td>1.5–4.1 (3.0)</td>
<td>Ellipsoid</td>
</tr>
<tr>
<td></td>
<td>Birch</td>
<td>7</td>
<td>8–31 (15)</td>
<td>9.1–24.0 (13.2)</td>
<td>2.8–10.3 (5.8)</td>
<td>1.2–3.5 (2.0)</td>
<td>Ellipsoid</td>
</tr>
<tr>
<td></td>
<td>Balsam poplar</td>
<td>6</td>
<td>12–47 (39)</td>
<td>8.5–27.1 (21.8)</td>
<td>2.9–10.0 (6.3)</td>
<td>1.2–3.1 (2.3)</td>
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<td>22.3–26.3 (23.6)</td>
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<td>1.6–3.3 (2.6)</td>
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<tr>
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<td>Birch</td>
<td>5</td>
<td>7–31 (20)</td>
<td>8.2–23.6 (14.9)</td>
<td>3.2–9.3 (5.7)</td>
<td>1.9–3.5 (2.4)</td>
<td>Ellipsoid</td>
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<td>2.7–5.0 (4.1)</td>
<td>0.8–3.0 (2.0)</td>
<td>Ellipsoid</td>
</tr>
</tbody>
</table>

² Data shown are minimum–maximum (mean).
³ Diameter at breast height (1.3 m).
⁴ Parameter describing the relative projection of leaf area in the vertical versus the horizontal direction assuming the leaves follow an ellipsoidal inclination distribution (Campbell and Norman, 1989). Calibrated for the dominant trees, aspen (leaf-on, n=10), overstory (n=10) and understory white spruce (n=10), only using the data of Constand (1995); all other species are assigned a value of 1 (foliage area inclined as if covering the surface of a sphere).
parameter, the ratio of vertical to horizontal projections, \( \chi_j \). When \( \chi_j = 1 \), the foliage inclines as if covering a sphere: smaller values indicate mostly vertical foliage area orientation, larger \( \chi_j \) indicate mostly horizontal foliage. The projection of an ellipsoid, having a surface area of 1, perpendicular to the light source zenith is given by Eq. (9) (Campbell and Norman, 1989).

\[
G[Z, \chi_j] = \frac{\sqrt{\chi_j^2 \cos^2 Z + \sin^2 Z}}{\chi_j + 1.774(\chi_j + 1.182)^{-0.733}} \tag{9}
\]

Campbell and Norman (1989) also demonstrated how the inclination \( \chi_j \) parameter can be obtained by measuring the gap fraction at a number of zenith angles (\( Z \)), then solving for foliage area and foliage inclination using inversion techniques. We used direct sunlight transmission as a measure of gap fraction in individual crowns then corrected for any change in the path length (\( S \)) with solar zenith angle (\( Z_s \)) to yield the dependent variable \(-\ln(T_{\text{direct}}/S)\). We found the inclination and foliage area density \( F_j \) parameters that offered the best fit to Eq. (10) by nonlinear regression of \(-\ln(T_{\text{direct}})/S\) on the independent variable \( Z_s \).

\[
-\ln \frac{T_{\text{direct}}}{S} = G[Z_s, \chi_j]F_j \tag{10}
\]

A similar approach suing sunlight transmission approach to obtaining \( G \) for particular solar angles when \( F \) is known is demonstrated by Oker-Blom et al. (1991).

Direct-beam light transmission data taken at a number of solar angles were obtained from a study by Constabel (1995). She sampled 10 aspen, 10 overstory white spruce and 10 understory white spruce trees on 22 June 1994 in a partially-cut mixed forest in central Alberta (53°40'N, 116°40'W). Each crown was sufficiently isolated that it cast a distinct shadow through most of the day. Shadows were sampled during five 1 h periods with mean solar zenith angles of 30.7, 32.9, 35.7, 51.2 and 65.3°. At each sampling time, nine light measurements \( (Q) \) were taken in the crown shadow of each tree, with the 80 cm radiometer wand held parallel to the stem. Three measurements were taken at random positions in the upper third of the crown, three in the middle, and three in the lower third, then averaged for each tree. Total \( (Q_0) \) and diffuse flux density \( (Q_{\text{diffuse}}) \) were measured in full sun adjacent to the crown shadow immediately before and after sampling each tree as outlined for the foliage area density measurements above. Tree height, \( h_l \), live-crown height, \( h_{lc} \), and crown width, \( cw \), were determined with a clinometer and measuring tape.

