Fuzzy multiple-criteria decision making for crop area planning in Narmada river basin

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Abstract

The present study dealt with the real-world problem of irrigation water management of evolving suitable cropping pattern, which should be in harmony with optimal operation of the multi-reservoir system in the basin. The Narmada river basin system, comprising 11 reservoirs either existing, planned or under construction, has been first simulated and analyzed. The time horizon used to simulate the monthly operation of the system corresponds to the past 30 years of history. A multi-objective fuzzy linear programming (MOFLP) area allocation model has been formulated to cope with the diverse/conflicting interests of different decision makers such as the irrigation authority (government) and the individual farmers involved. Simulation output in the form of optimal monthly releases for irrigation is one of the main inputs to the MOFLP area allocation model. Variable irrigation demand over the planning time horizon has been incorporated into the formulated model considering high variation in precipitation. Thus, varying cropping patterns in the command area, one for each year, have been analyzed which are based on the past 30 years of reservoir simulation. Besides this, a cropping pattern corresponding to 80% dependable releases and rainfall is also analyzed. Such analysis is extremely useful in deciding on an appropriate cropping pattern in any command area that minimizes the average crop failure risk in view of uncertain irrigation water availability, especially in dry years. © 2000 Published by Elsevier Science Ltd. All rights reserved.

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1. Fuzzy multiple-criteria decision-making model for crop area planning

1.1. General

The task of reservoir operation and planning remains incomplete without ensuring the beneficial use of obtained releases for irrigation. It requires that cropping pattern must be readjusted with respect to possible releases available. Therefore, the second step, following reservoir management, is the problem of irrigation water management. Essentially, three decisions are required in irrigation water management, namely optimal crop selection, optimal land allocation under different selected crops, and optimal amount of water to be allocated to each crop. Optimization techniques provide a powerful tool for analysis of problems
that are formulated with single, quantifiable objectives. However, the real-world decision-making problems usually require consideration of multiple, conflicting and non-commensurable criteria. Numerous single as well as multiple-objective crop area allocation models were developed in the past and are well described in the literature (Maass et al., 1962; Hall and Dracup, 1970; Rogers and Smith, 1970; Lakshminarayana and Rajagopalan, 1977, etc.).

The decision problems like crop area planning or agricultural planning also involve multiple, conflicting and non-commensurable criteria and a 'satisfying' decision is desired. These are called multiple-criteria decision problems, where the decision maker generally follows a satisfying solution rather than the maximization of objectives. Such a problem leads to numerous evaluation schemes and to the formulation of a vectormaximum problem in mathematical programming. Further, uncertainty due to the random character of natural processes can be dealt with quantitatively by various developed techniques and tools provided by probability, decision, control and information theories. The real-world decision problems can rarely be defined precisely in mathematical terms rather than in terms of the real world, which may often be imprecise, by nature (because of fuzziness). A fuzzy set is a class with unsharp boundaries, i.e. a class in which transition from membership to non-membership is gradual rather than abrupt. Fuzziness plays an essential role in human cognition because most of the classes encountered in the real world are fuzzy — some only slightly and some markedly so. It is more true in system modeling in which human aspects have to be included. A decision maker might not be able to express his goals or constraints precisely because his utility function is not defined or definable precisely, or phenomena of the decision problem might only be described in a

**Nomenclature**

**Subscript and superscripts**

<i>=1, 2, \ldots, n \quad \text{crop type index}

<i> = 1, 2, \ldots, 12 \quad \text{month type index}

**Variables and parameters**

<i> allocation of land to ith crop

<i> production (yield) per unit area of ith crop

<i> net return (in excess of field level costs) per unit area from production of ith crop

<i> labor requirement per unit area for production of ith crop

<i> total per unit area investment required for cultivation (production) of ith crop

<i> total per unit area energy (calories) available from production of ith crop

<i> overall crop ranking factor for ith crop

<i> volume of irrigation water requirement for ith crop in jth month

<i> total cultivable area available in planning unit

<i> maximum surface water availability in jth month obtained from reservoir simulation study for optimal releases

<i> maximum permissible groundwater availability in jth month

TAGW maximum permissible annual groundwater withdrawal

<i> minimum quantity of cereals (maize, paddy, wheat) requirement in the planning unit

<i> minimum quantity of pulses requirement in the planning unit

<i> minimum quantity of oil seeds requirement in the planning unit

<i> fraction of the total area under crop i exists on the field during month j (0–1; describing the composite crop period)

<i> fractional amount of edible food obtained from ith crop

<i> lower production bound of ith crops

<i> upper production bound of ith crops

<i>(x) membership function of ith objective
fuzzy way. Much of water resources management takes place in an environment in which the basic input information, goals, constraints, and consequences of possible actions are not known precisely. Therefore, water resource managers and modelers are bound to deal with imprecision — mostly due to insufficient data and imperfect knowledge — which should not be equated with randomness and the consequent uncertainty. Hence, it is more realistic to consider imprecise model constraint and goals. Fuzzy goals and/or fuzzy constraints are regarded as fuzzy criteria (Bellman and Zadeh, 1970).

In the last two decades, multiple-criteria decision making (MCDM) techniques have experienced spectacular growth (Cohon, 1978; Goicoechea, 1982; Zeleny, 1982; Harboe, 1992, etc.), but not many water resource planning studies have utilized these methods throughout the entire course of the planning process. There are several reasons behind this. Some multi-criteria methods require information that is difficult to obtain in a real-world setting. Sometimes the complexity of mathematical formulations alienates the potential users. Generally, prevailing institutional arrangements are also not favorable for their application. Thus, there is a dilemma. The single criteria alone do not satisfy every sphere of society any more and, on the other hand, multi-criteria approaches are difficult to apply in real practice. In this paper a simple, yet potential, tool in fuzzy multi-objective analysis has been explored and utilized. The fuzzy linear programming is used to formulate the multi-objective fuzzy linear programming (MOLFP) area allocation model considering various conflicting objectives involved in irrigation planning.

