Groundwater management for Lower Indus Basin

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Abstract

The present study aims to find out an optimum policy for pumping out the optimized volume of groundwater obtained from the authors’ earlier Two Level Optimization Model [N.K. Garg, A. Ali, 1998. Two level optimization model for Lower Indus Basin. Agric. Water Manage. 36, 1–21] for the Dadu Canal Command of the Lower Indus Basin. It is shown that the tubewells in this canal command may be operated at their maximum capacities to pump out the required volume of water to get the overall benefits. The groundwater hydraulics simulation model results show the development of serious waterlogging problem in the lower reaches of the command. It has also been studied that by suitably increasing and placing the tubewells, the waterlogging problem can be tackled very effectively. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Groundwater management model; Groundwater simulation model; Waterlogging; Conjunctive use

1. Introduction

The authors have proposed a two level optimization model in their earlier paper Garg and Ali (1998) to obtain the optimal cropping pattern and the sowing dates. The model was applied to the Dadu Canal Command of the Lower Indus Basin. The coupling of groundwater hydraulics with canal water was not considered in the optimization model because it was found from the irrigation project reports (Harza Engineering International, 1991) that the canal water was always much cheaper (actual operation and maintenance cost) than the tubewell water for the Lower Indus Basin. However, the interaction between surface water and groundwater was considered in the model by imposing a groundwater balance constraint on the groundwater withdrawals. Monthly optimal canal and groundwater withdrawals were also obtained by Garg and Ali (1998). However, no
policy for the optimal groundwater withdrawal was determined based on groundwater hydraulics. The present paper aims to find out the optimal pumping rates and the regional changes in hydraulic heads caused by the optimal groundwater withdrawals from the aquifer.

2. Previous work

Burt (1967) has developed an economic model for allocation of groundwater. The economic consequences of altering various parameters in the model were examined with respect to the effect on the groundwater equilibrium and rate of pumping. Gisser and Sanchez (1980) have discussed the problem of groundwater pumping. They conducted an analytical comparison between a free market behavior (no control) and optimal control and shown that if the storage capacity of the aquifer is relatively large then the two strategies perform equally well. Gorelick (1983) classified the groundwater hydraulic management models into the embedding and response matrix approaches. The embedding approach combines the finite difference or finite element approximations of the groundwater flow equations with an optimization model and therefore, suffers from the large dimensionality problem. In the response matrix approach, an external groundwater simulation model is used to develop unit response. Each unit response describes the influence of a pulse stimulus (such as pumping for a brief period) upon hydraulic heads at points of interest throughout a system. An assemblage of the unit response, a response matrix, is included in the management model, with a much smaller dimension than the discretized governing flow equations.

The use of embedding model was presented by Aguado and Remson (1974, 1980) to determine the optimal steady state pumping scheme to maintain groundwater levels below specified elevation for a dry dock excavation site. Maddock (1972) solved a quadratic programming management problem of minimizing pumping costs. A constraint called the ‘algebraic technological functions’ was developed as a response matrix that relates well pumping effects on drawdown in the aquifer system. Chaturvedi and Srivastava (1979) coupled a finite difference groundwater simulation model with a pumping cost minimization linear programming management model. Casola et al. (1986) developed an optimal control management model for spatial and temporal allocation of groundwater. Makinde-Obusola and Marino (1989) introduced the feedback method of optimal control as a method for solving a groundwater hydraulic management problem of maintaining a target piezometric surface within a confined aquifer. Jones et al. (1987) developed a differential dynamic programming algorithm for unsteady, nonlinear, groundwater management problems to reduce dimensionality and applied to some hypothetical problems. The computational efficiencies was most strongly influenced by the dimension of the state vector. The optimization groundwater simulation system developed by Wanakule et al. (1986) is closely related to the work by Gorelick et al. (1984) in combining simulation and optimization. Large computation times are required since the generalized reduced gradient method used to solve the nonlinear optimization model, requires a large number of functional evaluations using the simulation model. Kanazawa (1992) investigated the possibility that pumping groundwater may involve
increasing marginal costs. Yeh (1992) reviewed the state of the art of systems analysis and optimization techniques developed in the field of water resources for the planning and management of a groundwater system.

