A parameterization of cloud microphysics for long-term cloud-resolving modeling of tropical convection

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Abstract

This paper documents development of a simple cloud microphysical parameterization for use in long-term cloud-resolving simulations of maritime tropical convection. The parameterization is based on the bulk approach and considers two classes of liquid water (cloud water and rain) and two classes of ice (slowly falling ice A and fast-falling ice B). Ice A represents unrimed or lightly rimed ice particles whose spectral characteristics are assumed to follow aircraft observations in tropical upper-tropospheric anvil clouds. Ice B, on the other hand, represents heavily rimed ice particles (e.g., graupel) which occur in the vicinity of convective towers. Mixing ratios for these four classes of cloud condensate are used as model variables. Together with the mixing ratio for water vapor, five field variables are used to represent all forms of water in the tropical atmosphere. The parameterization is used in a prescribed flow model to illustrate development of tropical convective and stratiform precipitation. Application of the parameterization to the cloud-resolving simulations of cloud systems observed during the TOGA COARE field campaign is also presented. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Cloud modeling; Cloud microphysics; Tropical convection

1. Introduction

The increase of computational resources in recent years enables application of cloud models to problems directly related to the role of clouds in the climate system. This is especially relevant for the Tropics because diabatic effects associated with cloud processes can only be crudely captured in large-scale models, which have to rely on sub-gridscale parameterizations. Cloud-resolving simulations with spatial resolutions of

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~ 1 km are able to consider tropical cloud systems in large computational domains (up to $10^5$ km$^2$) and to study their evolution over extended periods of time (up to several weeks, see Grabowski et al., 1996 and references therein; Grabowski et al., 1998, Wu et al., 1998, 1999, among many others). This approach is referred to as cumulus ensemble modeling, or a cloud-resolving modeling (CRM) approach.

Representation of cloud microphysics in cloud models applied in such long-term simulations is arguably one of the major uncertainties of the CRM approach. The role cloud microphysical processes play in such simulations has been recently documented by Wu et al. (1999) and, in more general terms, by Grabowski et al. (1999). Because the CRM approach is computationally very demanding, it is unlikely that sophisticated approaches which consider details of the evolution of cloud condensate from very small water or ice particles up to precipitation-size particles can be included. Consequently, simple microphysical parameterizations, employing as few field variables as possible to represent cloud condensate, and firmly based on cloud observations, should be the starting point. This is the approach advocated by Grabowski (1998). It can also be argued that there is still not enough known about cloud microphysics (ice physics in particular) for trustworthy comprehensive parameterizations to be designed for use in climate-related cloud-resolving studies. From that perspective, a simple approach which includes as few tunable parameters as possible and at same time is capable of capturing the essential aspects of cloud microphysics, seems very appealing.

The purpose of this paper is to present a microphysical scheme that has been developed for long-term CRM of tropical convection. The emphasis is on cloud microphysics at cold temperatures, that is, involving the ice phase, because upper-tropospheric ice clouds have an important effect on radiative processes in the tropics (e.g., Ramanathan and Collins, 1991). In addition, traditional ice schemes used in cloud models are based on spectral characteristics of ice particles obtained in extratropical cloud systems (either convective or stratiform). As far as the ice microphysics is concerned, the scheme proposed herein follows a strategy suggested by Koenig and Murray (1976) (hereinafter KM76). It considers two classes of ice, referred to as ice A and ice B for which only conservation equations for mixing ratios are considered (KM76 solved for both mixing ratios and number concentrations of ice A and ice B in their approach). Ice A represents slowly falling ice particles produced by diffusional growth of pristine ice crystals. Its spectral characteristics are based on aircraft observations of upper-tropospheric tropical ice clouds (McFarquhar and Heymsfield, 1997, hereinafter MH97). Ice B, on the other hand, represents heavily rimed ice particles (graupel) which in the proposed scheme originate from the interaction of ice A with rain (KM76). These two mechanisms of precipitation formation are thought to mimic development of stratiform and convective precipitation as observed in the tropics (Houze, 1997).

Section 2 presents water and energy conservation equations applied by the microphysical scheme. Details of the microphysical transfer terms are given in Appendix A. Section 3 presents application of the scheme to the problem of precipitation development in a prescribed-flow framework. These highly idealized simulations illustrate development of tropical convective and stratiform precipitation. Application of the scheme to cloud-resolving simulations of tropical cloud systems is discussed in Section 4.
2. The parameterization

As discussed in Section 1, the approach is designed to capture essential aspects of the cloud microphysics using as few model variables as possible. As far as warm-rain processes are concerned, an approach similar to Kessler (1969) is used, that is, mixing ratios for the cloud water and for the rain water are predicted. In addition, mixing ratios for the two classes of ice, referred to as ice A and ice B, are considered. Altogether, four model variables are designated for four classes of the cloud condensate. The equations describing the moist precipitating thermodynamics are as follows:

\[
\frac{\partial \rho_A \theta}{\partial t} + \nabla (\rho_A \vec{u} \theta) = \mathcal{F}_\theta = -L_v \frac{\theta}{c_p T_e} (\text{COND} - \text{REVP}) + \frac{L_v \theta}{c_p T_e} (\text{DEPA} + \text{DEPB} + \text{HOMA1}) \\
+ \frac{L_v \theta}{c_p T_e} (\text{RIMA} + \text{RIMB} + \text{HOMA2} + \text{HETB1} - \text{MELA} - \text{MELB})
\] (1a)

\[
\frac{\partial \rho_A}{\partial t} + \nabla (\rho_A \vec{u} q) = \mathcal{F}_{q_A} = -\text{COND} + \text{REVP} - \text{DEPA} - \text{DEPB} - \text{HOMA1}
\] (1b)

\[
\frac{\partial \rho_A}{\partial t} + \nabla (\rho_A \vec{u} q_\Lambda) = \mathcal{F}_{q_\Lambda} = \text{COND} - \text{AUTC} - \text{RCOL} - \text{RIMA} - \text{RIMB1} \\
- \text{HOMA2} - \text{HETA}
\] (1c)

