A bin-microphysics cloud model with high-order, positive-definite advection

Alexandre A. Costa\textsuperscript{a,b,*}, Gerson P. Almeida\textsuperscript{b},
Antônio José C. Sampaio\textsuperscript{b,1}

\textsuperscript{a} Department of Atmospheric Science, Colorado State University, Fort Collins, CO, USA
\textsuperscript{b} Laboratorio de Fisica de Nucens e, Departamento de Física, Universidade Federal do Ceará, Fortaleza, CE, Brazil

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Abstract

An axisymmetric, anelastic model of a convective cloud is described. The model comprises prognostic equations for the azimuthal vorticity, the perturbation potential temperature, the perturbation water vapor mixing ratio, 44 categories of cloud condensation nuclei, and 100 categories of liquid-phase hydrometeors. Results from a control simulation show that the model is capable to reproduce realistically the life cycle of a convective cloud including the production of warm rain.

A discussion of the role of advection in bin-microphysics models is presented and sensitivity tests were performed regarding the order of advection. The results show that, although the global characteristics of all simulated clouds were similar, significant differences occur with respect to their microstructure, particularly close to the cloud edges, when the order of the advective scheme changes. The conclusion is that intermediate-order advection schemes can indeed be used in cloud-resolving simulations, as far only as the gross characteristics of the cloud/cloud system are being investigated, but not poor, low-order schemes. On the other hand, the sensitivity with respect to the advection suggests that the evaluation of cloud phenomena that occur in fine-scales, such as entrainment and certain microphysical and radiational processes, must require the use of accurate, higher-order schemes. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Bin-microphysics cloud model; Advection; Simulation

* Corresponding author. Tel.: +55-85-288-9904; fax: +55-85-288-9903.
E-mail address: alex@fisica.ufc.br (A.A. Costa).
1 Current affiliation: Universidade do Vale do Acaraú, Sobral, CE, Brazil.

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1. Introduction

In the past three decades, many efforts have been made to develop and use cloud models. From the early models of dry convection to the modern bin-microphysics (e.g., Kogan, 1991; Reisin et al., 1996, etc.) and mesoscale models (e.g., Pielke et al., 1992; Xue et al., 1995), too much progress has been made. For instance, the representation of physical processes, such as turbulence, microphysics, radiation, and surface fluxes, became more and more sophisticated.

Nevertheless, one should point out that atmospheric modeling in mesoscale and cloud-scale did not incorporate, in most cases, the latest developments on numerical techniques, for instance, modern advection schemes. In several current atmospheric models, old (and inaccurate) numerical schemes are still used (see, for instance, Schlüzen, 1994; Christensen et al., 1997; Cox et al., 1998). Of course, the introduction of new techniques in very complex models is not always timely, because it is not often clear if one gains more in accuracy than looses in computational cost. Also, changes in complicated models are not usually an easy task.

New numerical schemes can be tested in simple models with ease. Therefore, simple, low-dimension models can still be useful to simulate some specific phenomena as well as to test physical parameterizations and/or numerical schemes.

In this paper, a two-dimensional, axisymmetric cloud model containing a warm-phase bin-microphysics is presented. Although similar models exist in the literature (e.g., Soong, 1974; Takahashi, 1975; Tzivion et al., 1994), the present model uses more accurate numerical techniques for advection (Bott, 1989a,b; Easter, 1993; Costa and Sampaio, 1997). One of the purposes of this paper, along the description of the model itself, is to explore sensitivities regarding the advection schemes.

In Sections 2–4, the main characteristics of the model are described. In Section 5, results from a control simulation of a deep, isolated, tropical convective cloud are shown. Section 6 is dedicated to a discussion on the use of advective schemes in cloud models. Section 7 shows results from sensitivity tests using the cloud model, regarding the order of advection. In Section 8, an overall discussion is presented.

2. Basic equations

The present model is axi-symmetric, vorticity based, and anelastic, as those described by Soong and Ogura (1973), Soong (1974), Costa and Sampaio (1996) and Almeida et al. (1998).