Most of these papers have solved a hypothetical groundwater management problem and tried to minimize the computational efforts. It was also implicitly assumed that there exists the metering devices to control pumping rates and also to monitor the aquifer heads at various locations. It also calls for the existence of a central controlling authority to implement the optimal pumping policies. There are around 700 tubewells in the present study area of the Dadu Canal Command of 210,000 ha. Physical, operational, financial and institutional constraints along with the dimensionality problems may give hardly any advantages of coupling the simulation model along with groundwater management model for the study area. Therefore, it is considered appropriate to use a trial and error approach based on heuristic reasons to study the groundwater behaviour in Dadu Canal Command. The monthly groundwater withdrawals have been taken from the two level optimization model for the optimum cropping pattern (Garg and Ali, 1998). In order to pump this required volume of the groundwater, the optimum pumping rates are first obtained using a management model. The aquifer response in then obtained using a simulation model.

3. Formulation of groundwater simulation model

The finite element method is used to solve the Boussinesq equation by Galerkin weighted residual criteria. Shape functions are used to approximate the spatial distribution of the variables and the finite difference has been used to approximate the temporal distribution. Eight noded isoparametric elements are used for the domain subdivision. The resulting approximate equations have been solved by frontal solution technique.

The unconfined aquifer hydraulics are described by Boussinesq equation and for a two dimensional flow can be written as:

\[
C(h) = \frac{\partial}{\partial X} \left( K_X h \frac{\partial h}{\partial X} \right) + \frac{\partial}{\partial Y} \left( K_Y h \frac{\partial h}{\partial Y} \right) - S \frac{\partial h}{\partial t} + Q = 0
\]  

In which \(C(h)\) is the governing equation for the unknown variable \(h\); \(h\) the height of water table above a datum, in meter (m); \(K_X\) the hydraulic conductivity of the aquifer in \(x\)-direction (m/sec); \(K_Y\) the hydraulic conductivity of the aquifer in \(y\)-direction (m/sec); \(S\) the storage coefficient of the aquifer; \(Q\) the source/sink term (ms\(^{-1}\)) and \(X\) and \(Y\) are the principal directions of the flow.

Eq. (1) is a nonlinear one because of the product \(h(\partial h)/(\partial X)\). The product ‘\(K \times h\)’ can be regarded as the transmissivity ‘\(T\)’ of the unconfined aquifer. However, unlike the transmissivity in the confined aquifer, here it may vary as \(h\) may vary both in space and time. In order to linearize the equation, it can be assumed that \(T = \bar{T} + \tilde{T}; \bar{T}(\gg \tilde{T})\) is the average transmissivity of the unconfined flow and \(\tilde{T}\) is a deviation from the average (Bear, 1979). Therefore assuming \(T \simeq \bar{T} = K \times h\) Eq. (1) will reduce to:

\[
C(h) = \frac{\partial}{\partial X} \left( T_X h \frac{\partial h}{\partial X} \right) + \frac{\partial}{\partial Y} \left( T_Y h \frac{\partial h}{\partial Y} \right) - S \frac{\partial h}{\partial t} + Q = 0
\]  

(2)
The approximation involved in the linearization is justified in view of the relatively small changes in \( h \) (with respect to the total thickness \( h \)) in most of the unconfined aquifers and it is also found to be true in the present aquifer of the Lower Indus Basin. However, finite element analysis will pose no difficulty in solving the resulting nonlinear equation in \( h \) derived from Eq. (1). Simple iterative techniques or Newton–Raphson method can be effectively used for each time step with additional computational effort.

To solve \( C(h) = 0 \) by Galarkin method, the following trial solution is assumed.

\[
\hat{h}(X, Y, t') = \sum_{c=1}^{n} N_c(X, Y) h_c(t')
\]

(3)

In which, \( N_c(X, Y) \) are shape functions chosen beforehand; \( h_c(t') \) are undetermined coefficients at time \( t' \) that will be the solution of the Eq. (2) at specified points (nodes) in the domain; \( c \) is the index for nodes and \( n \) the number of nodes.

The approximation function \( \hat{h} \) will be an exact solution of the Eq. (2) only if \( C(\hat{h}) = 0 \). The expression \( c(\hat{h}) \) and the shape functions \( N_c(X,Y) \) will be orthogonal if

\[
\int \int_{\Omega} C[N_c(X,Y)\hat{h}(X,Y,t')] \, dX \, dY = 0
\]

(4)

In which \( \Omega \) is the solution domain.