With this data, we calculated direct sunlight transmission through the whole crown by subtracting the diffuse component as above (Eq. (6)). Since the measurement points were not mapped spatially (to speed sampling), we had to simplify the crown shape. We first determined the volume of each crown as either an ellipsoid (aspen) or a rocket (white spruce) using the crown length and width to establish the axis lengths. We then redefined the crowns as rectangular solids with one side facing the sun, one crown length deep and of equal length and width such that they contained the same volume as the original shape.

\[
S = \frac{w}{\sin Z_s} \tag{11}
\]

The path length of light through the crown \( (S) \) at the solar zenith \( (Z_s) \) was calculated using the width \( (w) \) of the rectangle (Eq. (11)). The negative logarithm of direct sunlight transmission was then divided by \( S \), to yield the dependent variable in Eq. (10). We used the derivative-free nonlinear regression method available in SAS release 6.12 (PROC NLIN METHOD=DUD, SAS Institute, Cary, NC) to regress \(-\ln(T_{\text{direct}})/S\) on \( Z_s \) (Eq. (10)) and obtain \( \chi_j \) and \( F_j \). These parameters were calculated for each tree, then averaged for the species. Since the measurement points were not well-mapped, only the inclination parameter \( \chi_j \) was used.

2.5. The MIXLIGHT model

2.5.1. Crown-scale modeling

The light transmission model was adapted from previous models developed in Scandinavia (Pukkala et al., 1993) and New England (Canham et al., 1994). The trees were projected mathematically in three-dimensional \((x, y, z)\) space with stems modeled as paraboloids truncated at the base of the crown and crowns of the shape appropriate to the species (ellipsoids, paraboloids, cylinders, cones or rockets). The elevations of the stems and crowns were adjusted for the stand slope and aspect.
At each measurement point, diffuse skylight transmission was calculated by averaging the transmission along at least 480 light paths (Eq. (4): na=24 equally spaced azimuths from 0 to 360°, and nc=20 zeniths, spaced by equal increments of the cosine of the zenith from 1 (0°) to 0 (90°)). Each path was checked for slope, stem and crown hits. If the light source was obscured by the sloping ground (Eq. (12)), a value of 0% light transmission was returned for that path.

\[
\cos(\Delta s) = \cos \beta \cos Z + \sin \beta \cos(\alpha - \Omega)
\]  

(12)

Here, \(\cos(\Delta s)\) is the cosine of the angle between a line perpendicular to the slope and the light source, \(\beta\) is the slope inclination, and \(\Omega\) the slope aspect. If \(\cos(\Delta s)\)<0, the source is obscured by the slope. Stem hits were determined by checking the paraboloid stems for an intersection with the light path using the simultaneous solution to the ray and paraboloid equation (Appendix A). Any stem hit caused 0% transmission to be returned for that path.

In the absence of these, ray transmission was determined by the length the light ray passed through each crown of the various trees between the simulation point and the sky. Again, the simultaneous solution to the ray and geometric crown equation (Appendix A) was used to determine intersection points, and Eq. (7) applied to obtain the length of the ray between these points (\(S\)). This ray length was multiplied by the foliage projection and foliage area density for the appropriate species to give an absorbance \(A_j=G[Z, F_j, S_j]\), which was summed for all crowns encountered on the path. The sum of these absorbance values along each light path was then converted back to a flux density \(q(Z, a)\), as shown in Eq. (2). We used the uniform overcast sky (UOC) assumption for diffuse light, i.e. light incident from all equal-area sectors of the sky was assumed to have equal flux density.

Direct sunlight absorbance was obtained in a similar way, but for only one light path using the solar zenith and azimuth angle at the date, time and location of measurement. The proportions of direct and diffuse light at the time of measurement were applied to the output to give the integrated light transmission prediction.

2.5.2. Canopy-scale modeling

To avoid excessive edge effects, crown scale modeling requires a large area of inventoried trees surrounding a much smaller central area in which predictions are made. This allows ray-tracing to proceed to wide zenith angles without encountering the plot edge. For example, in a 50 m×50 m flat inventory plot with 20 m tall trees and with the measurement point at plot center, 51° is the widest possible zenith angle that will still avoid the plot edge. Although the light ray transmission values are weighted by the cosine of their zenith angle, nearly 40% of the weight of the diffuse light measurement would be obtained from zenith angles larger than 51° since there is so much sky area beneath this angle. Light transmission is low at large zenith angles owing to the long path through the canopy, so ignoring these angles would inflate the transmission estimate.

A stand-level light modeling scale can be used when edge effects are large or when the input data lack a stem map. This scale assumes the foliage is carried randomly within the space above the entire stand. Leaf area index (LAI) measuring devices such as the plant canopy analyzer (LAI-2000, LI-COR, Lincoln, NB) use this model scale to calculate LAI and the mean inclination angle of the leaves from measurements of transmitted diffuse light. MIXLIGHT provides a provision to input plant canopy analyzer measurements directly.