1.2. Conflicting objectives

In any irrigation system, two distinct groups of decision makers are generally involved, namely irrigation administration representing the government who usually control the operation of the system and the individual farmers (cultivators) who are the actual producers of agricultural commodities. Different groups of decision maker have different goals, perspectives and values. Issues of equity, risk, redistribution of national wealth, environmental quality and social welfare are as important as economic efficiency. It is clearly impossible to develop a single objective that satisfies all interests, all adversities and all political and social viewpoints. Hence, six conflicting objectives have been identified to illustrate the potential methodology. The objectives are classified into three groups, namely social, economic and socio-economic. The objectives like benefit maximization, investment minimization are economic objectives whereas maximization of calories, labor employment and total area under irrigation are social objectives. The objective like maximize crop area in order of crop ranking reflects both the social as well as economic aspects of planning.

1.3. Crop selection

Generally, the choices of crop types to be sown are based on social and agronomical considerations only. Selection of the appropriate type of crops to be sown should be made by incorporating all conflicting objectives of agricultural planning, instead of arbitrarily out of convenience or out of tradition. Some criteria among others like domestic need of a particular crop commodity, productivity, market value, regional balance and resource requirements and their availability can significantly influence the crop selection decision. Under such circumstances, crop selection is a case of a multi-attribute decision-making problem. Candidate crops should be identified and short-listed, eliminating those that are obviously dominated. Crops that are inferior to at least one other crop, by all considered criteria, should not be considered further. A set of non-inferior crops should then be evaluated (Tabucanon, 1993). The Analytic Hierarchy Process, Multi-attribute Value Theory and Promethee are suitable techniques for crop selection problem.

1.4. Generalized framework of multi-objective crop area optimization model

1.4.1. Vector maximum problem

The vector maximum problem is defined as follows:
\[ \text{Max } Z_1 = \sum_i N_i \times A_i, \forall i. \] (1)

2. Calories (energy) maximization: in many developing countries malnutrition is widespread. Therefore, the government prefers a cropping pattern which results in maximum calorie production to prevent calorie deficiency in the country or community. Such a social objective can be formulated as:
\[ \text{Max } Z_2 = \sum_i f_i \times C_i \times A_i, \forall i. \] (2)

3. Labor employment maximization: the governments of under-developed or developing countries may advocate a labor-intensified cropping pattern to minimize unemployment as well as under-employment, especially in the agricultural sector. Hence, mathematically:
\[ \text{Max } Z_3 = \sum_i L_i \times A_i, \forall i. \] (3)

4. Investment minimization: the investment required to produce a particular crop also plays a significant role in crop selection, especially in a developing country like India where cultivators have very limited financial capacity. Usually, farmers prefer a cropping pattern which is less investment intensive, though it may fetch some smaller benefits. Thus, mathematically:
\[ \text{Min } Z_4 = \sum_i I_i \times A_i, \forall i. \] (4)

5. Maximize total area under irrigation: considering the government’s policy of providing irrigation to as large an area as possible rather than limiting to some specific crops that yield more returns or serves individual objectives:
\[ \text{Max } Z_5 = \sum_i A_i, \forall i. \] (5)

6. Maximize crop area in the order of crop ranking: at the outset all possible candidate crops suitable for sowing should be identified
and ranked based on several criteria. Therefore, among others one objective of the planning could be the maximization of the area under crops in order of crop ranking. Such an objective indirectly takes care of several other conflicting criteria. Mathematically, it can be written as:

\[
\text{Max } Z_6 = \sum_i R_i \times A_i, \forall i.
\]

(6)

1.4.2.3. Model constraints. The developed model is subjected to the following constraints:

1. Cultivable area constraint: in a planning unit, area allocated to different crops in any month is utmost equal to the total cultivable area, i.e:

\[
\sum_j \beta_j A_i \leq A, \forall j.
\]

(7)

2. Water requirement constraint: in any month irrigation water demand of all the crops should not exceed the water available in that particular month, i.e:

\[
\sum_i W_{ij} \times A_i \leq SW_j + GW_j, \forall j.
\]

(8)

3. Annual groundwater withdrawal constraint: the total groundwater use in the command area should not exceed the annual permissible groundwater withdrawal:

\[
\sum_j GW_j \leq TAGW.
\]

(9)

4. Upper and lower bound constraints: to cater the need of social, economic and regional factors of the dynamic system, upper and lower bound constraints are required to introduce into the model for better control on production of certain crops. These constraints are frequently needed for the sake of maintaining commodity prices in the region. Lower bound constraints are usually required to fulfill social obligations such as production of minimum food requirements, which is associated with food affinity or malnutrition of the society. Thus, constraints can be expressed in general form as follows:

- upper bound constraints:
  \[ f_i \times P_i \times A_i \leq UB_i \text{ for selected } i. \] (10)
- lower bound constraints:
  \[ f_i \times P_i \times A_i \geq LB_i \text{ for selected } i. \] (11)

Food requirement constraints are lower bound constraints of the model. They are usually required to produce a minimum quantity of desired food commodities for domestic consumption. Considering the food habits of the local population as well the recommended balanced diet by the Indian Council of Medical Research, certain minimum quantities of cereals, pulses and oil seed foods are required to be produced. The suggested cropping pattern should ascertain the supply of these minimum quantities of food commodities. These can be expressed as follows:

- cereals requirement:
  \[ \sum_i f_i \times P_i \times A_i \geq C_e \text{ index } i \text{ for cereal crops only.} \] (11a)

- pulses requirement:
  \[ \sum_i f_i \times P_i \times A_i \geq P_a \text{ index } i \text{ for pulse crops only.} \] (11b)

- oil seeds requirement:
  \[ \sum_i f_i \times P_i \times A_i \geq O_s \text{ index } i \text{ for oil seed crops only.} \] (11c)

5. Non-negativity restriction: the basic assumption underlying the model is that all decision variables must be non-negative, i.e.
\[ A_i \geq 0 \ \forall \ i \ 	ext{and} \ \text{GW}^j \geq 0 \ \forall \ j. \] (12)

