Aggregated hydraulic sensitivity indicators for irrigation system behavior

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Abstract

Knowledge of the way perturbations propagate and/or attenuate along free-flow irrigation delivery systems is crucial for monitoring and effectively operating a system. With this knowledge sections of the canal system which are identified as sensitive to flow perturbations should be targeted for more frequent inspection and operational decisions than others. The steady-state hydraulic approach for canal operation design is well understood by irrigation agencies, but it has limited practical application and is unsuitable for studying the propagation of perturbations. The unsteady flow approach is more powerful but rather complex. A third intermediate way is proposed here through the development of hydraulic sensitivity indicators. These indicators allow the characterization of the response of a structure, or a set of structures, to any input change. Local sensitivity indicators are first estimated for the main hydraulic structures, i.e. offtakes and cross-regulators, for delivery and conveyance. Aggregation processes are then proposed for the computation of lumped sensitivity indicators at reach and canal levels. These sensitivity indicators are designed to improve the knowledge of canal behavior and to lead to improved monitoring and operation of irrigation canals.

A case study of the application of a sensitivity analysis at system level is carried out, describing the behavior of two different irrigation canals in Sri Lanka. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Irrigation system; Sensitivity; Offtake; Operation; Perturbation; Regulator

1. Introduction

The adoption of new software and hardware solutions in irrigation system management is slow in less developed countries, therefore manual canal operation is still a major task...
for irrigation managers. Canals are complex systems and operations are often interactive. The impacts of operational changes vary in both space and time, and gaining empirical knowledge is a long process. Although, the effect of improper operation on water distribution and water productivity is well recognized, fine tuning the canal operation does not often receive sufficient attention. Reasons for this are linked to the level of difficulty of the task, and the need for managers to carry out many other important duties. In this context knowing *where* and *when* fine tuning should be made, is essential to the effectiveness of operation.

A common flaw in irrigation system operation, is the absence of a proper information system to monitor the system status and enhance the decision making for operation. One of the means for improving manual canal operation efficiency, is to design and implement a data collection network, and a procedure for data processing. However, an information system by itself is not sufficient to enhance decisions, it has to be combined with an improvement in the knowledge of the canal behavior. Decisions have to be based on ‘information’ regarding the current status of the system, on ‘targets’ for delivery and on ‘knowledge’ of the behavior of the system when operated. Furthermore, information systems can be cumbersome to handle, if information is not collected in a selective and effective way. Monitoring every structure, without any prioritization, can lead to difficulties in data processing and confusion in the decision making. Thus knowing *where* and *when* information has to be recorded, is also essential for improving efficiency.

Beyond the questions of *when* and *where*, it is also important to address *how* to operate and *what* effects will result. Here the knowledge of canal behavior becomes very important in anticipating the impact of operational decisions.

Answering these questions of *when* and *where*, and *how* and *what*, in operation management, is becoming more crucial today as irrigated agriculture is under pressure to generate improvements in the productivity of water resources. Also the users (farmers) have to increasingly bear the cost of irrigation, and particularly the cost of operating the system. Therefore, designing the most cost-effective operation strategy is becoming of wide spread importance.

It is proposed here to investigate the spatial distribution of canal behavior through a physical sensitivity analysis of its components. The first step of the investigation of canal behavior and operation is reported in this paper. It focuses on a description of the sensitivity concepts, the establishment of sensitivity indicators and applications to two different types of canals. The implications of sensitivity analysis for the design of information systems and in building efficient operation strategies will be the subject of a later paper.

## 2. Rationale and generalities on canal sensitivity

Computer modeling has recently generated significant improvements in the hydraulic assessment of irrigation systems, particularly in addressing unsteady situations and regulation strategies (Schuurmans, 1991; FAO, 1994; RIC, 1997). It allows the exploration of the consequences of alternatives in the decision making process. However,
models dedicated to irrigation canals are not generally simple to handle, and very seldom used by irrigation agencies even when available, as stated in a recent worldwide survey on modernization (Burt and Styles, 1998). Hence, there is still a need to develop simple and affordable methodologies, to investigate canal behavior. A sensitivity analysis, applied to the physical properties of an irrigation system, is one methodology which meets these requirements. It investigates the way the system responds when operated or stimulated by external influences.

The sensitivity of a process reflects the relationship between the magnitude of the OUTPUT response to given INPUT variations. This process can be modeled as a black box where results are derived from trials or the model used has no direct physical relevance. In irrigation canals, however, the response process is not a black box as the hydraulic laws of the system enable the prediction of the response of each component of the canal. The model may tend towards a black box type when looking at a system with numerous interconnected components, as the hydraulic behavior becomes complex.

Sensitivity analyses, applied to irrigation systems, focus primarily on the delivery and control structures (i.e., offtakes, outlets and cross-regulators), with the goal of assessing the system behavior resulting from a perturbation in the input. The perturbation can be either a scheduled delivery change or an unscheduled discharge variation. So far, most of the studies on this matter have dealt with local sensitivity analyses, either for a single structure or for a combined structure (an offtake and regulator close to each other) (Albinson, 1986). This is due to the hydraulic complexity of the interactions of multiple structures within irrigation systems. However, a local sensitivity analysis does not provide a clear understanding of the impact of perturbations on either the reach or on the whole system. For example, the water depth variation in a reach is often not only affected by the first downstream regulator, but by other downstream regulators having strong backwater effects. These water depth variations modify the withdrawals at offtakes within the affected reaches, making the variation in downstream discharge difficult to estimate.