However, since LAI measurements are seldom obtained in forest inventories, MIXLIGHT can also estimate the leaf area contained within the trees in the inventory list. Our measurements of foliage area density \(F_j\) did not suggest a change in \(F_j\) with tree size: trees of the same species maintained a similar foliage area density regardless of their crown volume. We therefore calculated stand-level foliage area density \(F_{\text{canopy}}\) as the sum of the foliage area contained in each tree crown (the product of crown volume, \(V_j\), and crown foliage density, \(F_j\) plus one half the surface area of each stem \(B_j\)), divided by the canopy volume \(V_{\text{canopy}}=\text{plot length} \times \text{plot width} \times \text{average canopy tree height} \times \cos \beta\), where \(\beta\) is the slope inclination) (Eq. (13)).

\[
F_{\text{canopy}} = \frac{\sum_{j=1}^{n} (V_j F_j + (1/2)B_j)}{V_{\text{canopy}}}
\]  

(13)
Half the stem surface area is used since the light rays can only hit one side of the stem at a time. The volume of the geometric crowns and the surface area of the paraboloid stems \((B_j)\) were calculated using standard equations (Glover, 1996).

The projection of the foliage for stand-level modeling was treated differently than for individual crowns. In order to incorporate the effect of the below-crown stems (which we treated as truncated paraboloids) the projection of a unit of canopy foliage \((G_{\text{canopy}}[Z])\) was determined separately for every zenith angle used in ray tracing (the angles used in Eqs. (3) and (4)). Again, the foliage projection was assumed constant with respect to the azimuth.

\[
G_{\text{canopy}}[Z] = \frac{\sum_{j=1}^{n} (V_j F_j G[Z, \chi_j] + (4/3) r_j h_j \sin Z)}{F_{\text{canopy}}}
\]  

(14)

The projection fraction for each zenith angle \((G_{\text{canopy}}[Z], \text{Eq. (14)})\), is the sum of the projected area of the crown foliage at this angle (crown volume \(\times\) crown foliage area density \(\times\) crown foliage projection) plus the projected area at the same angle of the below-crown stem of each tree, divided by the total one-sided foliage and stem surface area \((F_{\text{canopy}}, \text{Eq. (13)})\). For simplicity, we treated the stem projection as the projection of a parabola with one face to the light source, which underestimates the projected area significantly only for very small zenith angles. In Eq. (14), \(r_j\) is the radius at the base of the paraboloid stem and \(h_j\) is the tree height.

Light transmission is calculated as for the crown-scale model, except that there is only one object \((n=1 \text{ in Eq. (2)})\) and the path length through the canopy is simpler to determine.

If the canopy is the height of an average canopy tree \((h_{\text{canopy}})\) and extends ‘horizontally’ (or following a constant ground slope) to infinity, the path length of a ray through the canopy \((s_{\text{canopy}}, \text{Fig. 1C})\) is given by Eq. (15), where \(z_0\) is the height of the measurement point, \(Z\) and \(\alpha\) are the zenith and azimuth of the light ray, \(\phi\) is the apparent slope in the \(\alpha\) direction, and \(\beta\) and \(\Omega\) are the slope inclination and aspect, respectively.

\[
s_{\text{canopy}} = \frac{(h_{\text{canopy}} - z_0) \cos \phi}{\cos(Z - \phi)} \\
\phi = \arctan [\tan \beta \cos(\alpha - \Omega)]
\]  

(15)

2.6. Stand level validation

Alberta Forest Service permanent sample plots (PSPs) were used to apply and test the instantaneous predictions of MIXLIGHT on a stand level. We selected 17 upland PSPs, with species compositions ranging from dominance by pure aspen, white spruce, or lodgepole pine, to mixtures of these and balsam poplar, white birch and balsam fir. These PSPs were square plots, 900–1600 m² in area, with stem densities of 335–1420 stems ha⁻¹, and stand age (all sites were fire-initiated) from 69 to 159 years. We sampled 16 plots during the deciduous leaf-on period (mid June–early September) in 1996 and 1997, and remeasured eight, plus one new one, in the leaf-off period (April 1997). Light measurements were made with the same linear radiometer as the calibration measurements, either during mostly cloudy (no direct sunlight) or mostly clear (>80% of light from the sun) periods. Light was sampled above shrub height (~1.5 m) for 36–49 points following a 5 m \(\times\) 5 m grid pattern. At each point, 12 light readings were taken at 30° increments while sweeping the radiometer wand around a horizontal circle at arm’s length: the mean of these 12 flux density measurements was recorded. These values were expressed as a percentage of the above-canopy light averaged for 5 min before and 5 min after the within canopy value was taken. Above-canopy light was measured by a point quantum sensor (LI-190SA, Li-COR, Lincoln, NB) and datalogger (CR21X, Campbell Scientific, Logan, UT) in a nearby clearing. These 36–49 transmission measurements were then averaged to give the stand-level light transmission value.