As only \( n \) shape functions have been selected, there are \( n \) undetermined coefficients and only \( n \) conditions of orthogonality can be satisfied.

Applying the Green’s theorem, to reduce the second derivatives into first order derivatives, the weak formulation is obtained. Following the standard procedure, it can be expressed in the matrix form as:

\[
[F] h + [G] \frac{dh}{dt'} + D = 0
\]

(5)

In which \([F]\) and \([G]\) are \( n \times n \) matrix and \( D \) is a vector with

\[
F_{cd} = \int \int_{\Omega} \left[ T_X \frac{\partial N_c}{\partial X} \frac{\partial N_d}{\partial X} + T_Y \frac{\partial N_c}{\partial Y} \frac{\partial N_d}{\partial Y} \right] \, dX \, dY
\]

(6)

\[
G_{cd} = \int \int_{\Omega} SN_c N_d \, dX \, dY
\]

\[
D_c = -\int \int_{\Omega} Q N_c \, dX \, dY - \int \int_{\Omega} N_c \sum_{d=1}^{n} \left( T_X \frac{\partial N_d}{\partial X} l_x + T_Y \frac{\partial N_d}{\partial Y} l_y \right) h_d \, dE
\]

(7)

where \( B \) is boundary of the domain, \( l_x \) and \( l_y \) are direction cosines of the outward normal to the boundary with the \( x \) and \( y \) axes, respectively.

Once the matrices of Eq. (5) have been determined, the \( n \) undetermined coefficients are still to be solved for which a finite difference scheme is applied to the time domain in a
standard way. As the time dimension is of the infinite extent, finite domain of the time are
dealt with and calculations are repeated for the subsequent time domain with new initial
conditions.

4. Formulation of groundwater management model

An algebraic technological function (ATF) relates in a simple algebraic manner the
draw down to the pumping stress, which is the behavior that stress produces in an aquifer.
Groundwater and economic models can be tied together in a management model through
the use of ATF. An ATF relating draw down in wells to the pumping was determined by
Maddock (1972). The same concept has been used here.
The draw down at the \( p \)th well at the end of \( q \)th time period assuming pulse pumping,
\( s(p,q) \) is given by the ATF

\[
s(p,q) = \sum_{u=1}^{NT} \sum_{v=1}^{M} \lambda(p,v,q-u+1)Q(v,u)
\]  

(8)

In which \( \lambda(p,v,q-u+1) \) = is the change in draw down at the \( p \)th well at the end of the
\( q \)th time period due to a unit quantity of water pumped from the \( v \)th well (\( v \) may equal to
\( p \)) during the \( u \)th time period; \( Q(v,u) \) is quantity of water pumped from the \( v \)th well during
the \( u \)th time period; \( M \) the number of wells penetrating the aquifer. \( NT \) the number of
pumping periods

The ATF given by Eq. (8) indicates a linear relation between draw down and pumping.
The \( \lambda(p,v,q-u+1) \), which are called response functions are independent of pumping
rates.
The response functions are related to the well hydraulics, being function of the
distance between the wells, the well radii, the transmissivity of the aquifer, the storage
coefficient of the aquifer, the boundary conditions, the initial conditions, and the type of
the linear partial differential equation used in the model (Maddock, 1972). The response
function values can be determined using the simulation model following the standard
procedure.

5. Objective function

The objective function is to minimize the pumping cost which comprises of the
variable energy cost and is function of the total lift (draw down plus initial lift), and rate
of pumping. The objective function can be expressed as:

\[
\text{Minimize } \sum_{p=1}^{M} \sum_{q=1}^{NT} [s(p,q) + L(p)]Q(p,q)
\]  

(9)

In which \( M \) is the number of wells penetrating the aquifer; \( NT \) the number of pumping
periods; $L(p)$ the initial lift at $p$th well; $s(p,q)$ the draw down at $p$th well at the end of $q$th
time period and $Q(p,q)$ the pumping at $p$th well during $q$th time period.