The set of basic equations comprises a diagnostic equation for the streamfunction (1), and prognostic equations for the azimuthal vorticity (5), the perturbation potential temperature (6), the perturbation water vapor mixing ratio (7), 44 categories of cloud condensation nuclei (8), and 100 categories of liquid-phase hydrometeors (9). All symbols are listed in Appendix A.

\[
\frac{\partial}{\partial t} \left( \frac{1}{r} \frac{\partial \phi}{\partial r} \right) + \frac{1}{r} \frac{\partial^2 \phi}{\partial z^2} = \zeta, 
\]  
(1)
where

\[
\zeta = \frac{\partial (p_0 u)}{\partial z} - \frac{\partial (p_0 w)}{\partial r},
\]
(2)

\[
u = \frac{\partial \psi}{\rho_0 r \partial z}, \quad w = -\frac{\partial \psi}{\rho_0 r \partial r},
\]
(3)

and

\[
\frac{\partial (p_0 ru)}{\partial r} + \frac{\partial (p_0 rw)}{\partial z} = 0;
\]
(4)

\[
\frac{\partial \zeta}{\partial t} = -u \frac{\partial \zeta}{\partial r} - w \frac{\partial \zeta}{\partial z} + \left( 2w \frac{d \rho_0}{\rho_0} \frac{d}{d z} + u \right) \left( \zeta - u \frac{d \rho_0}{d z} \right) + u w \frac{d \rho_0}{d z^2} - \frac{g \rho_0}{\theta_0} \frac{d}{d r} \left( \theta' \frac{\theta_0}{\theta_0} + 0.608 d' - q_i \right) + \tau _\zeta;
\]
(5)

\[
\frac{\partial \theta'}{\partial t} = -u \frac{\partial \theta'}{\partial r} - w \frac{\partial \theta'}{\partial z} - w \frac{d \rho_0}{d z} + \frac{L}{C_p} (C - E) + \tau _\theta';
\]
(6)

\[
\frac{\partial q_i'}{\partial t} = -u \frac{\partial q_i'}{\partial r} - w \frac{\partial q_i'}{\partial z} - w \frac{d q_i}{d z} - C + E + \tau _{q_i};
\]
(7)

\[
\frac{\partial n_j}{\partial t} = -u \frac{\partial n_j}{\partial r} - w \frac{\partial n_j}{\partial z} - \left( \frac{\partial n_j}{\partial t} \right)_{\text{nuc}} + \tau _n;
\]
(8)

\[
\frac{\partial f_i}{\partial t} = -u \frac{\partial f_i}{\partial x} - w \frac{\partial f_i}{\partial z} + \left( \frac{\partial f_i}{\partial t} \right)_{\text{nuc}} + \left( \frac{\partial f_i}{\partial t} \right)_{\text{con}} + \left( \frac{\partial f_i}{\partial t} \right)_{\text{coa}} + \left( \frac{\partial f_i}{\partial t} \right)_{\text{shik}} + \tau _{f_i};
\]
(9)

where \( j = 1, \ldots, 44; i = 1, \ldots, 100, \) and \( \tau \) represents turbulent terms. For the numerical experiments performed in this paper, the subgrid-scale transport terms of CCNs and cloud droplets were set to zero, in order to emphasize the model sensitivity to the numerical diffusion associated with the advection schemes.

### 3. Microphysics

Microphysical processes are represented as source/sink terms in the prognostic equations for temperature, water vapor, CCN and droplet distributions.
CCN are activated as the supersaturation exceeds the critical value determined by Köhler’s curve. Because only the small nuclei reach their equilibrium size rapidly (Mordy, 1959), the procedure proposed by Kogan (1991) is used to determine the radius of the activated nuclei at the cloud base, i.e., only small aerosol particles are assumed to reach its equilibrium size.

Condensational growth is calculated according to Mordy (1959). The solute term is calculated only for nuclei at a single microphysical timestep (see Section 4) and the curvature term is neglected for raindrops (radius greater than 50 μm). The amount of condensed/evaporated water \((C - E)\) term in Eqs. 6 and 7 and the latent heat released are calculated based on \(\frac{d r}{d t}\).

Changes due to collision–coalescence, collision-breakup and spontaneous breakup are also considered. Coalescence and collisional breakup probabilities are calculated according to Low and List (1982a). Filament, sheet and disk breakup are represented, according to the formulas by Low and List (1982b) for the distribution-function of droplet fragments. Spontaneous breakup is incorporated, following a procedure similar to the one proposed by Srivastava (1971), but with formulas derived from more recent experiments (Kamra et al., 1991).