The behavior of irrigation canals may be analyzed with regards to two processes: the reactive and active processes. In the reactive process, the propensity of the canal to attenuate and/or propagate input perturbations is considered. In the active process, the propensity of each component to generate perturbations is concerned. Only the reactive process is addressed in this paper. In general, both steady and unsteady hydraulic states are important in characterizing the behavior of irrigation systems and canal operation. In this paper, only the hydraulic steady state is addressed, focusing on the lasting hydraulic effects of perturbations.

Irrigation systems are made up of three main components, canal reaches to convey water, cross-regulators to control water depth within the canal and offtakes to distribute water to dependent canals and downstream users. Operating an irrigation system consists essentially of acting on specific structures (i.e. cross-regulators and offtakes), to ensure targeted change in deliveries, and to react to unexpected perturbations occurring along the system. Accordingly, sensitivity analyses need to be undertaken considering a range of units, from a single structure to an irrigation system, and with regard to both delivery and conveyance.
Components can be grouped into:

- **Highly sensitive points** which are highly affected by perturbations, hence they must be monitored carefully to:
  
  detect ‘unscheduled perturbations’ (Information); and
  readjust the setting of those points and prevent deviation from targets in their area of influence (Operation).

- **Points of low sensitivity** where perturbations in the input have little effect on the output, and therefore may be monitored less closely.

- **Points of medium sensitivity** where perturbations have medium effects.

A primary requirement in a sensitivity analysis is a clear definition of the physical system considered and its boundaries. It can be a singular point like an offtake, a bifurcation point, a cross-regulator; or a combination of singular points (for example a set of contiguous offtakes). It can also be a reach, as an association of offtakes and a cross-regulator, or an irrigation system or subsystem, as an association of reaches. The INPUT changes (stimuli) have to be defined either as hydraulic variables, discharge and/or water depth (reactive process), or as structure setting variables (active process). The sensitivity analysis must be oriented towards either conveyance or distribution. The direction of the perturbation process has also to be defined. Perturbations may propagate upwards as well as downwards. A top–down effect deals with the impact of an upstream perturbation in the parent canal. A bottom–up effect, inversely focuses on the impact on the parent canal, of a downstream perturbation, occurring either in the dependent canal or in the parent canal.

### 3. Background on irrigation system sensitivity

#### 3.1. Offtake flexibility and sensitivity

Flexibility and sensitivity of irrigation outlets (offtakes) has been analyzed by Mahbub and Gulhati (1951), Horst (1983).

The flexibility \( F \) at an offtake or a bifurcation point, is commonly defined (ICID, 1967), as the relative change of the offtake flow \( q \) to the relative change of the ongoing flow \( Q \), as follows:

\[
F = \frac{dq}{q} / \frac{dQ}{Q}.
\]  

(1)

Flexibility in Eq. (1), refers to an implicit relationship between on-going discharge \( Q \) and offtake discharge \( q \). Discharge \( Q \) is not a primary input for \( q \), but a secondary input that influences water depth and subsequently \( q \). Therefore, flexibility cannot be derived without defining the relationship between discharge and water depth, which might be complex in some cases.

Shanan et al. (1986) (see also ICID, 1967), defined the sensitivity of an offtake as the relative change of offtake flow \( q \) to the relative water depth change in the parent...
canal ($H_{US}$):

$$S = \frac{dq}{dH_{US}/H_{US}} \tag{2}$$

Assuming a power relationship between discharge and acting head ($H$) at the offtake, as follows:

$$q = c.H^n, \tag{3}$$

then the derivative of the logarithm of Eq. (3) leads to the offtake sensitivity indicator:

$$S = u. \tag{4}$$

The analytical solution in Eq. (3), refers implicitly to an offtake having a free flow downstream condition, however an extension to any kind of downstream conditions can be made.

A slightly different indicator worth considering is the sensitivity to absolute deviation of water depth. It is defined as the fractional increase, or decrease, of offtake discharge per 0.1 ft rise (or fall) in the parent canal. This sensitivity indicator (called ‘Sensitiveness’ in Mahbub and Gulhati (1951)), is written:

$$S_1 = \frac{dq}{dH_{US}} \quad \text{(with } dH_{US} = 0.1 \text{ ft}). \tag{5}$$

One benefit of this indicator is that it relates sensitivity to the precision of control ($dH_{US}$) in canal operation. This precision is generally a physical variable readily understood by local operators and managers.

The previous sensitivity indicators (Eqs. (2) and (5)) only consider the water depth as an input and the delivery as an output. Renault and Hemakumara (1999) proposed a more comprehensive framework to analyze offtake sensitivity which included the conveyance effect (i.e. the impact of the offtake flow variation on the on-going discharge). They also considered the setting of the structure as an input to the process and analyzed the corresponding sensitivity. They addressed the upward sensitivity, i.e. the effect of a variation of the downstream water depth in the dependent canal on offtake flow.