\[
\frac{\partial \rho_A}{\partial t} + \nabla [\rho_A (\vec{u} - V_A \hat{k}) q] = \mathcal{F}_{q_A} = -\text{REVP} + \text{AUTC} + \text{RCOL} + \text{MELA} \\
+ \text{MELB} - \text{HETB1} - \text{RIMB2}
\] (1d)

\[
\frac{\partial \rho_A}{\partial t} + \nabla [\rho_A (\vec{u} - V_B \hat{k}) q_B] = \mathcal{F}_{q_B} = \text{HOMA} + \text{HETA} + \text{DEPA} + \text{RIMA} \\
- \text{MELA} - \text{HETB2}
\] (1e)

\[
\frac{\partial \rho_A}{\partial t} + \nabla [\rho_A (\vec{u} - V_B \hat{k}) q_B] = \mathcal{F}_{q_B} = \text{HETB} + \text{DEPB} + \text{RIMB} - \text{MELB}
\] (1f)

where \( \theta \) is the potential temperature, \( q \) is the water vapor mixing ratio, and \( q_A, q, q_\Lambda, \) and \( q_B \) are the cloud water, rain water, ice A, and ice B mixing ratios. The sinks and sources due to microphysical transfer terms are depicted as \( \mathcal{F} \); the sedimentation velocities for the rain, ice A, and ice B are \( V_V, V_A, \) and \( V_B \), respectively, and \( \hat{k} \) is the unit vector in the vertical direction. Other symbols and constants are listed in Appendix B. Note that, in general, Eqs. (1a), (1b), (1c), (1d), (1e) and (1f) should include additional terms on the right-hand-side (e.g., due to subgrid-scale turbulence parameterization or gravity wave absorbers in the vicinity of model boundaries); these terms are omitted for simplicity.
The sources/sinks of water substance in Eqs. (1a), (1b), (1c), (1d), (1e) and (1f) describe phase changes and they are parameterized based on the following assumptions (see Appendix A for details). Cloud water is formed when the air becomes supersaturated with respect to water. The condensation instantaneously brings the air back to saturation. The cloud water moves with the air and evaporates instantaneously in undersaturated conditions. The cloud water is a source of rain. Rain moves relative to the air, collects cloud water as it falls through the cloud, and can evaporate in the undersaturated conditions outside clouds. Raindrops are assumed to follow the size distribution of Marshall and Palmer (1948) with the prescribed intercept and the slope dependent upon the rain mixing ratio. When temperatures fall below freezing, ice A forms due to either heterogeneous or homogeneous (for temperatures colder than $-40^\circ$C) nucleation of ice crystals. The heterogeneous nucleation rate depends on the availability of ice-forming nuclei, which in the current formulation depends on the air temperature alone. Ice A particles are assumed to follow size distributions observed in upper-tropospheric tropical anvil clouds (MH97). MH97 consider two separate size distributions for small and large ice particles and such an approach is included in the parameterization of ice A. Interaction of ice A with raindrops results in formation of ice B which is assumed to have Marshall–Palmer type size distribution typical for graupel particles as in Rutledge and Hobbs (1984). Growth of either ice A or ice B particles is represented by the parameterization developed by KM76 which relates the ice particle growth rate (by either water vapor diffusion or accretion of supercooled liquid water, or both) to environmental conditions. Both ice A and ice B move relative to the air with sedimentation velocities depending on the ice mixing ratio and the air density. Upon crossing the melting level, ice A and ice B melt and are converted into rain.

The microphysical water transfer terms in Eqs. (1a), (1b), (1c), (1d), (1e) and (1f) describe the following processes (the sink and the source of the water are shown in the parentheses):

- **COND** ($q_a \rightarrow q_c$): condensation of water vapor to form cloud water;
- **AUTC** ($q_c \rightarrow q_i$): autoconversion of cloud water into rain (initiation of the rain field);
- **RCOL** ($q_c \rightarrow q_r$): collection of cloud water by rain water;
- **REVP** ($q_r \rightarrow q_s$): evaporation of rain;
- **HETA** ($q_c \rightarrow q_h$): heterogeneous nucleation of ice A (freezing of cloud droplets);
- **HOMA = HOMA1 + HOMA2** ($q_c \rightarrow q_h$): homogeneous nucleation of ice A;
- **HETB = HETB1 + HETB2** ($q_r \rightarrow q_h$): nucleation of ice B due to interaction of rain with ice A;
- **DEPA** ($q_c \rightarrow q_h$): growth of ice A due to deposition of water vapor;
- **DEPB** ($q_r \rightarrow q_h$): growth of ice B due to deposition of water vapor;
- **RIMA** ($q_i \rightarrow q_h$): growth of ice A due to accretion of cloud water (i.e., growth by riming);
- **RIMB = RIMB1 + RIMB2** ($q_i \rightarrow q_h$): growth of ice B by accretion of cloud water and rain;
- **MELA** ($q_h \rightarrow q_i$): melting of ice A to form rain; and
- **MELB** ($q_h \rightarrow q_i$): melting of ice B to form rain.

Detailed expressions for these terms are given in Appendix B.
The treatment of the sources and sinks on the rhs of Eqs. (1a), (1b), (1c), (1d), (1e) and (1f) is similar to that used by Grabowski and Smolarkiewicz (1996): only condensation is treated to the second order in time and all other sources apply uncentered (i.e., first order in time) approach.

Because of the finite time step, sources/sinks of water variables can result in negative values of a condensate variable. A simple solution is to limit the lhs of Eq. (1c) as

\[ F_{q_c}' = \max \left( F_{q_c}, \frac{q_c}{\Delta t} \right) \]  

(2a)

where \( \Delta t \) is the model time step and similar expressions hold for Eqs. (1d), (1e) and (1f) and to apply \( F' \) in Eqs. (1a), (1b), (1c), (1d), (1e) and (1f). Unfortunately, such an approach results in a lack of conservation of total water and energy, as the adjustment (2) affects only a sink of one variable and not the corresponding source of another variable in the system (1). To ensure conservation of water and energy, the adjustment (2a) is supplemented with the adjustment of the water vapor and temperature sources according to:

\[ F_{q_c}' = F_{q_c} - \Delta F_{q_c} - \Delta F_{q_t} - \Delta F_{q_h} - \Delta F_{q_a} \]  

(2b)

\[ F_{q_t}' = F_{q_t} + \frac{L_v \theta_v}{e_p T_e} (\Delta F_{q_t} + \Delta F_{q_c}) + \frac{L_v \theta_v}{c_p T_e} (\Delta F_{q_h} + \Delta F_{q_a}) \]  

(2c)

where \( \Delta F_{q_c} = F_{q_c}' - F_{q_c} \) and similar expressions hold for \( \Delta F_{q_t}, \Delta F_{q_h} \) and \( \Delta F_{q_a} \).