1.5. Generalized fuzzy decision model in multi-objective analysis

For each non-redundant objective function \( Z_t(x) \) there exists a uniquely determined functional efficient optimal solution \( x_t^* \) that means:

\[ \bigwedge_{x \in X} Z_t(x) \leq Z_t(x_t^*) = Z_t^*. \] (13)

For the individual optimal solutions holds: \( X_t^* \neq X_j^* \) for \( i \neq j \) and \( i, j = 1, \ldots, k. \)

Furthermore, we define for \( t = 1, \ldots, k. \)

\[ Z_t^m = \min(Z_t(X_1^*), \ldots, Z_t(X_{t-1}^*), Z_t(X_{t+1}^*), \ldots, Z_t(X_k^*)) \] (14)

and

\[ d_t^* = Z_t^* - Z_t^m > 0. \] (15)

1.5.1. Membership functions

Membership function acts as surrogate characterization of preference in determining the desired outcome for each of the objectives within the multi-objective framework.

Membership function, denoted by \( \mu_{z_t}(x) \) for the \( t \)th objective, should at least satisfy the following conditions:

\[ \mu_{z_t}(x) = \begin{cases} 1 & \text{if } Z_t(x) \geq Z_t^* \\ 0 < \mu_{z_t}(x) < 1 & \text{if } Z_t^m < Z_t(x) < Z_t^* \\ 0 & \text{if } Z_t(x) \leq Z_t^m \end{cases} \] (16)

where \( Z_t(x) \) is the outcome of the \( t \)th objective.

Several membership functions as described below could be employed in fuzzy linear programming:

1. Linear membership function: a linear membership function \( \mu_{z_t}^{LP}(x) \) for the \( t \)th objective function, \( t = 1, \ldots, k \), is defined as follows:

\[ \mu_{z_t}^{LP}(x) = \frac{Z_t(x) - Z_t^m}{d_t^*}. \] (17)

2. Piecewise linear membership function: for each objective function \( Z_t(x) \), the corresponding piecewise linear membership function \( \mu_{z_t}^{PL}(x) \) is defined as follows:

\[ \mu_{z_t}^{PL}(x) = s_{ij}Z_t(x) + \beta_{ij} \quad \text{for} \quad q_j \leq Z_t(x) \leq q_{ij-1}. \] (18)

The interval \([Z_t^m, Z_t^*]\) has been divided into sub-intervals \([q_{ij}, q_{ij-1}]\), \( j = 1, 2, \ldots, N \), such that \( Z_t^m = q_{ij} \leq q_{ij-1} \leq q_{ij,0} = Z_t^* \). The \( s_{ij} \) is the slope and \( \beta_{ij} \) is the \( y \)-intercept for the section between \( q_{ij} \) and \( q_{ij-1} \).

Nonlinear membership function: the rate of increase of membership of satisfaction must not always be constant as it is in the case of linear membership functions. Hersh and Caramazza (1976) also empirically show this.

3. Hyperbolic membership function: for each objective function \( Z_t(x) \), the corresponding hyperbolic membership function \( \mu_{z_t}^{HP}(x) \) is defined as follows (Leberling, 1981):

\[ \mu_{z_t}^{HP}(x) = \frac{1}{2} \left( \tan(h(y) + 1) \right), \] (19)

where \( y = \alpha_i(Z_t(x) - (Z_t^m + Z_t^*)/2) \), and \( \alpha_i \) is a parameter.

4. Logistic membership function: for each objective function \( Z_t(x) \), the corresponding membership function \( \mu_{z_t}^{LO}(x) \) is defined as follows (Sakawa and Yano, 1985):

\[ \mu_{z_t}^{LO}(x) = \begin{cases} P_t & \text{for } Z_t(x) < Z_t^m \\ \frac{1 + \exp(-\alpha_t - \beta_tZ_t(x))}{P_t} & \text{for } Z_t^m < Z_t(x) < Z_t^* \\ 0 & \text{for } Z_t(x) > Z_t^* \end{cases} \] (20)

where, \( P_t \) and \( P_u \) represent the degree of the decision maker's preference corresponding to the lowest and highest attainable values for the \( t \)th objective; and \( \alpha_t \) and \( \beta_t \) are constants in the membership function which can be determined analytically by:
\[
\alpha_i = \left(\frac{U_i}{d_i}\right) \times \ln\left(\frac{P_i}{(1 - P_i)}\right) - \left(\frac{L_i}{d_i}\right) \times \ln\left(\frac{P_u}{(1 - P_u)}\right);
\]

\[
\beta_i = \left(\frac{1}{d_i}\right) \left[ \ln\left(\frac{P_u}{(1 - P_u)}\right) - \ln\left(\frac{P_i}{1 - P_i}\right) \right].
\]

Since, the chances that a decision maker has full satisfaction or absolutely no satisfaction are rare, in general, the values for \(P_u\) and \(P_1\) are selected between 0.95–0.99 and 0.10–0.05, respectively.

5. Exponential membership function: the exponential membership function can be defined as:

\[
\mu_{Z_i}^E(x) = \alpha_i \left[ 1 - \exp\left\{ -\alpha_t (Z_i(x) - Z_i^m) / (Z_i^* - Z_i^m) \right\} \right],
\]

where \(\alpha_i > 1, \alpha_t > 0\), or \(\alpha_i < 0, \alpha_t < 0\).

6. Hyperbolic inverse membership function: for each objective function \(Z_i(x)\), the corresponding hyperbolic inverse membership function \(\mu_{Z_i}^{HI}(x)\) is defined as follows:

\[
\mu_{Z_i}^{HI}(x) = \alpha_t \tan h^{-1}(y) + 1/2,
\]

where \(y = \alpha_t \{Z_i(x) - (Z_i^m + Z_i^*)/2\}\); and \(\alpha_t\) is a parameter.

Fig. 1 illustrates different types of membership functions.