3.2. Sensitivity at aggregated level

Structures along free-surface delivery system do not behave independently. For example cutting off the discharge at an offtake is likely to create a rise of water depth at the downstream regulator which in turn generates an increase of discharge in all offtakes within the backwater curve of this regulator. Understanding the final response of the reach requires an aggregation of the sensitivity indicators:

Albinson (1986) made a comprehensive study of the sensitivity and flexibility of common structures (offtakes and regulators). Offtakes were analyzed either under the influence of a close regulator or under normal flow. This approach considers the aggregation of two structures in close proximity but does not address the reach behavior or the whole system.
Horst (1983) presented a qualitative global approach of flexibility (Eq. (1)). In studying the propagation of hydraulic perturbations along a canal, he showed that perturbations would be spread equally for \( F = 1 \), would be felt most strongly at the upper end when \( F > 1 \), or at the lower end for \( F < 1 \) (Fig. 1). Constant discharge offtakes, such as baffles, present a flexibility almost equal to zero; perturbations are propagated, almost unchanged, all the way down to the tail for a canal equipped with such delivery structures. Alternatively, overshot offtakes have a high value of flexibility, and perturbations are absorbed in the upper part of a canal equipped with this type of offtakes. Although this approach is useful to gain an overview of the system behavior, it cannot be used to address aggregated sensitivity in quantitative terms.

The author, in a recent paper (Renault, 1999), presented a study in which irrigation systems are characterized by summing offtake sensitivities for delivery. This leads to the production of an aggregated indicator which can be linked to the potential performance of the irrigation system, through the precision of control.

4. Analytical approach of system sensitivity

The proposed methodology for assessing system sensitivity is progressive. It starts by establishing indicators of sensitivity for each type of structure; then aggregates the sensitivity at reach level and finally the reach indicators are grouped at subsystem level.

4.1. Hydraulic laws of irrigation structure

To relate the sensitivity indicator proposed in Eqs. (2) and (5) to physical properties of the structure, we need first to identify the hydraulic laws of the structure. Whatever the
structure considered, offtake or cross-regulator, the hydraulic process can be split into two 
successive hydraulic stages. The first stage depicts the flow through the structure itself 
between the upstream section in the parent canal and the section downstream of the 
structure. The second stage depicts the flowing conditions downstream of the structure in 
the parent canal for a cross-regulator or in the dependent canal for an offtake. This second 
stage considers the immediate water level downstream of the structure and a further 
downstream reference point. This second stage is required to take consideration of the 
submergence condition at the structure.

The discharge through the structure and the physical features of these two stages are 
linked by hydraulic equations:

\[ Q_{\text{out}} = aA(w)(H_{US} - H_{DS})^\alpha \quad \text{stage 1}, \]  
\[ Q_{\text{out}} = b(H_{DS} - H_{REF})^\beta \quad \text{stage 2}, \]  

where \( A(w) \) is the cross-section through the structure expressed as a function of the 
setting \( w \); \( a, b \) the discharge coefficients; \( H_{US} \) the water height in the parent canal 
upstream of the structure; \( H_{DS} \) the water height in the downstream section of the 
structure; \( H_{REF} \) a fixed downstream level taken further down in the parent or dependent 
canal; \( \alpha, \beta \) are the exponent coefficients of hydraulic formula.

\( H_{REF} \) is a constant reference level within the parent or dependent canal (e.g., the canal 
bed level or a weir-crest level). If free-flow conditions occur at the structure, then Eq. (7) 
is irrelevant and the problem reduces to Eq. (6) with \( H_{DS} \) being a fixed height (axis of the 
orifice of the gate or crest level).

For a large rectangular canal, with normal flow prevailing downstream of the 
structure, the value of \( \beta \) is set to 5/3, the exponent in the Manning Strickler equation 
(Levi, 1995). In all other cases (a narrow section, a non-rectangular section or a section 
with water depth controlled by a backwater curve from a downstream regulator) the 
parameters of Eq. (7) have to be found from physical experiments or from hydraulic 
models.

Inverting Eqs. (6) and (7), subtracting to eliminate \( H_{DS} \) and differentiating with respect 
to \( H_{US} \), leads to:

\[ dH_{US} = \left[ \frac{1}{\alpha} \left( \frac{Q_{\text{out}}}{aA(W)} \right)^{\frac{1}{\alpha}} + \frac{1}{\beta} \left( \frac{Q_{\text{out}}}{b} \right)^{\frac{1}{\beta}} \right] \frac{dQ_{\text{out}}}{Q_{\text{out}}}, \]  

substituting with Eqs. (6) and (7), gives:

\[ dH_{US} = \frac{1}{\alpha} H_{E} \frac{dQ_{\text{out}}}{Q_{\text{out}}}, \]  

where \( H_{E} \) is the head loss equivalent through the structure. This is the head loss, through 
an equivalent free-flow structure, that would lead to the same sensitivity, (i.e. the same 
relationship between the variation of upstream water depth and the variation of 
discharge). \( H_{E} \) is equal to:

\[ H_{E} = (H_{US} - H_{DS}) + \frac{\alpha}{\beta} (H_{DS} - H_{REF}). \]
4.2. Offtake sensitivity

The more commonly used offtake sensitivity indicator as defined in Eq. (5), reflects the response of the structure in terms of delivery to the command area. Combining Eqs. (5) and (9) the offtake sensitivity of delivery can be derived:

\[ S_1 = \frac{\alpha}{H_E}. \]  