The model applies accurate formulations for the saturated water vapor pressure over water and over ice for a wide range of ambient temperatures. Tables of the saturated vapor pressure with respect to water and ice are created during model startup using accurate but expensive formulas discussed by Flatau et al. (1992). The tables provide saturated vapor pressure using the temperature interval of 0.1 K. Linear interpolation using the tabulated values is applied during model run to deduce the saturated vapor pressure for a given temperature.

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Fig. 1. Horizontal velocities [thin lines, solid (dashed) for positive (negative) values, contour interval 3 m s\(^{-1}\)] and vertical velocities (thick lines, contour interval 1 m s\(^{-1}\)) used in the kinematic test mimicking development of convective precipitation.
3. Kinematic framework tests

Prescribed flow simulations (Szumowski et al., 1998) represent an attractive way of testing microphysical parameterizations before they are used in a dynamic model. In tests presented in this paper, Eqs. (1a), (1b), (1c), (1d), (1e) and (1f) is solved using a two-dimensional \((x-z)\) flow prescribed to mimic development of either convective or stratiform precipitation (Houze, 1997). In the former case, precipitation develops mostly due to accretional growth of water and ice particles, whereas diffusional growth of the ice phase dominates in the latter one. The tests illustrate that the parameterization is capable of reproducing these two modes of precipitation development.

![Isolines of the condensate fields for the kinematic test mimicking development of convective precipitation at time \(t = 4\) h. The panels show cloud water (a), rain (b), ice A (c) and ice B (d) mixing ratios with contour intervals of 0.2 g kg\(^{-1}\) (a), 1.0 g kg\(^{-1}\) (b, c), and 2.0 g kg\(^{-1}\) (d). The dashed contours are for mixing ratios of 0.01 g kg\(^{-1}\).]
The initial temperature and moisture profiles are taken from the 00 GMT September 1st, 1974 GATE sounding applied in simulations described by Grabowski et al. (1996, 1998, 1999). These profiles are also used on inflow lateral boundaries. Zero-gradient conditions are applied on lateral outflow boundaries. The advection is performed using a monotone MPDATA algorithm (Smolarkiewicz and Margolin, 1998) and 15-s time step is used in the calculations. The simulations are carried out for 4 h for the convective situation and for 8 h for the stratiform situation, which are sufficient to established steady-state thermodynamic fields inside the domain.

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\(^2\) GATE [Global Atmospheric Research Programme (GARP) Atlantic Tropical Experiment] was conducted in the tropical eastern Atlantic in summer 1974.
In the convective precipitation case, the computational domain is 90 km long and 16 km deep with a uniform grid of size of 500 m in the horizontal dimension and 250 m in the vertical. The flow is prescribed using a steady streamfunction given by (cf. Grabowski, 1998)

\[
\Psi(x, z) = A \frac{\rho_0}{\rho_{eo}} \sin \left( \frac{\pi \hat{x}}{Z} \right) \sin \left( \frac{\pi \hat{z}}{X} \right) - \frac{1}{2} Sz^2
\]

where \( A = 4.8 \times 10^4 \) kg m\(^{-1}\) s\(^{-1}\), \( S = 2.5 \times 10^{-2} \) kg m\(^{-3}\) s\(^{-1}\), \( \rho_{eo} = 1 \) kg m\(^{-3}\), \( \hat{x} = \min(z, Z) \), \( \hat{z} = \max(-X, \min(X, x - X_s)) \), \( Z = 15 \) km, \( X = 10 \) km, \( x_s = 30 \) km, and \( x, z \) are the coordinates which cover the entire domain (i.e., \([0, 90 \text{ km}] \times [0, 16 \text{ km}]\)).

The streamfunction provides a narrow zone of the lifting throughout the entire troposphere centered at \( x_s \) (with corresponding low-level convergence and upper-level divergence) superimposed with the vertical shear of the horizontal wind which allows mean advection of the rising air from the left to the right of the domain. The velocity components are deduced from Eq. (3) as \( \rho_w u = -\partial \Psi/\partial z \) and \( \rho_w w = \partial \Psi/\partial x \) and they are shown in Fig. 1. The updraft speed reaches a maximum of about 7.5 m s\(^{-1}\) at a height of about 7.5 km.

Fig. 2 shows the isolines of the steady-state condensate fields (cloud water, rain, ice A, and ice B) for this case, whereas Fig. 3 shows the horizontal distribution of the

![Fig. 3. Distribution of the surface precipitation intensity in mm h\(^{-1}\) across the domain at time \( t = 4 \) h for the kinematic test mimicking development of convective precipitation. Precipitation rates smaller than 0.01 mm h\(^{-1}\) are not shown.](image)
surface precipitation rate. As shown in Fig. 2, precipitation development occurs through
the conversion of cloud water into rain in the lower part of the updraft, and through
growth of the graupel (ice B) field above. The trace of cloud water extends as high as 11
km, which is approximately the level at which the temperature drops below $-40^\circ C$ and
homogeneous nucleation becomes possible. Fig. 2 also documents an extensive anvil
cloud formed by the horizontal advection of ice A from the convective updraft. The
horizontal distribution of surface precipitation shows high values for the precipitation
intensity, a characteristic of convective rainfall, giving way to more moderate intensities
as one moves downstream from the location of the strong updraft.