1.5.2. Generalized fuzzy linear programming model

The central idea behind fuzzy linear programming is that ill-defined problems are first formulated as fuzzy decision models. Crisp models can then be designed which are equivalent to the fuzzy models and could be solved by using existing standard algorithms. This approach is particularly suitable for decision problems which have the structure of linear programming. The fuzzy programming approach to multi-objective linear programming problems was first introduced by Zimmermann (1978) and further developed by several researchers including Leberling (1981), Hannan (1981), Sakawa and Yano (1985) and...
Sakawa (1987). Following fuzzy decision or minimum operator proposed by Bellman and Zadeh (1970) together with linear, hyperbolic, or piecewise-linear membership functions, authors proved the existence of an equivalent linear programming problem.

The fuzzy objective function is characterized by its membership function, and so are the constraints. To satisfy (optimize) the objective functions as well as the constraints, a decision in a fuzzy environment is a defined analogy to a non-fuzzy environment as the selection of activities which simultaneously satisfy objective function(s) and constraints. The ‘decision’ in a fuzzy environment is the intersection of fuzzy constraints and fuzzy objective function(s). The relationship between constraints and objective functions in a fuzzy environment is therefore fully symmetric, i.e. there is no longer a difference between the former and latter (Bellman and Zadeh, 1970).

If one defines the solution with the highest degree of membership to the fuzzy ‘decision set’ as the maximizing decision, then the fuzzy optimization problem can be defined as follows (Zimmermann, 1978; Leberling, 1981):

$$\max Z(x) = (Z_1(x), \ldots, Z_k(x))^T$$

such that:

$$AX \leq b$$

$$X \geq 0$$

where ‘~’ means that all objective functions are characterized by corresponding membership functions:

<table>
<thead>
<tr>
<th>Objective function $Z_t(x)$</th>
<th>Membership function $\mu_{z_t}(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1(x)$</td>
<td>$\mu_{z_1}(x)$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$Z_k(x)$</td>
<td>$\mu_{z_k}(x)$</td>
</tr>
</tbody>
</table>

The task is then to satisfy simultaneously all objective functions via their corresponding membership functions. For a general aggregate function:

$$\mu_D(x) = \mu_D(\mu_{z_1}(x), \ldots, \mu_{z_k}(x)).$$

The general optimization problem will be:

$$\max \mu_D(x),$$

such that:

$$AX \leq b$$

$$X \geq 0$$

**Definition:** $\bar{x} \in X$ is called optimal fuzzy decision of the fuzzy linear vector optimization problem, iff $\bar{x}$ is an optimal solution of Eq. (23), that means:

$$\bigwedge_{x \in X} \mu_D(x) \leq \mu_D(\bar{x}).$$

Using Zadeh’s mini-operator (Zadeh, 1965) to specify the general aggregation function $\mu_D(x)$:

$$\bigwedge_{x \in X} \mu_D(x) = \min(\mu_{z_1}(x)) = \min(\mu_{z_1}(x), \ldots, \mu_{z_k}(x)).$$

Eq. (23) then can be written as:

$$\max \min(\mu_{z_t}(x)).$$

such that:

$$AX \leq b$$

$$X \geq 0$$

Zimmermann (1978) showed that Eq. (24) is equivalent to the following linear programming problem:

$$\max \lambda$$

such that:

$$\lambda \leq \mu_{z_t}(x), \quad t = 1, 2, \ldots, k$$

$$AX \leq b$$

$$X \geq 0$$

(25)
1.6. Multi-criteria decision model for crop area planning using fuzzy linear programming

In formulation of a fuzzy linear programming-based multi-criteria decision model for crop area allocation, it is quite reasonable to allow the membership function to be of hyperbolic nature. This is particularly true to real-world problems, as marginal satisfaction of the decision maker decreases as the level of satisfaction (grade of membership) with respect to attainment of objective increases.

Assuming the following nonlinear tangent hyperbolic membership function for representing the fuzzy goals of the decision maker:

\[ H_t(Z_t(x)) = \frac{1}{2} \left( \tanh(\frac{(Z_t(x) - b_t)\alpha_t}{2}) + 1 \right), \quad t = 1, 2, \ldots, k \]

where \( \alpha_t \) is a shape parameter and \( b_t \) represents the value of \( Z_t(x) \) such that \( H_t(Z_t(x)) = 0.5 \). Taking worst value and best value of \( t \)th objective function as \( Z_{m,t} \) and \( Z_{t} \), respectively,

\[ b_t = \frac{Z_{m,t} + Z_{t}}{2}. \]

Using the above-mentioned hyperbolic membership function for representing the fuzzy objectives of the decision maker together with the fuzzy decision of Bellman and Zadeh (1970), the general form of the problem can be stated as:

\[ \begin{align*}
\max & \lambda \\
\text{s.t.} & \lambda \leq H_t(Z_t(x)), \quad t = 1, 2, \ldots, k \\
& AX \leq b \\
& X \geq 0 \text{ and } \lambda \geq 0
\end{align*} \]

(26)

In this formulation \( \lambda \leq H_t(Z_t(x)) \) is a nonlinear function hence it is a nonlinear programming problem. Leberling (1981) showed that such nonlinearity by introduction of nonlinear hyperbolic membership function could be equivalently converted to a conventional linear programming problem. Eq. (26) can be written as:

\[ \begin{align*}
\max & \lambda \\
\text{s.t.} & \lambda \leq H_t(Z_t(x)), \quad t = 1, 2, \ldots, k \\
& AX \leq b \\
& X \geq 0 \text{ and } \lambda \geq 0
\end{align*} \]

(27)

This is equivalent to:

\[ \begin{align*}
\max & \hat{\lambda} \\
\text{s.t.} & (Z_t(x) - b_t)\alpha_t \geq \tanh^{-1}(2\hat{\lambda} - 1), \quad t = 1, 2, \ldots, k \\
& AX \leq b \\
& X \geq 0 \text{ and } \hat{\lambda} \geq 0
\end{align*} \]