(11)

There are offtakes along canals that, although having a low sensitivity value for delivery, may have a strong impact on the on-going discharge in the parent canal because their withdrawal is relatively high. To account for this effect it is worth also considering the offtake sensitivity of conveyance. This indicator is the ratio of on-going discharge variation \((dQ/Q)\) to the water depth deviation \((dH_{US})\). Considering that discharge variations are equivalent \((dQ=dq)\), this ratio is given by:

\[ S_2 = \frac{\alpha \cdot q}{H_E \cdot Q}. \]  

(12)

4.3. Regulator sensitivity

Cross-regulators are irrigation structures controlling the water depth along a canal, by adjusting local head losses. The degree of control, and the magnitude of variation of water depth between zero and full discharge, vary with the type of structure. A fixed weir, for example, has no control on water depth, but the magnitude of resulting variation with respect to discharge change is very low. Inversely, gated regulators have a high control level, but improperly managed the magnitude of variation of water depth can be high.

For cross-regulators the distinction between delivery and conveyance makes no sense, however the upward effect as well as the downward effect have to be considered.

4.4. Downward cross-regulator sensitivity

For a cross-regulator, the local on-going discharge \((Q_{out})\) is usually considered as the hydraulic INPUT, and the water level upstream of the structure \((H_{US})\) is the OUTPUT when considering the downward effect. In this situation, the sensitivity indicator \((S_G)\), is the ratio of absolute water depth deviation to the relative change of on-going discharge, \(Q_{out}\). It can be expressed as:

\[ S_G = \frac{dH_{US}}{dQ_{out}/Q_{out}}. \]  

(13)

Note that the expression of the sensitivity for a regulator (13) is inverse to that of an offtake in (5).

Substituting Eqs. (9) and (10) into Eq. (13) gives the downward sensitivity indicator for discharge variation at a cross-structure:

\[ S_G = \frac{H_E}{\alpha}. \]  

(14)
This indicator is equal to the head loss equivalent through the structure divided by the exponent of the discharge equation through the structure.

Some cross-regulators are composite with both undershot gates and side weirs. This is the case for most cross-regulators in irrigation systems in Sri Lanka. The combination of both undershot flow and overshot flow requires a different sensitivity analysis. Two sensitivity indicators should be defined for a composite structure, depending on whether there is a spill or not ($S^+$ and $S^-$). The problem is complicated further as the spilling flow might significantly modify the hydraulic behavior for the undershot flow.

4.5. Upward effect

The upward sensitivity of a cross regulator expresses the link between the downstream height ($H_{DS}$), taken as the INPUT, and the level upstream of the structure ($H_{US}$) taken as the OUTPUT. It has a meaning only for submerged conditions and is important when assessing the way a backwater curve propagates upstream.

When the upstream reach is not affected by the change of water depth at the cross-regulator the upward sensitivity is simply equal to 1. The same variation will be expected for $H_{US}$ and $H_{DS}$, provided the hydraulic conditions at the structure are invariant. In most cases, the on-going discharge in the upstream reach will vary because a change of upstream water depth causes a variation of the withdrawals in the upstream reach. The analysis of the upward sensitivity cannot be simply local but has to be done at aggregated level. Development for this situation is given in Appendix A.

5. Aggregated sensitivity

5.1. Aggregation of discharge variations within a reach

A reach is considered as the pool between two consecutive hydraulic control cross-structures. It is assumed to be composed of “n” offtakes and a downstream cross-regulator.

To aggregate perturbation impacts within a reach, it is necessary to consider the water depth deviation along the reach. Here we assume that the change in water depth at any location [$\Delta H_{US}(i)$], along a reach controlled by a cross-regulator (G), is linearly related to the change occurring at the cross-regulator [$\Delta H_{US}^{G}$], as:

$$\Delta H_{US}(i) = m_i \Delta H_{US}^{G}. \quad (15)$$

The approximation, expressed in Eq. (15), has been validated for this study using computational outputs of hydraulic simulations in typical conditions of Sri Lanka. It is also supported by the outputs of a recent study made by Strelkoff et al. (1998) on the steadiness of the backwater curves once put into non-dimensional form.

Depending on the distance between the offtake and the regulator, the coefficient $m_i$ may vary with on-going discharge, as the backwater curve from the regulator is modified. However, for a given discharge it is relatively constant for small variations of water depth.
about a target (say plus or minus 20 cm). It can be considered as a site specific parameter ($0 < m_i < 1$), and constant for each offtake within a limited range of on-going discharge variation. Its value depends on the position of the point along the backwater curve and has to be determined by hydraulic computation or experimental measurements. Values of $m_i$ range between 0 for offtakes fed by a canal section under normal flow and 1 for offtakes close to a regulator.

From Eq. (5) we write the sensitivity of the delivery for offtake ‘$i$’:

$$ S_1(i) = \frac{dq_i}{dH_{US}(i)}. \tag{16} $$

Then the total deviation of discharge delivered $\Delta Q_{del}$ within the reach, is computed by summing the offtake deviations, using Eqs. (15) and (16):

$$ \Delta Q_{del} = \Delta H_{US}^G \sum_{i=1}^{n} q_i m_i S_1(i). \tag{17} $$

Eq. (17) expresses the delivery deviation within the reach, as the product of the deviation in water depth at the cross-regulator, and the weighted sum of the offtake sensitivities, the weight being the product of the offtake discharge and the parameter $m_i$.