On sunny days, the proportion of diffuse light was determined by sampling a single radiometer sensor outside the stand with and without a shade disk before and after the plot measurements. Trees were tallied to check for new mortality or ingrowth since the last forest service inventory, and measurements were taken of height, live-crown height, crown radius and breast height diameter (DBH) for 8–12 trees in the plot. A polynomial regression between these crown parameters versus DBH, overstory species density and composition was developed for each species (all regressions had \(R^2 > 0.76\)). For each PSP we then created a complete list of all the trees, containing species, stem position (x and y coordinates), DBH,
and estimated height, height-to-live-crown, and crown radius for each tree. The tree lists were loaded into the model along with the appropriate site information (latitude, longitude, slope, aspect, plot dimensions, date and time of measurement, percentage of diffuse light at time of measurement) and the species-specific parameters (crown shape, crown foliage area density and crown foliage inclination). Canopy-scale model predictions were then compared to the measured values of light transmission on a stand-average basis.

2.7. Model sensitivity and foliage inclination

The model was evaluated for the sensitivity of its light transmission predictions to the inputs on a stand level. This sensitivity analysis was performed using tree lists from three stand types: deciduous-dominated, mixed conifer-deciduous with some large (80 m²) gaps, and a conifer stand. For each sensitivity run, we altered one of four key parameters across all species. These parameters were the species-specific foliage area density ($F$) and ellipsoidal leaf inclination parameter ($\chi_j$), the crown radius (cr), and the crown length (cl). Note that this was a test of the sensitivity of MIXLIGHT’s stand-level predictions, so varying the crown size parameters altered the model predictions through their effect on the calculated canopy foliage area and projection. They were included nonetheless, since the model is driven by these individual tree measurements. The other parameters not tested are measured with a much higher degree of accuracy (latitude, longitude, slope, aspect, plot dimensions, date and time of measurement, percentage of diffuse light at time of measurement, tree coordinates). The tested parameters were varied one at a time through a set of logarithmic multiples over seven runs: $2^{-2}, 2^{-1}, 2^{-0.5}, 2^0, 2^{0.5}, 2^1$, and $2^2$ (25, 50, 71, 100, 141, 200, and 400% of the original value). Instantaneous light transmission predictions on cloudy (100% diffuse light) and sunny (90% direct sunlight) days were compared.

Since other models have not attempted to account for the effects of foliage inclination (Grace et al., 1987; Pukkala et al., 1993; Canham et al., 1994; Brunner, 1998), and some effort is required to calibrate this parameter, we also tested MIXLIGHT on the validation sites with the foliage inclination parameter set to spherical ($\chi_j=1$) for all species. This required small corrections to the foliage area density estimates, since they were adjusted for projection during calibration (Eq. (8)).

3. Results

3.1. Calibration

Calibration trees were sampled in 10 harvested areas. Generally, each species was sampled in at least at two different sites, except for lodgepole pine for which only one suitable partial-cut site was available. Tree size spanned the range of sizes which might be expected in mature stands (Table 1). Selection of the appropriate geometric crown shape was straightforward. Early self-pruning and die-back of the lower branches made an ellipsoid shape appropriate for the deciduous and pine crowns. Continuous branch growth with the development of a slight sag made a paraboloid fit the fir crowns well, and the rocket (cone over cylinder) shape was superior for white spruce because of the increasingly pronounced branch sag from the mid to lower crown.

Individual fir crowns transmitted the least light, followed by overstory white spruce, understory spruce, pine, and leaf-on birch, aspen, and balsam poplar (Table 1). During the leaf-off season, light transmission through deciduous crowns was roughly doubled. In 50 of the 57 trees sampled, and without apparent bias as to species or size, the light absorption value calculated at each measurement point within the crown shadow was significantly correlated with the calculated path length of the sun ray through the geometric crown to that point ($P<0.05$), indicating that path length can account for at least some of the variation in transmitted light. Regression analysis indicated that the effect of path length on light absorbance ($-\ln T$) was linear, so Eq. (8) should provide appropriate estimates of foliage area density.

Crown foliage area densities generally followed similar patterns as crown light transmission: species that transmitted little light through their crowns had the highest foliage density (Table 1). That this is not necessarily so is shown by birch, which, owing to the short light path through its narrow crown, transmitted more light than aspen, yet had more foliage per unit crown volume. There was no effect of stem diameter or crown volume on the foliage density
within each species (data not shown); this parameter appears to be constant within the size range of trees measured. Foliage inclination, measured in leaf-on aspen and white spruce only, tended from mostly vertical for overstory spruce ($\chi = 0.10$ as much straight-down as sideways projection) to half as much horizontal projection as vertical ($0.53$) for understory spruce, to close to spherical ($0.82$) for aspen (Table 1).

### 3.2. Validation

MIXLIGHT predictions of instantaneous stand-level light transmission were well-correlated with measured light transmission (Fig. 3, Pearson’s $r = 0.86$, $n = 25$, $P < 0.0001$). The slope of the predicted versus measured relationship ($1.03$) for a regression constrained to pass through the origin was not significantly different from the ideal slope of $1$ ($P = 0.5606$). For some medium-light sites, MIXLIGHT predicted higher light levels than were actually measured by as much as $22\%$ absolute transmission ($44\%$ transmission predicted for a site with $22\%$ measured light). There was also a tendency to underestimate transmission to high-light sites (which were measured under leaf-off conditions): the worst was $48\%$ transmission predicted for a site in which $64\%$ was measured.