(28)

if we define:

\[ x_{n+1} = \tanh^{-1}(2\hat{\lambda} - 1), \]

then

\[ \hat{\lambda} = \frac{\tanh^{-1}(x_{n+1}) + 1}{2}. \]

Since, \( \tanh(x) \) is a strictly monotone increasing function with respect to \( x \), the maximization of \( \hat{\lambda} \) is equivalent to the maximization of \( x_{n+1} \). Hence, fuzzy vector-valued multi-objective optimization problem can be transformed to the following crisp model:

\[ \begin{align*}
\max & x_{(n+1)} \\
\text{s.t.} & \alpha_t Z_t(x) - x_{(n+1)} \geq \alpha_t b_t, \quad t = 1, \ldots, k \\
& AX \leq b \\
& X \geq 0 \text{ and } x_{(n+1)} \geq 0.
\end{align*} \]

(29)

Let \((x^*_{(n+1)}, x^*)\) be an optimal solution of Eq. (29), then optimal solution of the original problem can be obtained by:

\[ (\lambda^*, x^*) = \left( \frac{\tanh(x^*_{n+1}) + 1}{2}, x^* \right). \]

(30)
Therefore, the multi-objective fuzzy area allocation model with hyperbolic membership function can be formulated as follows:

\[
\text{maximize } A_{(n+1)},
\]

subjected to:

1. Original constraints [Eqs. (7) – (12) of multi-objective area allocation model formulated in Section 1.4.2).

2. Hyperbolic membership constraints one for each considered objective:
   - Net benefit maximization:
     \[
     -\alpha_1 \sum_{i=1}^{n} N_i A_i + A_{n+1} \leq -\frac{1}{2} \alpha_1 (Z_1^m + Z_1^*)
     \]
   - Calories maximization:
     \[
     -\alpha_2 \sum_{i=1}^{n} f_i C_i A_i + A_{n+1} \leq -\frac{1}{2} \alpha_2 (Z_2^m + Z_2^*)
     \]
   - Labor employment maximization:
     \[
     -\alpha_3 \sum_{i=1}^{n} L_i A_i + A_{n+1} \leq -\frac{1}{2} \alpha_3 (Z_3^m + Z_3^*)
     \]
   - Investment minimization:
     \[
     -\alpha_4 \sum_{i=1}^{n} I_i A_i + A_{n+1} \leq -\frac{1}{2} \alpha_4 (Z_4^m + Z_4^*)
     \]
   - Maximization of total area under irrigation:
     \[
     -\alpha_5 \sum_{i=1}^{n} A_i + A_{n+1} \leq -\frac{1}{2} \alpha_5 (Z_5^m + Z_5^*)
     \]
   - Maximization of crop area in order of crop ranking:
     \[
     -\alpha_6 \sum_{i=1}^{n} R_i A_i + A_{n+1} \leq -\frac{1}{2} \alpha_6 (Z_6^m + Z_6^*)
     \]
   
   where:

\[
\alpha_t = \frac{6}{Z_t^* - Z_t^m} \text{ for } t = 1, \ldots, 6.
\]

3. Non-negativity constraints:

\[
A_{(n+1)} \geq 0.
\]

2. Multiple-criteria decision model for crop area planning — application, results and discussion

2.1. General

In order to demonstrate the applicability as well as analytical capabilities of the model developed earlier, the model was applied to a command area of Kolar Reservoir in the Narmada River Basin (India). The model addresses the issue of establishment of most appropriate cropping pattern in a command area in view of surface water availability via optimal operation of the reservoir system in the form of available optimal releases and multiple conflicting and non-commensurable objectives framework which are usually involved in irrigation planning. The developed model determines the type of crops to be grown, their extent and the level of water application such that all other available resources are efficiently utilized so as to result in a most satisfactory decision to the multi-faceted, conflicting, non-commensurable and imprecise objectives. A generalized computer module FMCDM-CP has been developed in FORTRAN, that can be used independently or coupled with an external multi-reservoir operational model. The flowchart of solution procedure used for a multi-criteria decision-making model based on fuzzy linear programming is given in Fig. 2.

2.2. Description of study area

The irrigated command area of Kolar reservoir has been selected to illustrate the applicability of the formulated multiple-criteria decision model for crop area planning. Kolar reservoir is one of the 31 major reservoirs planned in the Narmada river basin, India. Kolar dam is under construction on the Kolar river, which is a tributary of Narmada.
The river rises in the Vindhya Range at an elevation of 600 m and flows 100 km in a south-westerly direction to join the Narmada river south of Nasrullahganj. Its total catchment area is 1347 km². Kolar is purely an irrigation project with limited water supply for domestic and industrial uses in nearby Bhopal city. It has been planned to irrigate 29 835 ha of cultivable land by Left Bank and Right Bank Canals of the Kolar reservoir. The average annual inflow to the Kolar reservoir is 425 Mm³. The average monsoon rainfall in the Kolar command area is approximately 985 mm. The location of study area is depicted in Fig. 3.

2.3. Assumptions used in model application

The following assumptions are made owing to the lack of reliable and sufficient information:

1. Time scale of the model is taken as a month, which is considered to be appropriate for such planning. All resource requirements and their availability have been evaluated on the same reference.
2. Overall project efficiency is applied to consider actual water available in the field.
3. All parts of the land under consideration receive the same management practices.
4. Land and soil are uniform throughout the command area; hence, suitable for all the crops under cultivation.
5. Annual availability of groundwater is considered which can be utilized at any time in a year. Further, it is also ensured that before using any groundwater in a period, all surface water available in that period should be utilized.

2.4. Determination of crop ranking factors

Ten common crops have been identified which are considered to be suitable for cultivation in the selected Kolar command area. In order to determine the crop ranking factors, multi-attribute value theory (MAVT) has been used.