The value of the deviation of the outflow leaving the reach, is derived from Eq. (13):

$$ \Delta Q_{out} = \frac{Q_{out} \Delta H_{US}^G}{S_G}. \tag{18} $$

Then writing the governing balance equation for the perturbation:

$$ \Delta Q_{in} = \Delta Q_{del} + \Delta Q_{out} \tag{19} $$

and replacing Eqs. (17) and (18) in Eq. (19), leads to:

$$ \Delta Q_{in} = \Delta H_{US}^G \left\{ \sum_{i=1}^{n+1} q_i m_i S_1(i) \right\}. \tag{20} $$

Eq. (20) links, at reach level, the resulting deviation in water depth to the change in the inflow ($Q_{in}$). In Eq. (20), the downstream cross-regulator is simply considered as offtake ($n+1$) of the reach, having the following properties:

$$ q_{n+1} = Q_{out}, \quad m_{n+1} = 1, \quad S_{1(n+1)} = \frac{1}{S_G}. \tag{21} $$

Eq. (20) is the basic equation of the reach from which aggregated sensitivity indicators can be derived.

6. Single reach sensitivity indicators

In the reactive process, the perturbation (INPUT) is defined as a variation of discharge at the boundaries of the pool. This variation can be physically provoked by a change in
the on-going flow coming from the upstream reach, or a change in the discharge balance within the reach. Both cases can be assimilated to a perturbation of discharge entering the reach \( \Delta Q_{\text{in}} \).

The corresponding OUTPUTS for reach sensitivity, are the water depth \( \Delta H \) in the reach (upstream of the regulator), deliveries within the reach \( Q_{\text{del}} \), and the downstream discharge \( Q_{\text{out}} \). A specific sensitivity indicator can be defined for each output variable.

The sensitivity indicator of water depth \( (S_{\text{RH}}) \) is defined as:

\[
S_{\text{RH}} = \frac{\Delta H^G_{\text{US}}}{\Delta Q_{\text{in}}/Q_{\text{in}}},
\]

the value of which is derived directly by substituting \( \Delta Q_{\text{in}} \) from Eq. (20):

\[
S_{\text{RH}} = \frac{1}{\sum_{i=1}^{n+1}(q_{i}/Q_{\text{in}})m_{i}S_{i}(i)}.
\]

The sensitivity indicator for conveyance \( (S_{\text{RC}}) \) is expressed as the ratio of the perturbations leaving and entering the system. It can also be called the propagation factor and is defined as:

\[
S_{\text{RC}} = \frac{\Delta Q_{\text{out}}}{\Delta Q_{\text{in}}}, \quad (S_{\text{RC}} \leq 1).
\]

Substituting Eqs. (18) and (22) in Eq. (24) leads to:

\[
S_{\text{RC}} = S_{\text{RH}} \frac{Q_{\text{out}}}{S_{G} Q_{\text{in}}},
\]

The sensitivity indicator for conveyance is, therefore, the product of the conveyance ratio \( (Q_{\text{out}}/Q_{\text{in}}) \) and the ratio of the water depth reach sensitivity and regulator sensitivity \( (S_{\text{RH}}/S_{G}) \).

The reach sensitivity of delivery, \( (S_{\text{RD}}) \) is the ratio of the delivery perturbation to the inflow perturbation:

\[
S_{\text{RD}} = \frac{\Delta Q_{\text{del}}}{\Delta Q_{\text{in}}}, \quad (S_{\text{RD}} \leq 1).
\]

It is:

\[
S_{\text{RD}} = 1 - S_{\text{RC}}.
\]

The flexibility indicator, as described previously, Eq. (1), can also be derived using Eq. (26):

\[
F = S_{\text{RD}} \frac{Q_{\text{in}}}{Q_{\text{del}}},
\]

7. Subsystem sensitivity

For a subsystem made of \( “k” \) reaches, the ratio \( \Delta Q_{\text{out}}(k)/\Delta Q_{\text{in}}(1) \) represents the sensitivity for conveyance. Computation is started using (Eq. (24)), at the reach \( k \):

\[
\Delta Q_{\text{out}}(k) = S_{\text{RC}}(k)\Delta Q_{\text{in}}(k).
\]
By definition, the outflow deviation at reach \((k-1)\) is equal to the inflow deviation of reach \((k)\):

\[
\Delta Q_{\text{in}}(k) = \Delta Q_{\text{out}}(k - 1). \tag{30}
\]

Then replacing Eq. (30) successively from the reach \((k-1)\) up to reach (1) in Eq. (29), it becomes:

\[
\Delta Q_{\text{out}}(k) = S_{\text{RC}(k)}S_{\text{RC}(k-1)} \ldots S_{\text{RC}(1)} \Delta Q_{\text{in}(1)}. \tag{31}
\]

The *aggregated sensitivity for conveyance* for a canal made of \(k\) reaches is equal to the product of the individual reach conveyance sensitivity indicators:

\[
S_{\text{KC}} = \prod_{i=1}^{k} S_{\text{RC},i}, \tag{32}
\]

which can be rewritten replacing Eq. (25) in Eq. (32):

\[
S_{\text{KC}} = \frac{Q_{\text{out}}(k)}{Q_{\text{in}}(1)} \prod_{i=1}^{k} \frac{S_{\text{RH}(i)}}{S_{\text{G}(i)}}. \tag{33}
\]

Given the balance of perturbation Eq. (19), the *subsystem sensitivity for delivery* is derived from the previous equation:

\[
S_{\text{KD}} = 1 - S_{\text{KC}}. \tag{34}
\]

Expressions of aggregated sensitivity indicators are given in Table 1.