The residual plot suggests a curvilinear relationship (Fig. 2B). Indeed, a quadratic equation with intercept had a better fit (adjusted $R^2 = 0.81$) than the linear, zero-intercept equation (adjusted $R^2 = 0.69$). Analysis of the residuals from a 1:1 relationship also showed that the deviations were negatively correlated with increasing site moisture class: overstory light transmission tended to be overestimated in wetter sites and underestimated in drier sites ($P = 0.0065$). No effect of site nutrient status was detectable ($P = 0.6194$).

### 3.3. Model sensitivity and foliage inclination

Results of the sensitivity analysis of MIXLIGHT at the canopy level were similar under sunny and cloudy conditions. Only the overcast day results, which had slightly more variation, are shown (Fig. 4). Predictions were most responsive to foliage area density and crown radius. Doubling or halving the species’ foliage densities (the range indicated by the calibration data, see Table 1) caused a change in the light prediction of about half the original transmission value in the aspen and mixedwood stands, and slightly more in the spruce stand. Crown radius had an effect on transmission roughly similar in magnitude to foliage density, but more asymmetric. An increase in radius tended to have a greater effect than a decrease, except in the spruce stand. The effect of varying the tree crown length was smaller than the effect of varying foliage density or crown radius. Changes in the foliage incli-
Fig. 4. Sensitivity analysis of the (\( m \)) foliage area density, (\( r \)) foliage inclination, (\( d \)) crown radius and (\( j \)) crown length parameters of MIXLIGHT for three stands: (A) aspen dominated, (B) white spruce/aspen mixedwood with gaps, (C) white spruce dominated. Parameters were multiplied by the percentage shown. Light transmission predictions are instantaneous stand-level predictions for a cloudy day during the leaf-off season.

The results of assuming the foliage inclination was spherical for all species are shown in Fig. 5. Light in all sites was underestimated, but the relative error was particularly high for the low-light sites. Model predictions are still precise and capture slightly more of the site-to-site variation (adjusted \( R^2 = 0.82 \) when the spherical assumption was used, versus 0.69 with the measured foliage inclination parameters), but consistently predict lower values.

4. Discussion

Models for predicting forest light regimes are well-developed, ranging from relatively simple stand models to complex microsite models which rely on detailed measurements of individual-crown or shoot architecture (Cescatti, 1997a, b; Brunner, 1998). To our knowledge, none of these models have been extensively tested in numerous forest stands, a necessary prerequisite for applying a light model to track forest regeneration and stand development. This is likely due to model inflexibility or the quantity and complexity of the data required to drive these models. MIXLIGHT, on the other hand, is designed to apply existing theory to the data presently available to a forest manager, and make understory light predictions at a scale appropriate to the data and the goals of the manager.

On the stand scale, MIXLIGHT predicts light transmission very well for a wide range of forest types during both the leaf-on and leaf-off season and under
sunny and cloudy conditions (Fig. 3A). Validation stands varied from highly structured mixtures of tree species in patches with gaps to closed, single-species, single-strata canopies. It appears that MIXLIGHT can capture much of the site-to-site variation in light transmission based on inventory data and the two species-specific foliage parameters. It should thus be an effective modeling tool for managing understory regeneration and vegetation in extensive forest areas.

For canopy-scale modeling, MIXLIGHT requires only a list of the trees, with species, stem diameter, height and crown dimensions (length and width), which are often available in standard forest inventory data, plus species-specific foliage area and inclination parameters. Stem diameter to height functions are available for tree species in many forest districts; we were also able to develop reasonable diameter to live-crown height relationships if stand characteristics are included. A predictive function for crown width or radius has proved more elusive. This parameter is affected by crown abrasion from snow and wind (Grier, 1988), as well as by stand density and composition. Good within-stand relationships were obtained using only stem diameter and species as predictive variables, but we are currently collecting more extensive data on crown width in hopes of developing a site-independent equation. In principle, these relationships could reduce MIXLIGHT data requirements to the two foliage parameters, site attributes (plot location and size, slope, and aspect) and a list of species with their stem diameter.