2.4.1. MAVT

The preference function $V$ is decomposed into some simpler forms if the decision maker's preference for the various combinations of the level of criteria satisfies certain conditions. A simple and most widely used form is the weighted additive form (Zeleny, 1982) which is used in the present case:

$$V(f_1, f_2, \ldots, f_k) = w_1 \times v_1(f_1) + w_2 \times v_2(f_2) + \ldots + w_k \times v_k(f_k),$$

where $w$'s are the weights and $v$'s are the value function for each criterion.

2.4.2. Assessment of $v_k(f_k)$

A simple method is to use a 100-point rating scale on which 0 indicates the worst level and 100 indicates the best level. The different levels of an
Fig. 3. Map of the Narmada river basin showing location of the existing and planned reservoir projects.
attribute are rated on this scale. The value of a level of a criterion is simply the rating divided by 100.

Five criteria were employed in ranking the crop alternative, namely, maximization of labor \( (f_1) \), maximization of net benefit \( (f_2) \), minimization of cost of cultivation \( (f_3) \), maximization of benefit/cost ratio \( (f_4) \) and maximization of available calorie \( (f_5) \).

2.4.3. Criteria weights

In the absence of subjective weights all the five criteria, i.e. \( f_1 - f_5 \) are assigned equal weightage (i.e. \( w_1 = w_2 = w_3 = w_4 = w_5 = 0.20 \)).

The overall value \( (V) \) for each crop alternative can be computed as:

\[
V = w_1 \times v_1(f_1) + w_2 \times v_2(f_2) + w_3 \times v_3(f_3) + w_4 \\
\times v_4(f_4) + w_5 \times v_5(f_5).
\]

2.5. Methodology

The multi-reservoir system of the Narmada river basin is first simulated under different operational strategies. After analyzing the simulation trade-off results of firm energy generation and irrigated area, the most preferred operational alternative has been chosen which provides 86.40% irrigation reliability in the Kolar command area as compared to 86.11% irrigation reliability achieved for the whole Narmada river basin. The seasonal firm on-peak energy generation in the system is to the tune of 328.5 GWH with 91.4% reliability. The output of the simulation model (optimal releases for irrigation at Kolar reservoir) has been taken as the input to crop area planning model (as surface water availability). Although the simulation model determines the optimal irrigation releases from a reservoir corresponding to the imposed demand which has been computed using given cropping pattern by the project authorities and assuming whole cultivable area under irrigation. In fact this is the maximum irrigation demand which can be used and desired but the system may not be in a position to fulfil such demand. The simulation model attempts to satisfy as much as possible of this imposed demand considering temporal and spatial priorities and energy generation targets in the system. Therefore, it is most likely that full crop water demand may not be satisfied by the system in all time periods of the planning horizon. Thus, the immediate problem of concern is how best to utilize these obtained irrigation releases from the reservoir? This problem can be solved by finding ‘how much area should be put under irrigation and under which crops to fulfill the aspiration of decision maker in the framework of available water and land resources?’ Moreover, the desirability of incorporating multiple imprecise goals of different decision maker(s) further complicates the problem. The developed crop area-planning model provides the most preferred satisfying solution to the present problem. The crop area-planning model determines the cropping pattern for each year of the considered 30 years planning horizon (1949–78). The model also computes and employs varying irrigation water requirements of crops in each year because the precipitation varies over time. In order to establish an optimal cropping pattern corresponding to some reliable irrigation releases and rainfall, different probability distributions have been fitted into the monthly series of optimal irrigation releases at Kolar reservoir (obtained through an external simulation model) and kriged rainfall in its irrigation command area. The fitted distributions are normal, log-normal, gamma 2-parameter, pearson 3-parameter and log pearson 3-parameter. The best-fitted distribution in each month is then identified and used to calculate 80% dependable irrigation releases and rainfall.

Lower bound constraints on food requirement are imposed for cereals, pulses and oil seed crops. In the case of cereal food requirement constraint, since cereals include paddy, maize, and two varieties of wheat crop, a proportionality constraint has been introduced to specify minimum proportion among them which ensures minimum desired production of each type of cereal. The developed computer program is also very flexible as it can automatically readjust the lower bound food requirement constraints, if the solution becomes infeasible due to violation of these constraints. Such flexibility is essential, since models have been
used for a long planning horizons period (30 years) which also contains extreme dry periods. During dry periods enough surface water (irrigation release) is not available to satisfy lower bound food requirement constraints and thus the solution may become infeasible.

2.6. Results and discussion

The basic statistics and results of distribution fitting in monthly series of irrigation releases at Kolar reservoir and kriged rainfall in Kolar command area are presented in Tables 1 and 2, respectively. The Gamma family distributions could represent all the series except the April month releases, which were best represented by normal distribution. Multi-attribute value technique was used to rank the crop alternatives and finding out the corresponding ranking factor. In the present analysis all the considered criteria used for crop ranking carry an equal weightage. The overall value ($V$) for each crop alternative along

### Table 1
Statistical analysis of monthly irrigation releases obtained from Kolar reservoir by multi-reservoir operational model for the simulation period of 1949–78

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (Mm$^3$)</td>
<td>12.93</td>
<td>12.76</td>
<td>8.08</td>
<td>5.26</td>
<td>13.13</td>
<td>9.35</td>
<td>0.06</td>
<td>0.05</td>
<td>4.20</td>
<td>10.09</td>
<td>15.87</td>
<td>15.48</td>
<td>107.28</td>
</tr>
<tr>
<td>Standard deviation (Mm$^3$)</td>
<td>5.16</td>
<td>7.16</td>
<td>6.70</td>
<td>5.35</td>
<td>3.57</td>
<td>5.19</td>
<td>0.35</td>
<td>0.21</td>
<td>6.73</td>
<td>4.07</td>
<td>4.32</td>
<td>6.18</td>
<td>31.73</td>
</tr>
<tr>
<td>Skewness coefficient</td>
<td>$-2.27$</td>
<td>$-1.33$</td>
<td>$-0.43$</td>
<td>0.00</td>
<td>$-3.66$</td>
<td>0.20</td>
<td>5.48</td>
<td>4.78</td>
<td>1.62</td>
<td>$-1.10$</td>
<td>$-3.66$</td>
<td>$-2.27$</td>
<td>$-1.82$</td>
</tr>
</tbody>
</table>