4. Numerical procedure

In the present version of the cloud model, advection is usually evaluated using the Area-Preserving Flux-Form (APF) scheme (Bott, 1989a,b). Options from zero-th (which is equivalent to the common forward-upstream scheme) to eight-order polynomials in the APF advection scheme are available, as in Easter (1993) and Costa and Sampaio (1997), with distinction to the third- and fifth-order schemes. In the control simulation, the fifth-order scheme (APF5) is used.

An iterative solver (Stone, 1968; Jacobs, 1972) is used to calculate the streamfunction from the vorticity.

A simple first-order closure (Smagorinsky, 1963) is used in turbulence parameterization, with turbulent coefficients calculated as in Soong and Ogura (1973). The same value of those coefficients was used for both the transport of momentum and scalars (with the exception of the distribution-function of drops).

Cloud condensation nuclei are divided into 44 categories, with dry radii ranging from approximately 0.006 to 7.59 μm and critical supersaturations ranging from almost 0 (largest nuclei) to 2.8% (smallest nuclei). Six out of the 44 categories generate CCN with a moist radius greater than 10 μm at the cloud base, although at small concentrations (less than 12 l⁻¹ for categories 39 to 44). The present microphysical model also comprises a set of 100 discrete bins, in which the liquid-phase hydrometeors are categorized, according to their radius, which varies exponentially from 1 μm to 5 mm.

Since microphysical processes (condensational growth, collision–coalescence, and break-up) alter the distribution-function of drops, mass has to be redistributed among the discrete bins. It is well known that the method by Kovetz and Olund (1969) artificially broads the hydrometeor distributions. Therefore, a modified, semi-Lagrangian version of
this method is adopted, as in Kogan (1991). It can be shown that such an approach reduces drastically the errors associated with the Kovetz–Olund method, and provides realistic values of supersaturation.

The large, dynamical timestep ($\Delta t$) is divided into small microphysical timesteps ($\Delta \tau$). For a given variable, the dynamical tendency (changes due to resolved advection and subgrid-scale transport) is assumed to be uniform within a large timestep, while microphysical tendencies are calculated in each small timestep.

All boundaries are rigid, free-slip. To avoid numerical instabilities, absorbing layers are imposed at the top and lateral boundaries.

5. Control simulation

The domain of integration contains $80 \times 80$ grid points with a 120-m grid spacing in both the horizontal and vertical directions and a dynamical timestep of 5 s. A total of 32 absorbing layers was used in both directions.

A horizontally homogeneous, resting basic state is defined from upper-air temperature and humidity data collected on 22 June 1994, during a cloud-microphysics field experiment (Ceará experiment, Costa et al., 2000). Aircraft data was not available above

![Skew-T diagram](image.png)

Fig. 1. Skew-T diagram depicting the basic state of temperature and dew-point temperature used in the cloud simulations. The sounding was obtained blending the 22 June 1994, 1200Z NCEP analysis over Mossoró, Northeast Brazil, with aircraft measurements from the Ceará experiment.
3000 m, therefore, they were blended with the NCEP analysis for 12 UTC. The skew-T diagram for the blended sounding is depicted in Fig. 1. The lifting condensation level, the level of free convection and the equilibrium level were found at 910 mb (975 m), 850 mb (1565 m) and 635 mb (3970 m), respectively. As discussed by Sampaio et al. (1996) and Costa et al. (1998), those atmospheric conditions favored the formation of precipitating warm cumuli.

The nuclei are assumed to be composed by NaCl only. The total nucleus concentration is assumed to be constant below 500 m. Above that level, the concentration decays exponentially with height.

Convection was triggered, introducing a warm bubble with a maximum perturbation potential temperature of 1 K, in which the relative humidity was increased by 5%.

Fifth-order APF advection is adopted for the control run. Flux correction is imposed to the advection of CCN and hydrometeors.

Table 1 shows some parameters for the simulated cloud and compares them with actual observations. The modeled values in Table 1 are physically reasonable and some of them (droplet concentration, cloud base and top heights) are in good agreement with observations from a warm, precipitating cumulus cloud that was penetrated in five different levels, on 22 June 1994, during the Ceará experiment.