Leaf area is the most critical driving variable in light transmission modeling (Sampson and Smith, 1993), but is also a difficult parameter to measure. Allometric equations are commonly used to relate leaf area to stem diameter or sapwood area (Pierce and Running, 1988; Oker-Blom et al., 1991; Pukkala et al., 1993) but require considerable effort to calibrate. Branch and stem area must be also be estimated, as these also extinguish light. Light interception techniques (Welles and Cohen, 1996) provide a simpler method to estimate shading leaf, branch and stem area. Generally these techniques are applied to entire canopies, but Lang and McMurtrie (1992), Acock et al. (1994) and Villalobos et al. (1995) have demonstrated how these methods can be applied to individual plants. For small plants, particularly where the foliage distribution is non-random, this method may underestimate foliage area (Lang and McMurtrie, 1992). Villalobos et al. (1995), however, found reasonable estimates of single-tree foliage area using light interception methods with larger olive trees. Our calibration trees had large (>2 m long) crowns and all species demonstrated a linear relationship between the negative logarithm of direct-beam light transmission and path length, which suggests a random foliage distribution. We did not independently verify our foliage density estimates, but our data do meet the theoretical requirements for effective estimation by the light interception technique.

Our leaf-on foliage area estimates are slightly higher than values reported for similar species. One-sided lodgepole pine needle area density, calculated from the data of Oker-Blom et al. (1991), was 0.4–1.8 m² m⁻³, which includes a lower limit than we found in our data for trees of similar crown volume. Engelmann spruce (Picea engelmannii Parry ex Engel.), in the same study, had a needle area density of 0.6–1.9 m² m⁻³, while our white spruce (which are similar in profile), had a foliage area density of 0.8–2.6 m² m⁻³. Our estimates include within-crown branch and stem area, which largely accounts for the differences (Oker-Blom et al. (1991) assumed the branch area was 15% of the needle area). An interesting contrast is with the New England forest species studied by Canham et al. (1994). They report ‘absolute path length extinction coefficients’ which are mathematically equivalent to projected foliage area density. This parameter can be obtained from our calibration data by dividing the negative logarithm of direct-beam light transmission by path length and assuming the projection (G) remains a constant for all light source angles (a spherical distribution of foliage inclination angles). The range of projected leaf area density was 0.07 m² m⁻³ for red oak (Quercus rubra L.) to 0.32 m² m⁻³ for white pine (Pinus strobus L.) while the range for our boreal species was 0.20 m² m⁻³ for balsam poplar to 0.99 m² m⁻³ for balsam fir. Boreal trees appear to have more compact, denser crowns.

In several validation sites, MIXLIGHT’s prediction error for stand-level light transmission was nearly half the measured value. This error may be important where the goal is to determine critical light levels for the survival of particular species. Fortunately, errors of this magnitude occurred for medium to high-light sites (>14% transmission) rather than low-light sites.
where the relative error would be greater and where the light compensation points of boreal tree and understory species occur (e.g. Greenway, 1994; Man and Lieffers, 1997). The sensitivity analysis indicated that the prediction range corresponding to the absolute variation found in the foliage area density (0.5–2 times the average value, Table 1) was similar in magnitude to the largest prediction error for the validation (Fig. 4). No trends were found between foliage area density and tree size or site nutrient status; however, deviations from the measured values were correlated with site moisture index. Since leaf area for aspen in particular appears to be limited by moisture (Waring and Schlesinger, 1985; Messier et al., 1998), it is not surprising that wetter sites transmit less light than was predicted using the foliage area density of an ‘average’ tree. Fine-tuning improvements to MIXLIGHT could be made by further calibration of the species’ foliage area densities for sites of different moisture index.

Light scattering may provide another source of error in the estimation of foliage area and subsequent simulation of light transmission. We assumed that the foliage elements transmit and reflect no light, but clearly this is not the case. For simulating light penetration through crowns or a canopy, neglecting light scattering should result in underestimation of the actual light transmission. In calibration, scattering would cause an underestimate of foliage area density. These effects may cancel each other; however, scattering depends on the species mix, leaf age, and leaf area as well as the solar angle (Norman and Jarvis, 1975; Hutchison and Matt, 1976), so its effects are complex. Hutchison and Matt (1976) suggested the contribution of beam enrichment (foliage-scattered sunlight), should be highest at high solar elevations in fully-leafed canopies (i.e. at midsummer), and lowest in winter, but there is little evidence of such a trend in our data (Fig. 3). Light scattering appears to be a relatively small component of the understory radiation in our high latitude forests.

Although canopy-scale modeling is more rapid and does not require a stem map, the assumption of random foliage distribution throughout the canopy volume is one which may not be met. The most obvious level of foliage grouping is into crowns. In quite open stands, as might occur following partial-cutting, more light would penetrate the canopy than would be predicted by a model that spreads the foliage randomly throughout the site. The concave-downward trend in our validation data (Fig. 3) may be caused by foliage grouping in the high-light (i.e. open) stands. Crown-scale modeling may be more appropriate for these situations.

The sensitivity analysis illustrates the behavior of MIXLIGHT under systematic changes to its key parameters. It is clear that foliage area density and crown radius should be estimated with the most care. Crown length and foliage inclination are less critical, particularly if only a relative site-to-site comparison is required. If accurate predictions are required, however, it is clear that even foliage inclination should be measured, since setting the foliage inclination to spherical for the validation sites created a downward bias in the predictions (Fig. 5).