Exceedence probability

<table>
<thead>
<tr>
<th>Percentile irrigation releases (Mm$^3$)</th>
<th>60%</th>
<th>65%</th>
<th>75%</th>
<th>80%</th>
<th>85%</th>
<th>90%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.59</td>
<td>13.92</td>
<td>11.21</td>
<td>10.04</td>
<td>8.53</td>
<td>6.35</td>
<td>2.57</td>
<td></td>
</tr>
<tr>
<td>12.54</td>
<td>11.54</td>
<td>9.15</td>
<td>7.63</td>
<td>5.75</td>
<td>3.17</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>6.81</td>
<td>5.90</td>
<td>3.83</td>
<td>2.59</td>
<td>1.33</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>3.90</td>
<td>3.19</td>
<td>1.65</td>
<td>0.76</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>7.87</td>
<td>7.19</td>
<td>5.75</td>
<td>4.93</td>
<td>3.99</td>
<td>2.81</td>
<td>1.11</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>1.10</td>
<td>0.48</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>9.81</td>
<td>9.24</td>
<td>7.90</td>
<td>7.06</td>
<td>6.03</td>
<td>5.64</td>
<td>2.39</td>
<td></td>
</tr>
<tr>
<td>17.13</td>
<td>16.74</td>
<td>15.54</td>
<td>14.62</td>
<td>13.31</td>
<td>11.27</td>
<td>7.43</td>
<td></td>
</tr>
<tr>
<td>16.26</td>
<td>15.46</td>
<td>13.41</td>
<td>12.03</td>
<td>10.20</td>
<td>7.60</td>
<td>3.07</td>
<td></td>
</tr>
<tr>
<td>108.98</td>
<td>104.67</td>
<td>93.95</td>
<td>86.95</td>
<td>78.00</td>
<td>65.48</td>
<td>44.33</td>
<td></td>
</tr>
</tbody>
</table>

Best fitted distribution $P_3$, $NOR$, $G_2$.

### Table 2
Statistical analysis of kriged monthly rainfall in Kolar command area for the period of 1949–78

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (mm)</td>
<td>129.20</td>
<td>304.80</td>
<td>297.50</td>
<td>220.80</td>
<td>32.20</td>
<td>984.40</td>
</tr>
<tr>
<td>Standard deviation (mm)</td>
<td>72.95</td>
<td>154.83</td>
<td>122.23</td>
<td>167.88</td>
<td>36.95</td>
<td>301.08</td>
</tr>
<tr>
<td>Skewness coefficient</td>
<td>0.24</td>
<td>2.87</td>
<td>0.48</td>
<td>1.26</td>
<td>1.25</td>
<td>0.84</td>
</tr>
</tbody>
</table>

Exceedence probability

<table>
<thead>
<tr>
<th>Percentile rainfall (mm)</th>
<th>60%</th>
<th>65%</th>
<th>75%</th>
<th>80%</th>
<th>85%</th>
<th>90%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>112.36</td>
<td>103.43</td>
<td>84.28</td>
<td>73.47</td>
<td>61.13</td>
<td>45.96</td>
<td>24.21</td>
<td></td>
</tr>
<tr>
<td>253.31</td>
<td>244.35</td>
<td>227.72</td>
<td>219.71</td>
<td>211.60</td>
<td>203.09</td>
<td>193.29</td>
<td></td>
</tr>
<tr>
<td>263.16</td>
<td>249.51</td>
<td>221.20</td>
<td>205.89</td>
<td>189.02</td>
<td>169.14</td>
<td>142.50</td>
<td></td>
</tr>
<tr>
<td>145.00</td>
<td>128.76</td>
<td>97.75</td>
<td>82.54</td>
<td>67.01</td>
<td>50.68</td>
<td>32.23</td>
<td></td>
</tr>
<tr>
<td>16.55</td>
<td>12.67</td>
<td>5.02</td>
<td>1.10</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>867.72</td>
<td>835.01</td>
<td>768.36</td>
<td>732.75</td>
<td>693.71</td>
<td>648.21</td>
<td>587.11</td>
<td></td>
</tr>
</tbody>
</table>

Best fitted distribution $P_3$, $LP_3$, $G_2$.
with crop ranking in the order of the decreasing value of $V$ is presented in Table 3. The pigeon-pea crop stood at first rank followed by high yielding wheat variety, while the paddy is ranked at eighth position and cotton occupied the last position. With 80% dependable surface water (irrigation releases at Kolar reservoir) and rainfall in the Kolar command area, the optimal cropping patterns corresponding to different objectives as well as fuzzy compromise cropping pattern are reported in Table 4. The payoff matrix for these solutions (Table 5) clearly indicates that all the objectives under study are in direct conflict with each other. Attainment of a particular objective is only possible by sacrificing other objectives. For example, maximum net benefit can be achieved with reduced labor employment, calorie production, ranking factor and with increased investment in crop production. Similarly, minimum investment intensive cropping plan can only be achieved with corresponding reduced values of other objectives compared with their optimal values.