Figs. 2–4 show snapshots of the simulated cloud for 10, 18, and 24 min, corresponding roughly to its growing, mature, and precipitating/decaying stages, respectively. Fig. 5 shows the time evolution of selected fields at the central axis of the cloud.

During the early stages, the cloud top rises at a ratio of about 3 ms⁻¹, which is less than the maximum vertical velocity found at the major cloud updraft.

At 10 min, the cloud expanded, with its top reaching 2.8 km for a horizontal dimension of about 5 km. The maximum cloud water content and droplet concentration are now 2.04 g kg⁻¹ (Fig. 3) and 446. cm⁻³. The rainwater content is still less than 0.01 g kg⁻¹, for a maximum updraft of 3.61 ms⁻¹.

At 18 min, the cloud is, in essence, completely developed (although the cloud top continues to rise for two more minutes). Most of the maximum field values shown in Table 1 occurred approximately at that time. High cloud water mixing ratio (> 3 g kg⁻¹) is found, particularly between the 2.5 and 3.0 km levels (Fig. 4a). In the upper

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Modeled value</th>
<th>Observed value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum updraft (ms⁻¹)</td>
<td>4.95</td>
<td>–</td>
</tr>
<tr>
<td>Maximum downdraft (ms⁻¹)</td>
<td>2.40</td>
<td>–</td>
</tr>
<tr>
<td>Maximum cloud water mixing ratio (g kg⁻¹)</td>
<td>3.99</td>
<td>1.2</td>
</tr>
<tr>
<td>Maximum rainwater mixing ratio (g kg⁻¹)</td>
<td>6.54</td>
<td>3.1</td>
</tr>
<tr>
<td>Maximum droplet number concentration (cm⁻³)</td>
<td>563</td>
<td>541</td>
</tr>
<tr>
<td>Maximum cloud top height (mb)</td>
<td>623</td>
<td>720</td>
</tr>
<tr>
<td>Minimum cloud base height (mb)</td>
<td>904</td>
<td>908</td>
</tr>
<tr>
<td>Maximum precipitation rate (mm h⁻¹)</td>
<td>10.8</td>
<td>–</td>
</tr>
</tbody>
</table>
portion of the cloud, the cloud water content was significantly depleted, due to the conversion to rainwater (Fig. 4b). The greatest value of the droplet concentration (563 cm$^{-3}$, see Table 1) occurred 2 min before; however, a significant portion of the cloud (right above the cloud base) is still filled with number concentrations greater than 500 cm$^{-3}$. A few points exhibit values of rainwater mixing ratio greater than 6 g kg$^{-1}$ (Fig. 4b); however, the drops are still not able to overcome the strong updraft (upward winds with speeds close to 5 ms$^{-1}$) occupy significant portions of the cloud, as seen in Fig. 4c). The structure of the perturbation potential temperature is depicted in Fig. 4d, where the heating associated with the condensation process produced departures from the basic state greater than 3.5 K. On the other hand, evaporative cooling at the cloud top generated negative perturbations with absolute values greater than 2 K.

Both the liquid water fields and the cloud extension drop, as the end of simulation approaches. At 24 min, the maximum cloud water mixing ratio was reduced to 0.73 g kg$^{-1}$ (Fig. 5a), and the maximum droplet concentration dropped down to 321 cm$^{-3}$. The water field is now dominated by raindrops (maximum rainwater mixing ratio of 3.48 g kg$^{-1}$), which, since the cloud updraft decayed, were able to reach the ground.