In the stratiform precipitation case, the computational domain is chosen as 180 km
long and 16 km deep with a uniform grid of size of 1000 m in the horizontal and 250 m
in the vertical. In this case, the flow is prescribed using a streamfunction pattern

$$\Psi(x,z) = A \frac{\rho_0}{\rho_{\infty}} \sin \left( \frac{\pi \hat{z}}{Z_1} \right) \sin \left( \frac{\pi \hat{x}}{X} \right) - Q$$

with $A = 4 \times 10^4$ kg m$^{-1}$ s$^{-1}$, $Q = 3$ kg m$^{-2}$ s$^{-1}$, $\hat{z} = \min(Z,\max(0,z-Z_0))$, $\hat{x} = \max(-X,\min(X,x-x_0))$, $Z_1 = 10$ km, $Z_2 = 3$ km, $X = 70$ km, and $x_0 = 72$ km.

The streamfunction provides a wide zone of gentle lifting superimposed with a weak
flow across the domain flow from left to right. The updraft speed reaches a maximum of
about 0.8 m s$^{-1}$ at a height of about 8 km. The horizontal and vertical velocity isopleths
are shown in Fig. 4.

Fig. 5 shows the isolines of the steady-state condensate fields (cloud water, rain, ice
A, and ice B) for the stratiform precipitation case, whereas Fig. 6 shows the horizontal
distribution of the surface precipitation rate. Precipitation development in this case
occurs mainly by the depositional growth of ice A. Small amounts of cloud water (a few
tenths of grams per kilogram) occur only near the inflow into the updraft zone ($z \sim 60$
km) and in the melting layer ($90 \text{ km} \leq x \leq 130 \text{ km}$). Only traces of the ice B field

Fig. 4. As Fig. 1, but for the kinematic test mimicing development of stratiform precipitation. Contour interval
is the same for the horizontal velocities and for the vertical velocities it is 0.1 m s$^{-1}$. 
(graupel) are present. Consequently, surface precipitation rates (Fig. 6) are between 1 and 10 mm h$^{-1}$ which are typical for stratiform precipitation.

Water conservation in the kinematic framework simulations is illustrated in Figs. 7 and 8. The figure compares time step-by-time step changes of the volume-integrated total water (a sum of the water vapor and all forms of the condensate) with the total flux of water across the domain boundaries (see Section 4 of Szumowski et al., 1998 for details). In the case when both advection and microphysical adjustments conserve the total mass of water, the agreement between the two should be perfect. As Fig. 8 illustrates, this is indeed the case for the simulation of stratiform precipitation. For the convective precipitation, on the other hand, a small difference between the two estimates

![Cloud Water](attachment:image1)

![Rain Water](attachment:image2)

**Fig. 5.** As Fig. 2, but for the kinematic test mimicking development of stratiform precipitation at time $t = 8$ h. The panels show cloud water (a), rain (b), ice A (c) and ice B (d) mixing ratios with contour intervals of 0.1 g kg$^{-1}$ (a, b, d), and 1.0 g kg$^{-1}$ (c).
of the total water change develops in the second half of the simulation. However, the
difference is comparable to the fluctuations of the total water calculation due to roundoff
errors which are apparent as fluctuations in the bottom panels of Figs. 7 and 8.

4. Application to the tropical convective systems during TOGA COARE

The parameterization of moist precipitating thermodynamics described in this paper
was applied to the GCSS (GEWEX [Global Energy and Water-Cycle Experiment] Cloud
System Study) Working Group 4 (Precipitating Convective Cloud Systems) Case 2 test
(see Krueger and Lazarus, 1999 and references therein). A few examples of model
results are presented below to illustrate the performance of a cloud model which applies
the microphysical scheme described in this paper.
The numerical model is the two-time-level, nonhydrostatic Eulerian/semi-Lagrangian anelastic fluid model EULAG of Smolarkiewicz and Margolin (1997). The Eulerian version of the model is used. The 2D computational domain is 800 km long and 30 km deep with a uniform grid with 2 km resolution in the horizontal direction and 0.3 km resolution in the vertical. The time step is 10 s. The large-scale advective tendencies of the temperature and moisture (usually referred to as large-scale forcing terms) as well as the evolution of horizontal winds and the sea surface temperature (SST) are prescribed to the model based on the Intensive Flux Array (IFA) observations. The dynamic model includes the TOGA COARE surface flux algorithm of Fairall et al. (1996) with the cool-skin and warm-layer corrections omitted for simplicity. The NCAR Community Climate Model radiation code (Kiehl et al., 1994) is used for radiative transfer calculations which are performed every 10 min of model time. Cloud water and ice A are the only forms of condensate used as input into the radiation code. The effective radius $r_{\text{eff}}$ of condensate particles is assumed as 10 $\mu$m for cloud water and for ice A the effective radius is expressed as a function of the ice water content (IWC) based on the aircraft observations of MH97:

$$\log_{10} r_{\text{eff}} = aX^2 + bX + c$$

where $X = \log_{10}(10^3 \text{ IWC})$ (IWC in $\text{kg m}^{-3}$, $r_{\text{eff}}$ in $\mu$m) and the coefficients in Eq. (5) are $a = 1.913 \times 10^{-2}$, $b = 0.3588$, and $c = 2.146$. Eq. (5) gives $r_{\text{eff}}$ of 10.4, 32, and 140 $\mu$m for the IWC equal to $10^{-3}$, $10^{-2}$, and 1 g $\text{m}^{-3}$, respectively. Values of 10.4
Fig. 7. Comparison between changes of the total water expressed as a mass of water in a unit distance in the third spatial direction, i.e., in kg m$^{-1}$) during the model time step derived from the total water inside the domain (upper panel, solid line) and derived from the total water fluxes across the model boundaries (upper panel, dashed line) for the kinematic tests mimicking development of convective precipitation. The lower panel shows the difference between the two derivations of the total water change.

and 140 μm are used for IWC smaller than $10^{-4}$ g m$^{-3}$ and larger than 1 g m$^{-3}$, respectively.

Fig. 9 shows the Hovmöller ($x$-$t$) diagram of the surface precipitation rate to illustrate the organization and movement of convection. During the 5-day simulation, three periods of strong convection are separated by two periods of little or no convection. The movement of the surface precipitation from east to the west in the first 2 days and from west to east in the second half of the simulation match the results of Wu et al. (1998) (Fig. 7).