MIXLIGHT provides a flexible predictive light transmission model which predicts instantaneous stand-level light well for a wide range of stand conditions. Calibration of species’ foliage area density and inclination parameters can be quickly accomplished by direct sunlight transmission measurements on isolated trees. Since so many forest processes are driven by light, we believe MIXLIGHT will provide a useful tool for evaluating stand tending and regeneration options in forests of diverse structure with limited inventory data. Work is currently under way to evaluate the microsite-level predictions of MIXLIGHT on instantaneous and long-term bases. Copies of Visual Basic code for MIXLIGHT are available from the authors.

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Appendix A

A.1. Light ray equation

The equation of a line containing the light measurement point \( P_0 = (x_0, y_0, z_0) \), with vertical (zenith) angle, \( Z \), and orientation with respect to north (azimuth), \( \alpha \), is given by Eq. (A.1)

\[
x = x_0 + at, \quad y = y_0 + bt, \quad z = z_0 + ct \quad (A.1)
\]

where \( a = \sin \alpha \), \( b = \cos \alpha \), \( c = \cot Z \), and \( t \) is any real number. The \( y \)-axis increases in a northward direction, the \( x \)-axis westerly, and the \( z \)-axis upward.

A.2. Crown and stem shape equations and intersections with light rays

Conical, paraboloidal, cylindrical and rocket shaped crowns are truncated at the base of the live-crown (tree base elevation plus the live-crown height \( z_b + h_{lc} \)) and at the top of the tree (base elevation plus tree height \( z_b + h_t \)), the apex of the cone or paraboloid. Crown centers (with respect to the crown length and width coordinates, not the center of mass) are at the point \((u, v, w)\) where \( u \) and \( v \) are the \( x \)- and \( y \)-coordinates of the tree base \((x_b, y_b)\), and \( w = z_b + h_{lc} + (1/2)cl \), \( cl = h_t - h_{lc} \). Stems are truncated at the tree base elevation \( z_b \) at the bottom and the live crown height \( z_b + h_{lc} \) at the top. To simplify calibration of the foliage area density and inclination parameters for each tree, the origin \((0, 0, 0)\) was set to the center of the crown shadow (i.e. midway along the crown shadow length and midway across the crown shadow width). For predicting light in a stand, the origin was set to the center of the stand.

Functions describing the intersection between the light ray and each of the geometric shapes were generated by substituting the three elements of Eq. (A.1) into each of the shape equations (i.e. substitute \( x_0 + at \) for \( x \), \( y_0 + bt \) for \( y \), etc.), then solving for \( t \). Since these were lengthy equations, we used mathematical software (Maple V Release 3, Brooks/Cole Publishing, Pacific Grove, CA) to obtain analytical solutions. Below, \( t \) is expressed as a function of \( x_0 \), \( y_0 \), \( z_0 \), \( u \), \( v \), \( w \), \( cr \), \( cl \), \( h_{lc} \) and \( h_t \). There are always two solutions for \( t \), corresponding to the two intersection points. If \( t \) is a complex number (i.e. \( m \) is negative), the light ray does not intersect the shape. Substituting \( t \) back into the light ray equation (Eq. (A.1)) yields the \( x \), \( y \), \( z \) coordinates of each intersection point.

Cylinders were described by Eq. (A.2) and their intersection with the light ray by Eq. (A.3).

\[
(x - u)^2 + (y - v)^2 = cr^2 \quad (A.2)
\]

\[
t = \frac{-x_0a + au - y_0b + bv \pm \sqrt{m}}{a^2 + b^2},
\]

\[
m = 2x_0ay_0b - 2x_0abv - 2auy_0b + 2aubv - a^2y_0^2 + 2a^2y_0v - a^2v^2 + a^2cr^2 - b^2x_0^2 + 2b^2x_0u - b^2u^2 + b^2cr^2 \quad (A.3)
\]

Cones were described by Eq. (A.4) and their intersection with the light ray by Eq. (A.5).

\[
(x - u)^2 + (y - v)^2 = \left[ cr - \frac{cr}{cl} (z - h_{lc}) \right]^2 \quad (A.4)
\]

\[
t = \frac{-x_0a + au - y_0b + bv - (cr^2/c/\pi)}{a^2 + b^2 - ((cr^2/c^2)/\pi^2)},
\]