The minimum and maximum area occupied under different crops was determined by analyzing the 30 years (planning horizon) fuzzy-compromised cropping patterns. These minimum and maximum crop areas were compared with fuzzy-compromised crop areas obtained by considering

### Table 3
**Computation of crop ranking factors (overall value) by multi-attribute value technique**

<table>
<thead>
<tr>
<th>Crop alternatives</th>
<th>$v_1(f_1)$</th>
<th>$v_2(f_2)$</th>
<th>$v_3(f_3)$</th>
<th>$v_4(f_4)$</th>
<th>$v_5(f_5)$</th>
<th>Overall value ($V$)</th>
<th>Crop ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pigeon pea</td>
<td>0.137</td>
<td>1.000</td>
<td>0.980</td>
<td>1.000</td>
<td>0.361</td>
<td>0.696</td>
<td>1</td>
</tr>
<tr>
<td>Wheat (HYV)</td>
<td>0.111</td>
<td>0.819</td>
<td>0.720</td>
<td>0.552</td>
<td>1.000</td>
<td>0.641</td>
<td>2</td>
</tr>
<tr>
<td>Gram</td>
<td>0.000</td>
<td>0.710</td>
<td>1.000</td>
<td>0.858</td>
<td>0.502</td>
<td>0.614</td>
<td>3</td>
</tr>
<tr>
<td>Pulses</td>
<td>0.054</td>
<td>0.539</td>
<td>0.924</td>
<td>0.625</td>
<td>0.362</td>
<td>0.501</td>
<td>4</td>
</tr>
<tr>
<td>Maize</td>
<td>0.156</td>
<td>0.253</td>
<td>0.886</td>
<td>0.434</td>
<td>0.731</td>
<td>0.492</td>
<td>5</td>
</tr>
<tr>
<td>Wheat (local)</td>
<td>0.063</td>
<td>0.285</td>
<td>0.825</td>
<td>0.406</td>
<td>0.735</td>
<td>0.463</td>
<td>6</td>
</tr>
<tr>
<td>Groundnut</td>
<td>0.524</td>
<td>0.351</td>
<td>0.318</td>
<td>0.251</td>
<td>0.403</td>
<td>0.369</td>
<td>7</td>
</tr>
<tr>
<td>Paddy</td>
<td>0.429</td>
<td>0.010</td>
<td>0.506</td>
<td>0.196</td>
<td>0.562</td>
<td>0.341</td>
<td>8</td>
</tr>
<tr>
<td>Fodder</td>
<td>0.168</td>
<td>0.000</td>
<td>0.806</td>
<td>0.269</td>
<td>0.000</td>
<td>0.249</td>
<td>9</td>
</tr>
<tr>
<td>Cotton</td>
<td>1.000</td>
<td>0.112</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.222</td>
<td>10</td>
</tr>
</tbody>
</table>

### Table 4
**Optimal cropping patterns under different objectives and fuzzy compromised solution considering 80% dependable surface water and rainfall**

<table>
<thead>
<tr>
<th>Crop</th>
<th>Area (ha) under different crops with respect to objective of …</th>
<th>Area (ha) under fuzzy compromise solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paddy</td>
<td>4655.15</td>
<td>4655.15</td>
</tr>
<tr>
<td>Cotton</td>
<td>1491.75</td>
<td>1491.75</td>
</tr>
<tr>
<td>Maize</td>
<td>3575.16</td>
<td>3575.16</td>
</tr>
<tr>
<td>Pulses</td>
<td>3928.74</td>
<td>3928.74</td>
</tr>
<tr>
<td>Groundnut</td>
<td>3928.74</td>
<td>3928.74</td>
</tr>
<tr>
<td>Fodder</td>
<td>3605.70</td>
<td>1491.75</td>
</tr>
<tr>
<td>Wheat (HYV)</td>
<td>2628.79</td>
<td>2628.79</td>
</tr>
<tr>
<td>Wheat (local)</td>
<td>3575.16</td>
<td>3575.16</td>
</tr>
<tr>
<td>Pigeon pea</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Gram</td>
<td>4977.39</td>
<td>4977.39</td>
</tr>
<tr>
<td>Annual</td>
<td>28437.84</td>
<td>28437.84</td>
</tr>
</tbody>
</table>
80% dependable irrigation releases and rainfall (Fig. 6). These comparative results indicate that minimum areas under different crops are biased to extreme dry hydrological years. It is further confirmed by observing the plotted figure of annual cropped area in different years (Fig. 4), where low peaks correspond to the dry years (1951, 1965, 1966, 1972, 1974). Therefore, the model was re-run
after excluding these five dry years from planning horizon and minimum and maximum crop area has been again determined from 25 years cropping pattern. A considerable improvement was observed in minimum area under some crops after excluding extremely dry periods from the analysis. According to a statistical analysis, the possibility of having a normal year is more than 83%; therefore, it is desirable that the cropping pattern should be established corresponding to the normal year’s hydrology instead of a dry or wet year. The fuzzy-compromise cropping pattern, obtained with 80% dependable irrigation releases and rainfall seems to be quite reasonable as it is in line with the 25-year minimum crop plan.

Water usage results are also as per expectations. Fig. 5 shows the annual conjunctive water use in fuzzy-compromise cropping pattern obtained in case of the 30-year historical period. Similarly, Fig. 6 shows the conjunctive water use in fuzzy-compromise cropping pattern obtained in the case of 80% dependable surface water and rainfall availability. The results of water use revealed that all the irrigation releases have not been fully utilized (in a particular month when they are available) while water shortage has been experienced in other months which were overcome by supplying groundwater. In the case of compromise solution with 80% dependable releases and rainfall, about 25% of surface water remain un-utilized. The possible reason is that the crop area planning model does not allow deficit irrigation. The required quantity of water should be available at the demanded time throughout the crop period if a crop has to be selected in solution. Therefore, in some months available releases could not be fully utilized in light of water shortages experienced in other months of the crop-sown period.

2.7. Recommendations and concluding remarks

It is essential to adjust water demand in the light of available releases for irrigation by selecting a suitable cropping pattern, which could incorporate multiple conflicting objectives of irrigation planning. Fuzzy linear programming is a rather simple but efficient technique that be applied to the multi-faceted, conflicting, non-commensurable criteria decision problems, such as agricultural planning, for obtaining the most satisfactory decision. The study result clearly indicates that if the decision maker is of risk aversion nature, the minimum area crop plan based on 25 years of historical period (excluding dry years) can be adopted. On the other hand, if the decision maker can tolerate small risk, the fuzzy-compromise cropping pattern corresponding with 80% dependable releases and rainfall can best serve the purpose. Further, the area under a particular crop should never exceed the obtained maximum area under that crop. The allocation model under net benefit maximization plan is more-or-less in concert with maximization of ranking factor plan.

References