Introducing Eq. (8) into Eq. (9) gives the objective function as:

$$\text{Minimize } \sum_{p=1}^{M} \sum_{q=1}^{NT} \left( \sum_{v=1}^{M} \sum_{u=1}^{NT} \lambda(v, v, q - u + 1)Q(v, u) + L(p) \right) Q(p, q)$$

subject to:

$$\sum_{p=1}^{M} \sum_{q=1}^{NT} [Q(p, q)] \geq Q(\text{demand}) \leq Q_{\text{max}}(p)$$

$$s(p, q) \leq s_{\text{max}}(p) \quad [p = 1, 2, \ldots, M]$$

In which, $Q_{\text{max}}(p)$ is the maximum pumping rate permissible at $p$th well; $s_{\text{max}}(p)$ the
maximum draw down permissible at $p$th well.

The objective function (10) and the set of constraints constitute a quadratic
programming problem. SUBROUTINE E04NAF of the NAG Fortran Library, is used to solve
this quadratic programming problem.

6. Application

The groundwater hydraulic management model has been applied to the Dadu Canal
Command of the Lower Indus Basin. Aquifer testing undertaken during Lower Indus
Project at 26 sites indicated that alluvial sediments constituted an aerially extensive, fairly
homogeneous unconfined aquifer system. The Dadu Canal Command consists of
210 000 ha irrigated area underlain by homogeneous unconfined aquifer. Mean lateral
hydraulic conductivity of the aquifer can be taken as 42 m/day and storage coefficient as
0.08. The general thickness of the alluvial sediments varies from 30 to 60 m. Seven
hundred pumps are installed in the command for pumping the groundwater, each having
capacity of 0.02835 m$^3$ s$^{-1}$. The groundwater management model has first been applied
to obtain the optimum pumping rates and simulation model is then applied to obtain the
new water table contours.

7. Groundwater management model

To assess the interfering effects on the optimal pumping rates, the groundwater
management model is applied to a well field consisting of four wells, chosen arbitrarily
from the Dadu Canal Command. Table 1 gives the monthly optimal water releases
from different sources for the Dadu Canal Command, obtained from the two-level
optimization model (Garg and Ali, 1998) corresponding to existing tubewell capacity
of the command. The tubewell capacity is exhausted in peak months, giving an
average maximum daily withdrawal of 1782 m$^3$ for a single tubewell. Since, we
are considering a cluster of four wells to start with, the optimal pumping rates are
determined to pump a daily withdrawal of 7128 m³ of water from these four wells. The initial lift
is taken to an average value of 15.5 m. From the results (Table 2), it is

Table 1
Monthly optimal water releases corresponding to existing tubewell capacity of the command

<table>
<thead>
<tr>
<th>Month</th>
<th>Canal (Ha-m)</th>
<th>Tubewell (Ha-m)</th>
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</thead>
<tbody>
<tr>
<td>January</td>
<td>17365.62</td>
<td>3860.56</td>
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<tr>
<td>February</td>
<td>15685.07</td>
<td>1685.35</td>
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<td>March</td>
<td>17365.62</td>
<td>438.86</td>
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<td>April</td>
<td>3916.30</td>
<td>0.00</td>
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<td>May</td>
<td>8406.05</td>
<td>3328.11</td>
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<td>June</td>
<td>16805.44</td>
<td>3736.02</td>
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<td>July</td>
<td>17365.62</td>
<td>2951.23</td>
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<tr>
<td>August</td>
<td>17365.62</td>
<td>3860.56</td>
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<td>September</td>
<td>16805.44</td>
<td>2263.78</td>
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<tr>
<td>October</td>
<td>17365.62</td>
<td>2331.45</td>
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<td>November</td>
<td>16805.44</td>
<td>2377.22</td>
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<tr>
<td>December</td>
<td>17365.32</td>
<td>3860.56</td>
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</table>