The deviations of model temperature and moisture profiles from the observations (Fig. 10) are typical for other cloud-resolving models (cf. Krueger and Lazarus, 1999,
Fig. 8. As Fig. 7, but for the kinematic test mimicking development of stratiform precipitation.

see also the WG4 website at http://www.met.utah.edu/skrueger/gcss/wg4.html) The approximately 2 K difference between the model results and observed temperature profiles is a subject of an ongoing investigation of the GCSS WG4 community (an error in the imposed large-scale forcing terms may be the culprit, see discussion in Krueger and Lazarus, 1999). The deviation in the moisture field is likely a result of the missing large-scale advective tendencies of cloud condensate (Grabowski et al., 1996, 1998; Wu et al., 1998).

Finally, Fig. 11 shows the evolution of the outgoing longwave radiation (OLR) and the top-of-the-atmosphere (TOA) albedo as predicted by the numerical model and the estimates of these quantities based on the satellite observations (the ISCCP (International

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7 The TOA albedo is defined as the ratio between the net solar flux at the top of the atmosphere (i.e., incoming minus outgoing) and the incoming solar flux.
Fig. 9. Hovmöller $(x-t)$ diagrams for the surface precipitation rate for the simulation of the TOGA COARE convection. Precipitation intensity larger than 1 and 10 mm h$^{-1}$ is shown using light and dark shading, respectively.

Satellite Cloud Climatology Project) Flux Cloud (FC) dataset for TOGA COARE, see Burks and Krueger, 1999 and references therein. The evolution of the OLR follows in general the observations and other model results (e.g., Wu et al., 1998; Krueger and
Fig. 10. Profiles of the 5-day mean difference between the model results and the observations for the temperature, the water vapor mixing ratio and the relative humidity for the simulation of the TOGA COARE convection.

Fig. 11. Evolution of the domain-average OLR and the TOA albedo for the simulation of the TOGA COARE convection. Estimates of these quantities using the FC data are shown as pluses. See text for details.
Lazarus, 1999; Wu et al., 1999, Fig. 1) with the values for periods with weak or no convection in excess of 250 W m$^{-2}$, and values smaller than 150 W m$^{-2}$ for periods of strong convection. However, it seems that the model tends to overpredict the OLR and to underpredict the TOA albedo, at least when compared to the FC data.

5. Concluding remarks

This paper documents a simple microphysical scheme designed for long-term cloud-resolving modeling of tropical convection. The scheme uses a traditional bulk approach for warm rain physics (Kessler, 1969) and considers two paths of precipitation development associated with ice physics. These two paths are related to the two modes of tropical rainfall: the convective mode, associated with accretional growth rain and ice, and the stratiform mode, dominated by the depositional growth of the ice field (e.g., Houze, 1997). Consequently, two classes of ice field are considered: ice A, representing unrimed or lightly rimed ice crystals, and ice B, representing the heavily rimed particles (graupel). The ice A is assumed to have spectral characteristics of ice particles observed in upper-tropospheric tropical anvil clouds (McFarquhar and Heymsfield, 1997). The formation and growth of the ice field is represented using a simplified approach of Koenig and Murray (1976).

The parameterization is applied in tests with prescribed flow (cf. Szumowski et al., 1998) which illustrate development of convective and stratiform precipitation. These tests also document conservation of water mass by the microphysical scheme. The Fortran code for these tests is available from the author and can be applied to any microphysical scheme to document its performance. The microphysical scheme is also applied to the TOGA COARE convection and is shown to produce results consistent with other cloud-resolving models.

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Appendix A

The microphysical transfer terms are derived using estimates of particle concentrations and growth rates as given by assumed particle size distributions and environmental conditions. For instance, DEPA is approximated as $DEPA = n_A (d m_A / dt)_{dep}$, where $n_A$
and \((dm_{\text{A}}/dt)_{\text{A}}\) are estimates of ice A particle concentration and the depositional growth rate of an ice A particle, respectively. The SI units are used throughout the Appendix unless otherwise stated.

Raindrops follow the size distribution of Marshall and Palmer (1948) with the constant intercept \(N_0\) and the slope \(\lambda_i\) given by

\[
\lambda_i = \left( \frac{\pi \rho_w N_0 i}{\rho_s q_i} \right)^{0.25}
\]

(A.1)

where \(\rho_w\) is the water density. It follows that the mean concentration \(n_i\), the mean size \(D_i = 2R_i\), and the mean mass \(m_i\) of raindrops, can be estimated as

\[
n_i = \frac{N_0 i}{\lambda_i}, \quad D_i = \frac{1}{\lambda_i}, \quad m_i = \frac{\pi \rho_w}{6 \lambda_i^2}.
\]

(A.2)

The raindrop terminal velocity is assumed to depend on its diameter \(D\) as \(v_t(D) = 130D^{0.5}\) (Kessler, 1969). The mass weighted terminal velocity \(V_t\) of the rain field is

\[
V_t = 251 \lambda_i^{-0.5} \left( \frac{1}{\rho_o} \right)^{0.5}
\]

(A.3)

where the reference air density in the last factor of Eq. (A.3) is taken as 1 kg m\(^{-3}\).