### 8. Application of sensitivity analysis in canal assessment

Application of the previous analytical approach has been carried out for two different canals in Sri Lanka:

- **Canal A** is a lined canal 44 km long (the Upper part of the Left Bank Main Canal of the Mahaweli System-B). It has a capacity of 65 m\(^3\)/s, and is equipped with 12 cross-regulators and 24 offtakes. Canal A distributes 22 m\(^3\)/s to branch and tertiary canals.
- **Canal B** is an earthen canal, 27 km long (the Right Bank Main Canal of the Kirindi Oya Irrigation and Settlement Project) with a capacity of 11 m\(^3\)/s, equipped with 19 cross-regulators and 37 offtakes.

These two canals are analyzed and compared for sensitivity. Firstly, values of structure sensitivity are examined. Secondly sensitivity is aggregated at reach level. Thirdly the behavior of the entire systems are estimated particularly looking at perturbation propagations.

#### 8.1. Analysis for Canal A

##### 8.1.1. Local sensitivity

The offtake sensitivities of delivery along Canal A are high and quite variable as shown in Fig. 2. (average=1.31 and CV=0.6) (see for more details Renault and Hemakumara, 2000).
Table 1
Aggregated sensitivity indicators for irrigation system

<table>
<thead>
<tr>
<th>Input</th>
<th>Physical situation</th>
<th>Output</th>
<th>Sensitivity indicator</th>
<th>Sensitivity expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta H_{US} )</td>
<td>Offtake</td>
<td>( \Delta q/q )</td>
<td>( S = \frac{\Delta q}{q} \frac{1}{\Delta H_{US}} )</td>
<td>( S_0 = \frac{\alpha}{H_E} )</td>
</tr>
<tr>
<td>( \Delta Q_{out} )</td>
<td>Regulator</td>
<td>( \Delta H_{US} )</td>
<td>( S_G = \frac{\Delta H_{US}}{\Delta Q_{out}/Q_{out}} )</td>
<td>( S_G = \frac{H_E}{\alpha} )</td>
</tr>
<tr>
<td>( \Delta H_{DS} )</td>
<td>Upward sensitivity</td>
<td>( \Delta H_{US} )</td>
<td>( S_{Uw}^G = \frac{\Delta H_{US}}{\Delta H_{DS}} )</td>
<td>( S_{Uw}^G = \frac{1}{1 + ((H_{US} - H_{DS})/\alpha S_G)} )</td>
</tr>
<tr>
<td>( \Delta Q_{in} )</td>
<td>Reach</td>
<td>( \Delta H_{US} )</td>
<td>( S_{RH} = \frac{\Delta H_{US}^G}{\Delta Q_{in}/Q_{in}} )</td>
<td>( S_{RH} = \frac{1}{\sum_{i=1}^{n+1} (q_i/Q_{in})m_i(i)} )</td>
</tr>
<tr>
<td>( \Delta Q_{out} )</td>
<td></td>
<td>( \Delta Q_{in} )</td>
<td>( S_{RC} = \frac{\Delta Q_{out}}{\Delta Q_{in}} )</td>
<td>( q_{n+1} = Q_{out}; m_{n+1} = 1; ) ( S_{1(n+1)} = \frac{1}{S_G} )</td>
</tr>
<tr>
<td>( \Delta Q_{del} )</td>
<td></td>
<td>( \Delta Q_{in} )</td>
<td>( S_{RD} = \frac{\Delta Q_{del}}{\Delta Q_{in}} )</td>
<td>( S_{RD} = \frac{S_{RH} Q_{out}}{S_G Q_{in}} )</td>
</tr>
<tr>
<td>( \Delta Q_{in} )</td>
<td>Canal</td>
<td>( \Delta Q_{out} )</td>
<td>( S_{KC} = \frac{\Delta Q_{out}(n)}{\Delta Q_{in}(1)} )</td>
<td>( S_{KC} = \prod_{i=1}^{n} S_{RC} )</td>
</tr>
<tr>
<td>( \Delta Q_{del} )</td>
<td></td>
<td>( \Delta Q_{in} )</td>
<td>( S_{KD} = \sum_{i=1}^{n} \frac{\Delta Q_{del}(i)}{\Delta Q_{in}(1)} )</td>
<td>( S_{KD} = 1 - \prod_{i=1}^{n} S_{RC} )</td>
</tr>
</tbody>
</table>
1999). The cross-regulators, made of undershot radial gates with two crested side weirs, are also quite sensitive as illustrated in Fig. 2. The average value for the eleven first regulators is 3.6 and the coefficient of variation is 0.3. The sensitivity indicators for cross-regulators are expressed in meters per relative change of discharge, therefore a value of 4 indicates highly sensitive regulators, as a variation of 5% of on-going discharge creates a depth variation of 20 cm. It is helpful to classify sensitivity into three categories of high, medium and low to facilitate the comparison of different structures. Table 2 summarizes a classification used here, together with a summary of the results for Canal A and B.