Table 2
Optimal pumping schedule to pump out 7128 m³ of water from four wells

<table>
<thead>
<tr>
<th>Well No.</th>
<th>Time (h)</th>
<th>20 h pumping rate (m³/h)</th>
<th>24 h pumping rate (m³/h)</th>
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</table>
observed that there is no interference between the wells and each well is behaving independently with the same optimal pumping schedule. Therefore, it is not considered necessary to take all the wells pumping together for interference purposes. This conclusion is also physically supported by the fact that the wells in Dadu Canal Command are, in general, located at least 1 km apart and will not cause any interference for such small drawdowns. There is also no infrastructure facilities available for transferring water of one well to the command of other well. It is therefore considered sufficient to obtain the optimal pumping rates for any single well. The relationship between duration of pumping and optimized operation (energy) cost for a single well for a water demand of 1200 m$^3$/day (average daily demand) is shown in Fig. 1. The energy cost is taken as per actual value of Rs 1.4 per unit (kilo-watt-hour) and the tubewell efficiency as 70%. The optimal pumping rates are given in Table 3. The energy cost to pump this volume of water with the maximum tube well capacity is Rs 115.20. It will take around 12 h to pump out 1200 m$^3$ of water. It can be seen from Fig. 1 that the optimized operation cost for the 24 h pumping duration is Rs 108.60. The difference in the operation cost between maximum and optimized rate of pumping is only Rs 6.6 which is not significant. If difference in labour cost for 12 h and 24 h is also considered, then the minimum duration of pumping will give more benefits as the pump operator costs in the region around Rs. 5/h. Further, additional expenditures would be incurred in order to install the metering devices as no metering devices are available to monitor the pumping rates in this region. Therefore, it can be safely concluded that the tube wells may be

![Fig. 1. Relationship between duration of pumping and optimized operation cost.](image)

116
114
112
110
108
106

10 12 14 16 18 20 22 24

Duration of pumping (Hours)
operated at their maximum capacities to optimize the overall expenditures till the system is automatized.

8. Groundwater simulation model

The simulation model has been applied to obtain the groundwater flow pattern for the pumping of optimal amount of groundwater. The monthly optimal groundwater withdrawals have been taken from the two level optimization model (Table 1). Optimal pumping rates are taken from the groundwater management model. The losses are taken to be the same as per the data given in Garg and Ali (1998). The water table contours have been obtained for each month considering the available groundwater recharge (from canal and irrigated areas) and required groundwater pumping in that month. The groundwater recharge is uniformly distributed over each element. The simulation model has been validated on a simple problem for which the analytical solution exists and the model results are found in total agreement with the theoretical results.

Table 3
Optimal pumping schedule to pump out 1200 m³ of water from a single well

<table>
<thead>
<tr>
<th>Well No.</th>
<th>Time (h)</th>
<th>12 h pumping (m³/h)</th>
<th>18 h pumping (m³/h)</th>
<th>24 h pumping (m³/h)</th>
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<td>1</td>
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<td>102.06</td>
<td>69.72</td>
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8.1. Domain discretization

The Dadu Canal Command consists of 210 000 ha CCA. The area has been divided into 2104, eight noded, isoparametric elements. The mesh is adjusted to place the nodes on the tubewell locations. The mesh consists of 6873 nodes and is shown in Fig. 2. The command stretches from 115 km horizontally to 147 km vertically.

Fig. 2. Finite element grid for Dadu Canal Command.
8.2. Initial and boundary conditions

The initial groundwater levels are taken as per the data given in Harza Engineering International, 1991 and are shown in Fig. 3. No groundwater recharge and the pumping is
considered before the start of the season. The command boundaries have canal at one side and river at the other side. Therefore, the boundary conditions are fixed at the existing groundwater levels.

8.3. New water level contours

Fig. 4 shows the new water level contours at the end of 10 years after pumping the optimal amount of water. There seems to be a sharp rise in groundwater levels in the lower reaches. The results indicate that the area is tending to be waterlogged if some remedial measures are not planned on urgent basis. This trend is also verified from the limited field data given in the irrigation reports.

9. Proposed groundwater development

It is clear from the Fig. 4 that the recharge is getting accumulated in the lower reaches over the years and is causing waterlogging problem. Hence, there is a necessity to increase the groundwater withdrawals. In order to estimate the increased groundwater withdrawals, Garg and Ali (1998) model was run on the same previous data but relaxing the limit on maximum groundwater withdrawals by tubewells. The groundwater withdrawals were allowed at the most equal to the annual groundwater recharge. The results about monthly optimal releases are given in Table 4. It can be seen from Tables 1 and 4 that the combined tubewell capacity of the command is now to be increased by 130%. The optimization model also gave a further increase of 32% in crop intensities alongwith 25% increase in the benefits. Now, the problem is reduced to find out the location of additional 130% tubewells in order to avoid waterlogging problem. Considering the huge dimensionality of the problem, we have tried to solve this problem more heuristically by trial and error procedure.