Parameterization of ice B (graupel) is similar to the formulation for rain and follows Rutledge and Hobbs (1984). Ice B particles are also assumed to follow the size distribution of Marshall and Palmer (1948) with the constant intercept \(N_{0B}\) and the slope \(\lambda_B\)

\[
\lambda_B = \left( \frac{\pi \rho_B N_{0B}}{\rho_s q_B} \right)^{0.25}
\]

(A.4)

where \(\rho_B\) is the graupel density. As in the rain case, the mean concentration \(n_B\), the mean size \(D_B = 2R_B\), and the mean mass \(m_B\) of ice B particles can be estimated as

\[
n_B = \frac{N_{0B}}{\lambda_B}, \quad D_B = \frac{1}{\lambda_B}, \quad m_B = \frac{\pi \rho_B}{6 \lambda_B^2}.
\]

(A.5)

The terminal velocity of an ice B particle is assumed to depend on its diameter \(D\) as \(v_b(D) = 19.3D^{0.37}\) (Rutledge and Hobbs, 1984). Consequently, the mass weighted terminal velocity \(V_B\) of the ice B field is

\[
V_B = 31.2 \lambda_B^{-0.37} \left( \frac{1}{\rho_o} \right)^{0.5}
\]

(A.6)

The formulation for ice A follows discussion in MH97. Based on in-situ aircraft observations, MH97 concluded that the ice particles in the upper tropospheric anvil clouds follow two separate size distributions: the first-order gamma distribution describing small ice particles (melted diameter smaller than 100 \(\mu\)m) and the lognormal distribution describing larger ice crystals. MH97 provide relationships between the
observed IWC and parameters of the distributions in terms of the ambient temperature. The contribution of the IWC content due to small crystals IWC$_S$ is approximated as

$$ IWC_S = \min \left( 10^{-3}, \rho_A q_A, 2.52 \times 10^{-4} \left( \frac{10^4}{\rho_0 q_A} \right)^{0.837} \right) $$

(A.7)

where a limit of 1 g m$^{-3}$ is imposed on the maximum IWC$_S$ in the case when Eq. (A.7) is applied to ice A mixing ratios higher than observed by MH97. The average mass of a small ice A particle $m_{AS}$ is calculated as

$$ m_{AS} = \frac{2\pi\rho_1}{\alpha^3} $$

(A.8)

where $\rho_1$ is the ice density and $\alpha$ is the parameter of the first-order gamma distribution (Eq. 6 in MH97):

$$ \alpha = \max \left[ 2.2 \times 10^4, -4.99 \times 10^3 - 4.94 \times 10^4 \log_{10} \left( 10^3 IWC_s \right) \right] $$

(A.9)

and the minimum value of $\alpha$ corresponds to an ice particle with a melted diameter of 100 µm. The mass-weighted terminal velocity of small ice A particles $V_{AS}$ is assumed constant $V_{AS} = 0.1$ m s$^{-1}$ (G. McFarquhar, personal communication).

The average mass of a large ice A particle $m_{AL}$ is calculated as

$$ m_{AL} = 1.67 \times 10^{12} \pi \rho_1 \exp \left( 3\mu + \frac{9}{2}\sigma^2 \right) $$

(A.10)

where $\mu$ and $\sigma$ are the parameters of the lognormal distribution (Eqs. 7 and 8 in MH97):

$$ \mu = a_\mu + b_\mu \log_{10} \left( 10^3 IWC_L \right) $$

(A.11)

$$ \sigma = a_\sigma + b_\sigma \log_{10} \left( 10^3 IWC_L \right) $$

(A.12)

with $IWC_L = \rho_0 q_A - IWC_S$. The coefficients $a$ and $b$ depend on the ambient temperature:

$$ a_\mu = 5.20 + 1.3 \times 10^{-3} T_C $$

(A.13a)

$$ b_\mu = 0.026 - 1.2 \times 10^{-3} T_C $$

(A.13b)

$$ a_\sigma = 0.47 + 2.1 \times 10^{-3} T_C $$

(A.13c)

$$ b_\sigma = 0.018 - 2.1 \times 10^{-4} T_C $$

(A.13d)

where $T_C = T - 273.16$ is the temperature in °C. The mass-weighted terminal velocity of large ice A particles $V_{AL}$ is estimated based on the aircraft observations discussed in MH97 (G. McFarquhar, personal communication):

$$ V_{AL} = 0.9 + 0.1 \log_{10} \left( 10^3 IWC_L \right) $$

(A.14)

Finally, the mass of the ice A particle and the mass-weighted terminal velocity are calculated as linear combinations of the appropriate values for small and large ice particles:

$$ m_A = \delta m_{AS} + (1 - \delta) m_{AL} $$

(A.15a)

$$ V_A = \left[ \delta V_{AS} + (1 - \delta) V_{AL} \right] \left( \frac{0.3}{\rho_0} \right)^{0.5} $$

(A.15b)
where $\delta = \text{IWC}_s / (\rho_a q_a)$ is the relative contribution of the small IWC and the reference air density in the last factor of Eq. (A.15b) is taken as 0.3 kg m$^{-3}$. When the ice particle size ($D$) or terminal velocity ($v$) as a function of the ice particle mass ($m$) is needed (for instance, to estimate the particle Reynolds number, see Eq. (A.25)), formulas representing rough fits to the observational data in the form $m = 2.5 \times 10^{-2} D^2$ and $v = 4D^{0.25}$ (Grabowski, 1988; Grabowski, 1998) are used.

The water transfer terms in Eqs. (1a), (1b), (1c), (1d), (1e) and (1f) are calculated in the following way.

- The condensation term COND is defined by the requirement to maintain saturated conditions with respect to the plane surface of water (cf. Grabowski and Smolarkiewicz, 1990, 1996).

- The autoconversion term AUTC is calculated using an approach proposed by Berry (1968) and applied by Simpson and Wiggert (1969) (Section 3) and Grabowski (1998) (Eq. 8):

\[
\text{AUTC} = 1.67 \times 10^{-5} (10 \rho_a q_c)^2 \left( 5 + \frac{3.6 \times 10^{-5} n_c}{d_c \rho_o q_c} \right)^{-1},
\]

where $n_c$ is the concentration of cloud droplets (expressed in number per cubic centimeter) and $d_c$ is the relative dispersion of cloud droplet spectrum, i.e., the ratio between standard deviation of cloud droplet spectrum and the mean droplet radius. Simpson and Wiggert (1969) suggested that the relative dispersion changes from 0.366 for maritime clouds with $n_c = 50$ cm$^{-3}$ to 0.146 for continental clouds with $n_c = 2000$ cm$^{-3}$. The relative dispersion is calculated from the prescribed $n_c$ and the two extreme values using the relation:

\[
d_c = 0.146 - 5.964 \times 10^{-2} \ln \frac{n_c}{2000}
\]

which gives the relative dispersion of 0.33, 0.26 and 0.19 for droplet concentrations of 100, 300 and 1000 cm$^{-3}$.