If we consider that highly sensitive regulators are those having an indicator above 2.5, low sensitive regulators those with an indicator below 1.5 and medium sensitive in between, then almost all cross-regulators (10 out of 11) in Canal A are highly sensitive (Table 2).

8.1.2. Reach sensitivity

The computed indicators of reach sensitivity for water depth ($S_{RH}$, Eq. (23)), are displayed in Fig. 3. Note that there is no cross-regulator no. 8, and accordingly no reach

Table 2

<table>
<thead>
<tr>
<th>Class of sensitivity</th>
<th>Sensitivity indicator</th>
<th>Approx. variation of height per percentage of discharge variation</th>
<th>Canal A</th>
<th>Canal B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>&lt;1.5</td>
<td>&lt;1.5 cm</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>Medium</td>
<td>1.5 &lt; and &lt;2.5</td>
<td>2 cm</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>High</td>
<td>2.5 &lt; ...</td>
<td>&gt;2.5 cm</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig. 2. Distribution of local sensitivity indicators along Canal A.
no. 8. All reaches have high values of sensitivity (above 2.5), with the exception of reach 12 having a low value (1.2). An indicator of 3 means that the water level varies at 3 cm per percent of variation of discharge entering the reach.

Values for the sensitivity indicator of conveyance ($S_{RC}$, Eq. (25)) are displayed in Fig. 4 along with the reach sensitivity of delivery ($S_{RD}$, Eq. (27)). Six reaches have a value for conveyance of less than 0.7, which means an incoming perturbation is highly attenuated.

Fig. 3. Aggregated sensitivity indicator for water depth at reach level along Canal A.

Fig. 4. Reach sensitivity indicators for delivery and conveyance along Canal A.
8.1.3. System sensitivity

The system response to perturbation can be seen in Fig. 5 where the evolution of a perturbation generated at the upstream end of the canal, is plotted along the canal. It creates a decay curve showing the attenuation of the perturbation. One decay curve has been computed using the aggregated sensitivity for conveyance, Eq. (33). The other decay curve is the result of hydraulic simulations under steady flow conditions with and without perturbations, using the SIC hydraulic model (Baume et al., 1993). It can be seen that the analytical approach (Eq. (33)) and the simulation do not deviate significantly when expressing the perturbation decay along the canal.

Reach 1 highly attenuates an incoming perturbation, while reaches 2, 3 and 4 have almost no effect. Reach 5 absorbs perturbation, and only 20% of the initial perturbation remains after reach 5.

The upward indicators along Canal A computed using Eq. (A.7) (Appendix A), have almost all low values (below 0.3). Only cross-regulator 2, has an indicator of one because there is no offtake in reach 2, hence the perturbation is transmitted upwards unchanged. These low values are the results of both a low submergence at many cross-regulators and a high sensitivity of offtakes along the canal. Globally Canal A does not propagate perturbations upwards.

8.1.4. Practical conclusions for operation of Canal A system operator

From the above description of the sensitivity indicators, it is clear that Canal A requires a lot of attention (frequent checking and accurate operational adjustments) to ensure a good performance. Offtake sensitivity indicators are high and variable, and regulator sensitivities are high and quite constant (with the exception of CR 12). An analysis of
Fig. 2 enables more specific recommendations and leads to dividing the system into two operational parts:

- For reaches controlled by CR1, CR5, CR6, CR7, CR9, CR10 and CR11 the control exercised by the system operator must be frequent and very precise. Not only do the cross-regulators require operation within a very limited tolerance of water depth ($\Delta H$), depending on the targets for performance, but also the offtakes must be verified and readjusted frequently.
- For reaches controlled by CR2, CR3, CR4 the control exercised by the system operator can be less strict than for the others because of low sensitivity either for the regulator or for the offtakes.

8.2. Analysis for Canal B

8.2.1. Local sensitivity

Offtake sensitivities of delivery along Canal B are low and quite constant (Renault and Hemakumara, 1999). The average is 0.47 and the coefficient of variation is 0.22. The cross-regulators are of the same type as for Canal A, undershot gates with two crested side weirs. Their sensitivities are lower than that of Canal A, i.e. average $=1.53$ and CV $=0.33$ compared to 3.6 and 0.3. According to the classes defined in Table 2., cross-regulators in Canal B are equally shared between the medium and low sensitivity classes.

8.2.2. Reach sensitivity

The computed indicators of reach sensitivity for water depth ($S_{RH}$, Eq. (23)) are displayed in Fig. 6. Note that there is no cross-regulator no. 1, and accordingly no reach no. 1. All reaches have low values of sensitivity (below 2.5).
Values for the sensitivity indicator of conveyance ($S_{RC}$, Eq. (25)) are quite high (all above 0.8), inversely values for the reach sensitivity of delivery ($S_{RD}$, Eq. (27)) are low. This means that reaches in Canal B have a tendency to propagate perturbations.