We assume that a real system may be working in the direction of attaining an optimal or a working stability under the prevailing constraints. Therefore, the existing policy

<table>
<thead>
<tr>
<th>Month</th>
<th>Canal (Ha-m)</th>
<th>Tubewell (Ha-m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>17365.62</td>
<td>3860.56</td>
</tr>
<tr>
<td>February</td>
<td>15685.07</td>
<td>1685.35</td>
</tr>
<tr>
<td>March</td>
<td>17365.62</td>
<td>0.00</td>
</tr>
<tr>
<td>April</td>
<td>8406.05</td>
<td>0.00</td>
</tr>
<tr>
<td>May</td>
<td>8406.05</td>
<td>2842.37</td>
</tr>
<tr>
<td>June</td>
<td>16805.44</td>
<td>3735.42</td>
</tr>
<tr>
<td>July</td>
<td>17365.62</td>
<td>2821.04</td>
</tr>
<tr>
<td>August</td>
<td>17365.62</td>
<td>8365.46</td>
</tr>
<tr>
<td>September</td>
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</tr>
<tr>
<td>October</td>
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<td>0.00</td>
</tr>
<tr>
<td>November</td>
<td>16805.44</td>
<td>2974.69</td>
</tr>
<tr>
<td>December</td>
<td>17365.32</td>
<td>5595.95</td>
</tr>
</tbody>
</table>
towards the tubewell installations in the Lower Indus Basin may be working in this direction. Keeping this view, we have run the simulation model on the previous data (Table 1) assuming no recharge from the canal and irrigation fields. The groundwater level contours are plotted in Fig. 5 after 5 years. These contours clearly show a very
heavy drawdown in the lower reaches with the existing tubewell patterns. It means that the existing tubewell density is very high in the lower reaches of the command as compared to the upper reaches but inadequate to stop the waterlogging problem when the recharge is considered.
Since the existing tubewell density pattern is non-uniform and is following a direction of having more dense in the lower reaches, we have followed the same ratio to install additional 130% tubewells i.e., if some location is having three tubewells per square km, then we have installed four additional tubewells there. The simulation model has been run

Fig. 6. Water level contours at the end of 15 years with increased tubewell capacity of the command.
for the water releases given in Table 4 with these additional tubewell capacity system. The groundwater contours are plotted after 15 years with these data and are shown in Fig. 6. There is no change observed in the groundwater level contours after 15 years and the groundwater system would attain stability. Results also show no abrupt rise or fall in the groundwater levels with the proposed management policies in the Dadu Canal Command. Therefore, it can be concluded that the tubewell capacity of the command may be increased to 130% to avoid waterlogging problem and will also provide extra benefits and crop production.

10. Conclusions

Groundwater management studies are carried out for the Dadu Canal Command aquifer of the Lower Indus Basin. It has been shown that the tubewells may be operated at their maximum capacities to pump the required volume of water to get an overall optimum management policy. The simulation model is applied to determine the waterlevel contours for the optimal pumping schedule. The groundwater level contours indicate a serious waterlogging problem in the lower reaches with the existing tubewell capacities. Based on a heurisitic approach, it has also been shown that the waterlogging problem can be tackled effectively by suitably placing the additional 130% tubewells in the command area.

Acknowledgements

The authors express their sincere thanks to Mr. Quamrul Hassan for helping in the preparation of this document.

Appendix

Notation

The following symbols are used in this paper:

- $B$: Boundary of domain
- $C(h)$: Governing equation for $h$
- Cumecs: Cubic meters per second
- $h$: Height of water table above a datum
- $\bar{h}$: Average height of water table above datum
- ha: Hectors
- ha-m: Hector-meter
- $K_x$: Hydraulic conductivity in $x$-direction
- $K_y$: Hydraulic conductivity in $y$-direction
- $L$: Initial lift at a well
Direction cosines of the outward normal to the boundary
Total number of wells penetrating in the aquifer
Meter
Shape function
Number of nodes in an element
Number of pumping periods
Index for well where draw down is observed
Quantity of water withdrawn
Index for time period when draw down is observed
Distance from a well
Pakistani rupees
Storage coefficient
Draw down at a well
Transmissivity of the aquifer
Index for time
Index for time period when pumping is done
Index for well where pumping is done
Summation
Solution domain
Change in the draw down
Time interval
Global coordinates
Local coordinates

References