- Collection of cloud water by rain RCOL is calculated using a continuous collection equation for a mean raindrop:

\[
\text{RCOL} = n_c \left( \frac{dm_t}{dt} \right)_{\text{col}} = \frac{N_{0t}}{\lambda_r} \pi R_t^2 v_i (R_i) E_i \rho_o q_c = 5.78 \times 10^7 \frac{\rho_o q_c N_{0t}}{\lambda_i^{1/2}}
\]

where $E_i$ is the collection efficiency.

- Evaporation of raindrops REVP is calculated based on the evaporation of the mean raindrop:

\[
\text{REVP} = n_r \left( \frac{dm_t}{dt} \right)_{\text{evp}} = \frac{N_{0t}}{\lambda_r} 4\pi R_c G(T_c)(S - 1) = 1.26 \times 10^{-6} \frac{N_{0t}}{\lambda_i} FG'(T_c)(S - 1)
\]
where \( F_r = 0.78 + 0.27R_e^{1/2} \) is the ventilation factor (\( R_e = D_r \nu/(D_r) \) is the Reynolds number), \( G(T_e) = 10^{-7} \) \( G'(T_e) = 10^{-7}(2.2T_e/\epsilon_{sw}(T_e) + 2.2 \times 10^2/T_e)^{-1} \), and \( S = q_v/q_{sw} \) is the saturation ratio.

- The heterogeneous nucleation of ice A HETA is assumed to occur through the freezing of cloud droplets. It occurs when the ice A mixing ratio is smaller than the mixing ratio obtained from the concentration of ice nuclei at a given temperature and the mass of the ice A particle (assuming it cannot be smaller than \( 10^{-12} \) kg which corresponds to a droplet with about 10 \( \mu \) m diameter). For computational reasons, the nucleation occurs over a finite time scale \( \tau_n \), where \( \tau_n \) can be specified equal to the model time step \( \Delta t \). The formula is

\[
\text{HETA} = \frac{1 - \exp\left(-\frac{\Delta t}{\tau_n}\right)}{\Delta t} \rho_v \min\left[q_v, \max\left(0, \frac{N_{\text{IN}}m_{\Lambda}}{\rho_v} - q_v\right)\right]
\]

(A.19)

where \( m_{\Lambda}' = \max(10^{-12}, m_{\Lambda}) \). The concentration of ice nuclei \( N_{\text{IN}} \) depends on the temperature alone and it is given by (Fletcher, 1962):

\[
N_{\text{IN}}(T) = \min(10^5, 10^{-2}\exp(0.6\Delta T))
\]

(A.20)

where \( \Delta T = 273.16 - T \) and the maximum concentration (100 per liter) is imposed to prevent unrealistically high concentrations in the upper troposphere where homogeneous nucleation dominates. If appropriate, another formulation for \( N_{\text{IN}} \) can easily be included.

- The homogeneous nucleation of ice A HOMA = HOMA1 + HOMA2 occurs only for temperature colder than 233.16 K. It transfers all cloud water and part of the water vapor into the ice A category; see discussion in Appendix B of Grabowski et al. (1996). The formulae are

\[
\text{HOMA1} = \frac{1 - \exp\left(-\frac{\Delta t}{\tau_n}\right)}{\Delta t} \rho_v \max\left[q_v, \frac{q_v - q_v^*}{\beta} \right]
\]

(A.21a)

\[
\text{HOMA2} = \frac{1 - \exp\left(-\frac{\Delta t}{\tau_n}\right)}{\Delta t} \rho_v q_v
\]

(A.21b)

where the adjusted vapor mixing ratio \( q_v^* \) is defined as \( q_v^* = \beta q_{sw} + (1 - \beta)q_v \) with \( q_{sw} \) and \( q_v \) denoting the saturated mixing ratios with respect to water and ice, and the coefficient \( \beta \) depends on the temperature \( T \) it decreases linearly from \( \beta = 1 \) for \( T = 233.16 \) K to \( \beta = 0.1 \) for \( T \leq 213.16 \) K, cf. Grabowski et al., 1996, Appendix B.

- Heterogeneous nucleation of ice B HETB = HETB + HETB2 occurs as a result of collisions between raindrops and ice A particles when the temperature is below freezing. The transfer terms are calculated using an estimate of the rate of collisions based on mean sizes and terminal velocities. The rate of collisions \( N_{\text{CA}} \) per unit volume between raindrops and ice A is based on the continuous collection model and it is approximated as

\[
N_{\text{CA}} = \frac{N_{\text{CT}}}{\rho_v} \sqrt{\frac{V_t - V_{\Lambda}}{\pi R_{\text{CA}}^2}} \frac{\rho_v q_v}{m_{\Lambda}}
\]

(A.22)
and the transfer terms are given by
\[ \text{HETB}_1 = N_{i_A} m_{i_A}, \quad \text{HETB}_2 = N_{i_B} m_{i_B} \]  
(A.23)

- Growth of ice A and ice B particles by deposition of water vapor (DEPA, DEPB) and by riming (RIMA, RIMB) is calculated as described in KM76 (Section 2). In general, the growth rate of a single ice particle is prescribed based on the particle size and environmental conditions (such as temperature, supersaturation, and the amount of cloud water and rain available for growth by riming, see Fig. 2 in KM76). An important aspect of the KM76 parameterization is that the diffusional growth of an ice particle at a given supersaturation is allowed to depend on the environmental temperature. Consequently, the parameterization attempts to represent dependence of the depositional growth rate of ice particles on the particle habit. As far as the growth by riming is concerned, it is assumed to occur through the interactions between ice A and the cloud water (RIMA), whereas both cloud water (via RIMB1) and rain (via RIMB2) are considered for the growth by riming of ice B. The growth by riming is taken as a difference between the total growth (as given by regimes A–B–C–D in Fig. 2 of KM76) and the diffusional growth (A–E in Fig. 2 of KM76). With the growth of a single particle derived, the change of the mixing ratios are derived as:

\[
\begin{align*}
\text{DEPA} & = \frac{\rho_o q_A}{m_A} \left( \frac{dm_A}{dt} \right)_{\text{dep}} \\
\text{RIMA} & = \frac{\rho_o q_A}{m_A} \left( \frac{dm_A}{dt} \right)_{\text{rim}} \\
\text{DEPB} & = \frac{\rho_o q_B}{m_B} \left( \frac{dm_B}{dt} \right)_{\text{dep}} \\
\text{RIMB} & = \frac{\rho_o q_B}{m_B} \left( \frac{dm_B}{dt} \right)_{\text{rim}}
\end{align*}
\]  
(A.24a-d)

Note that Eqs. (A.24a) and (A.24c) are also applied to calculate the sublimation of ice A and ice B particles in undersaturated conditions.