\[
m = a^2 cr^2 z_{c_0}^2 - 2a^2 cr^2 z_{h_{lc}} + a^2 cr^2 h_{lc}^2 + 2b^2 cl^2 x_0u - 2b^2 cl cr^2 z_0 + 2b^2 cl cr^2 h_{lc} + b^2 cr^2 z_{c_0}^2 - 2b^2 cr^2 z_{h_{lc}} + b^2 cr^2 h_{lc}^2 + cr^2 x_0^2 - 2cr^2 x_0u + cr^2 c_0^2 u + cr^2 c_0^2 u + cr^2 c_0^2 v - 2cr^2 y_0 v + 2x_0a cl cr^2 c - 2x_0a cr^2 z_{h_{lc}} + 2x_0a cr^2 c h_{lc} - 2au cl^2 y_0 b + 2au cl^2 v - 2au cl cr^2 c + 2au cr^2 z_{c_0} - 2au cr^2 c h_{lc} + 2y_0b cl cr^2 c - 2y_0b cr^2 z_{c_0} + 2y_0b cr^2 c h_{lc} - 2v_0 cl cr^2 c + 2bv cr^2 z_{c_0} - 2bv cr^2 c h_{lc} + 2a^2 cl^2 y_0 v - 2a^2 cl cr^2 z_0 + 2a^2 cl cr^2 h_{lc} + 2x_0a cl^2 y_0 b - 2x_0a cl^2 y_0 b - a^2 cl^2 y_0^2 - a^2 cl^2 v^2 + a^2 cl^2 cr^2 - b^2 cl^2 x_0^2 - b^2 cl^2 u^2 + b^2 cl^2 cr^2 + cr^2 c^2 v^2 \quad (A.5)
\]

For rocket-shaped crowns we looked for intersection points with the cylinder equation between heights \( z = (z_b + h_{lc}) \) to \( z = (z_b + h_{lc} + (3/4)cl) \). If one or both of the intersection points was higher than this, we looked for intersections with the cone equation from \( z = (z_b + h_{lc} + (3/4)cl) \) to \( z = (z_b + h_t) \).
Paraboloid stems and crowns were described by Eq. (A.6) and their intersection with the light ray by Eq. (A.7). For stems, r is the stem radius at the base, and l is the length of the stem (h_l). For crowns, r is the crown radius (cr) and l is crown length (cl).

\[(x - u)^2 + (y - v)^2 = \frac{r^2}{l}[z - (h_l + z_b)] \quad (A.6)\]

\[m = c^2 r^4 - 4 c^2 l a u + 4 c^2 l y_0 b - 4 c^2 l b v \]
\[+ 8 l^2 x_0 a y_0 b - 8 l^2 c x_0 a b v - 8 l^2 c y_0 b \]
\[+ 8 l^2 a a b v - 4 a^2 l^2 y_0 b - 4 a^2 l^2 b v \]
\[-4 b^2 l^2 x_0 b - 4 b^2 l^2 u - 8 a^2 l^2 y_0 v + 8 b^2 l^2 x_0 a \]
\[-4 a^2 l^2 z_0 r - 4 a^2 l^2 h r^2 - 4 l^2 x_0 b - 4 l^2 h r^2 \]
\[+ 4 c^2 l x_0 a \]
\[(A.7)\]

Ellipsoidal crowns were described by Eq. (A.8) and their intersection with the light ray by Eq. (A.9).

\[\frac{(x - u)^2}{cr^2} + \frac{(y - v)^2}{cr^2} + \frac{(z - w)^2}{(cl/2)^2} = 1 \quad (A.8)\]

\[t = 1 \left( \frac{(-x_0 a + au - y_0 b + bv)/cr^2}{(a^2 + b^2)/cr^2 + 4(c^2)/cl^2} \right), \]
\[m = 8 a^2 c^2 x_0 a - 4 c^2 x_0 a^2 u^2 - 4 c^2 x_0 a^2 y_0^2 \]
\[+ 8 c^2 x_0 a^2 y_0 v - 4 c^2 x_0 a^2 v^2 - 4 a^2 x_0 a^2 b v \]
\[+ 8 a^2 z_0 u c^2 w - 2 x_0 a^2 c^2 y_0 b - 2 x_0 a^2 c^2 b v \]
\[+ 8 x_0 a z_0 c^2 r - 8 x_0 a c w^2 r^2 - 2 a u c^2 y_0 b \]
\[+ 2 a u c^2 b v - 8 a u c z_0 c^2 r^2 + 8 a u c w^2 c^2 r^2 \]
\[+ 8 y_0 b z_0 c^2 r - 2 y_0 b c w^2 r - 8 b v z_0 c^2 c^2 r^2 \]
\[+ 8 b v c^2 w^2 r^2 + 2 a^2 c^2 l^2 y_0 v - 4 a^2 u^2 c^2 r^2 \]
\[+ 8 b^2 z_0 u c^2 r - 4 b^2 u^2 c^2 r^2 - 4 c^2 x_0^2 x_0^2 \]
\[+ a^2 c^2 y_0^2 + 2 b^2 c^2 x_0 a - 4 b^2 z_0^2 c^2 r^2 \]
\[+ a^2 c^2 v^2 + a^2 c^2 x_0^2 - b^2 c^2 x_0^2 \]
\[+ b^2 c^2 u^2 + b^2 c^2 x_0^2 + 4 c^2 c^4 \]
\[(A.9)\]

References