8.2.3. System sensitivity

The evolution of a perturbation generated at the upstream end of the canal is plotted along the canal, Fig. 7. Again two decay curves are shown, one as a result of the sensitivity approach, the other as outputs from hydraulic simulations. They clearly show that Canal B has a propensity to propagate perturbations downwards. After 27 km and 18 cross-regulators, the perturbation still has a magnitude of 40% of the initial value.

All the upward indicators along Canal B, computed using Eq. (A.7) (Appendix A), are close to one. Canal B propagates perturbations upwards as most cross-regulators are under submerged conditions and reaches have a low sensitivity.

8.2.4. Practical conclusions for operation of Canal B system operator

For the same level of performance, Canal B requires less care and less operational effort than Canal A. This is due to a much lower and also less variable sensitivity along Canal B than along Canal A. Canal B propagates perturbations downward as well as upwards. Some reaches of Canal B require a little more care than others (7–10 and 13–17) as they have higher reach sensitivity indicators. This additional care in practice is more important for the last set (13–17). The reaches of this set have the highest sensitivity values and are placed in the downstream part of the canal, which means the likelihood of
experiencing perturbations generated in the upstream reaches is high. This was confirmed by the variations of water level recorded at the offtakes for six seasons along Canal B, showing that the current homogeneous operation mode leads to higher fluctuations of water level in the downstream part of the canal.

9. Conclusions and perspectives

The analytical approach of the sensitivity of irrigation systems allows a description of the behavior of the irrigation system as a whole, or a consideration of the spatial distribution of local behavior. This analysis allows the identification of systems, or parts of systems, which required more care than others in operation, as they are more sensitive to perturbations. Sensitivity analysis also allows the prediction of global canal behavior such as its propagation/attenuation capacity. A sensitivity analysis does not require sophisticated means but only the available geometrical data of the system components. The validation of the analytical approach has been carried out for the propagation properties by comparing the results from the analytical solution with outputs from hydraulic simulations.

The application of a sensitivity analysis to aid in the design of a smart information system and to build cost-effective strategies for operation are under investigation and will be reported soon.

10. Nomenclature

\begin{itemize}
\item \(a\) discharge coefficient
\item \(A(w)\) cross-section area through the structure as a function of the setting \((w)\)
\item \(b\) discharge coefficient
\item \(c\) discharge coefficient
\item \(dH_{US}\) variation of water height upstream of the structure
\item \(dq\) variation of discharge through an offtake
\item \(dQ\) variation of discharge in the parent canal
\item \(F\) flexibility
\item \(H\) water height
\item \(H_{DS}\) water elevation downstream of the offtake
\item \(H_E\) head loss equivalent
\item \(H_{US}\) water elevation upstream of the structure
\item \(H_{REF}\) elevation of the reference point downstream of the cross-regulator
\item \(m_i\) coefficient of water depth variation within a backwater curve
\item \(q\) discharge through the offtake
\item \(Q\) discharge in the parent canal
\item \(Q_{del}\) discharge delivered within a canal reach
\item \(Q_{in}\) discharge entering a canal reach
\item \(Q_{out}\) discharge leaving a canal reach
\item \(S\) sensitivity indicator
\end{itemize}
Appendix A

Upward cross-regulator sensitivity

Consider a perturbation coming from a downstream reach, the INPUT is then the water depth deviation downstream of the cross-regulator \( \Delta H_{DS} \) and the OUTPUT is the water depth deviation upstream of the cross-regulator \( \Delta H_{US} \). The upward sensitivity indicator at the cross-regulator is as follows:

\[
S_{Uw}^{G} = \frac{\Delta H_{US}}{\Delta H_{DS}}. \tag{A.1}
\]

The change in water depth upstream of the cross-regulator, propagates upstream and may modify the discharge delivered within the upstream reach. We assume here the effects of a perturbation being limited to a single reach, computation can be established for several reaches but becomes rapidly cumbersome. Hence we assume for the upstream reach:

\[
\Delta Q_{\text{del}} = -\Delta Q_{\text{out}}. \tag{A.2}
\]

Then taking the derivative of the logarithm of Eq. (6), gives:

\[
\frac{\Delta Q_{\text{out}}}{Q_{\text{out}}} \alpha \frac{\Delta [H_{US} - H_{DS}]}{[H_{US} - H_{DS}]}.
\tag{A.3}
\]

Because variations of water depth are of the same sign, we can write:

\[
\Delta [H_{US} - H_{DS}] = \Delta H_{US} - \Delta H_{DS}, \tag{A.4}
\]
Furthermore we can rewrite Eq. (13) with consideration to the sign of variation as follows:

\[ S_G = \frac{-\Delta H_{US}}{(\Delta Q_{out}/Q_{out})}. \]  
\[ \text{(A.5)} \]

Replacing Eqs. (A.4) and (A.5) in Eq. (A.3) and dividing by \((\Delta H_{DS})\) gives:

\[ -\frac{S_{_{Uw}}}{S_G} = \alpha \frac{(S_G - 1)}{H_{US} - H_{DS}}. \]  
\[ \text{(A.6)} \]

Reorganizing Eq. (A.6), leads to the upward sensitivity indicator:

\[ S_{_{Uw}} = \frac{1}{1 + ((H_{US} - H_{DS})/\alpha S_G)}. \]  
\[ \text{(A.7)} \]

References