- Melting of ice A and ice B (MELA, MELB) for temperatures above freezing is a source of rain. Ignoring the evaporation and/or sublimation of a melting particle, the rate of the melting \( \left( \frac{dm_A}{dt} \right)_{\text{melt}} \) may be approximated as (Pruppacher and Klett, 1978, Section 16.7, Rasmussen et al., 1984, Section 5):

\[
\left( \frac{dm_A}{dt} \right)_{\text{melt}} = -\frac{4\pi K}{L_f} D_A \Delta TF_A
\]  
(A.25)

where \( \Delta T = 273.16 - T \) and \( F_A = 0.78 + 0.27R_c^{1/2} \) is the ventilation factor. The melting term is then given as:

\[
\text{MELA} = -\frac{\rho_o q_A}{m_A} \left( \frac{dm_A}{dt} \right)_{\text{melt}} \approx 4.5 \times 10^{-7} \frac{\rho_o q_A}{m_A} D_A \Delta TF_A
\]  
(A.26)

and a similar expression is used for MELB.
Appendix B

Below is a list of symbols and model coefficients used in the parameterization. The units and/or values of a given symbol are given in the parenthesis.

**Roman symbols**

- $a_\mu$, $a_\sigma$: parameters in the formulation of $\mu$ and $\sigma$ dependence on IWC and temperature
- $b_\mu$, $b_\sigma$: parameters in the formulation of $\mu$ and $\sigma$ dependence on IWC and temperature
- $c_1$: specific heat at constant pressure ($1005 \text{ J kg}^{-1} \text{ K}^{-1}$)
- $d_1$: dispersion of the cloud droplet spectrum
- $D_0$, $D_r$, $D_A$, $D_B$: diameter, diameter of rain, ice A, ice B particle (m)
- $e_{sw}$, $e_{si}$: saturation water vapor pressure with respect to water and ice (Pa)
- $E_r$: collection efficiency for rain collecting cloud water (0.8)
- $F_r$, $F_A$, $F_B$: ventilation factor for rain, ice A and ice B (0.78 + 0.27$R_{av}^{1/2}$)
- $G$, $G'$: thermodynamic functions in the formula for raindrop evaporation ($G = 10^{-7}G'$)
- $K_a$: thermal conductivity of air ($24 \times 10^{-2} \text{ J m}^{-1} \text{ s}^{-1} \text{ K}^{-1}$)
- $L_v$, $L_s$, $L_f$: latent heat of evaporation, sublimation, and freezing ($2.50 \times 10^6$, $2.83 \times 10^6$, $3.3 \times 10^5 \text{ J kg}^{-1}$)
- $m$, $m_r$, $m_A$, $m_{AL}$, $m_{AS}$, $m_B$: mass, mass of a raindrop, mass of an ice A particle, mass of a large ice A particle, mass of a small ice A particle, mass of an ice B particle (kg)
- $n_c$: concentration of cloud droplets (200 cm$^{-3}$)
- $n_r$, $n_A$, $n_B$: concentration of raindrops, ice A, ice B particles (m$^{-3}$)
- $N_{IN}$: concentration of ice nuclei (m$^{-3}$)
- $N_{AA}$: rate of collisions per unit volume between raindrops and ice A particles (m$^{-3}$ s$^{-1}$)
- $N_{AH}$, $N_{AB}$: intercept of the Marshall–Palmer distribution for raindrops and graupel ($10^7$, $4 \times 10^6 \text{ m}^{-1}$)
- $R$: radius, $D/2$ (m)
- $R_p$: Reynolds number, $Dv/\nu$
- $t$, $\Delta t$: time, model time step (s)
- $T$, $T_e$, $T_C$, $\Delta T$: temperature, ambient profile of the temperature, temperature in °C, $\Delta T = 273.16 - T = -T_C$ (K)
- $q_r$, $q_c$, $q_i$, $q_A$, $q_B$: mixing ratios for water vapor, cloud water, rain, ice A, and ice B (kg kg$^{-1}$)
- $q_{sw}$, $q_{si}$: saturated mixing ratio with respect to water and ice (kg kg$^{-1}$)
- $\bar{v}$: terminal velocity of a single precipitation particle (m s$^{-1}$)
- $V_r$, $V_A$, $V_{AL}$, $V_{AS}$, $V_B$: mass-weighted fall velocity of rain, ice A, large ice A, small ice A, and ice B (m s$^{-1}$)
Greek symbols

- $\alpha$: parameter in the small ice crystals size distribution (m$^{-1}$)
- $\beta$: temperature-dependent coefficient in the formulation of the homogeneous nucleation of ice A
- $\delta$: the relative contribution of small ice A particles to the total ice A water content
- $\theta, \theta_e$: potential temperature, ambient profile of the potential temperature (K)
- $\lambda_r, \lambda_B$: slope of the Marshall–Palmer distribution for raindrops and graupel (m$^{-1}$)
- $\mu$: parameter in the lognormal distribution of large ice crystals
- $\nu$: kinematic viscosity of air ($2 \times 10^{-5}$ m$^2$ s$^{-1}$)
- $\rho_a, \rho_r, \rho_B$: density of water, density of ice, density of ice B particle ($10^3, 9 \times 10^2, 4 \times 10^2$ kg m$^{-3}$)
- $\sigma$: parameter in the lognormal distribution of large ice crystals
- $\tau_n$: time scale of the ice A nucleation (s)

References