Fig. 6 shows the droplet spectra at 18 min of simulation. In contrast with the spectra at the early stages of the simulated cloud, precipitation formation and mixing processes introduced important changes in the hydrometeor distribution during the mature phase. As expected, the mean diameter tends to increase with height. The existence of a broad variety of spectrum shapes can be observed, in agreement with observations in cumulus
Fig. 3. Cloud water mixing ratio (a), rainwater mixing ratio (b), perturbation potential temperature (c), and wind vectors (d) for the simulated cloud control run, 18 min of simulation. The thick black line in panels (b), (c) and (d) is the 0.01 g kg\(^{-1}\) cloud water mixing ratio contour.
Fig. 3 (continued).
Fig. 4. Cloud water mixing ratio (a), and rainwater mixing ratio (b) for the simulated cloud (control run, 24 min of simulation).
Fig. 5. Time evolution of the cloud water mixing ratio (a), and rainwater mixing ratio (b) at the central axis of the cloud (control run).
Fig. 6. Cloud droplet spectra of the simulated cloud (APF5/control run, 18 min of simulation). The abscissa corresponds to the droplet radius, ranging from 0 to 50 μm. The scale is linear. The ordinate is the number concentration correspondent to each bin. The scale is logarithmic, ranging from 0.1 to 100 cm⁻³.

... Close to the cloud boundaries, and especially at its top, bi- and multimodal spectra occurred. In the upper kilometer of the cloud, bimodal spectra are, in fact, dominant.

6. Cloud modeling and advection schemes

It has been recognized that, as far as advection accounts for a significant part of the problem, accurate solutions for the advective terms are necessary in atmospheric modeling. Several advection schemes produce unattractive results as large numerical...
diffusion, or negative values for an expected positive-definite quantity. For instance, the simple upstream, forward-in-time scheme, produces a very significant damping, while the very popular leapfrog scheme, despite being amplitude-preserving, produces spurious oscillations and phase errors.

As shown by Tremback et al. (1987), high-order advection can reduce the numerical diffusion dramatically. On the other hand, flux correction can provide oscillation-free solutions (e.g., Smolarkiewicz, 1984; Bott, 1989a,b). Costa and Sampaio (1997) showed that a high-order positive-definite scheme could indeed be useful to obtain very accurate solutions for the advection equation.

The weak development of numerical techniques and the lack of understanding on the role of advection schemes in cloud modeling led early cloud modelers to use simple (and poor) schemes. The most popular scheme during the 1970s was certainly the forward-upstream (e.g., Ogura and Takahashi, 1971; Soong and Ogura, 1973), but other modelers used the leapfrog scheme (e.g., Murray and Koenig, 1972) or other simple algorithms.

As more accurate numerical schemes were developed (e.g., Purnell, 1976; Arakawa and Lamb, 1981; Smolarkiewicz, 1984; Bott, 1989a), they were being incorporated into atmospheric models. In recent cloud modeling studies, many authors adopted positive-definite schemes, particularly if bin-microphysics schemes are present. For instance, Brenguier and Grabowski (1993) used the scheme proposed by Smolarkiewicz (1984) and Smolarkiewicz and Grabowski (1990) to investigate entrainment as simulated by a two-dimensional bin-microphysics cloud model. Feingold et al. (1994) used sixth-order polynomial fitting (Tremback et al., 1987), plus flux renormalization and positive-definite constraints (Bott, 1989b) in large-eddy simulations of stratocumulus clouds.

One of the first articles addressing the importance of advection in cloud modeling was presented by Orville and Sloan (1970), hereafter referred to as OS. They verified the influence of different orders of advection in a two-dimensional model of orographic clouds, comparing results obtained with the standard, first-order, forward-upstream scheme, and Crowley’s (1968) second- and fourth-order schemes. Their results show that, for a symmetric case with no ambient wind, the three schemes produced similar results. Because in their kinetic energy analysis, the truncation error associated with the forward-upstream scheme was found to be significant, and since the second-order Crowley’s scheme is obviously superior to the fourth-order scheme in terms of computational efficiency, the authors recommend the use of the second-order scheme for advection. The results shown in the next section of this manuscript are in agreement with some of the conclusions by OS. In particular, as far as the model resolution is fine enough to resolve the cloud using a large number of grid points, the advective scheme is not very important to determine the gross structure of the cloud. However, this does not guarantee that the advection cannot influence other aspects of the cloud, such as its microstructure.

In multidimensional, non-linear, atmospheric modeling, non-linear interactions, aliasing and source terms often produce shortwave features. When clouds are present, no matter how small is the grid spacing, the issue of the short modes is even more important. According to Brenguier (1993), sharp discontinuities can be observed in cloud edges down to the spatial scale of centimeters. Because clouds are essentially
discontinuities in the water field (that often present sharp edges), atmospheric simulations involving clouds are necessarily rich in “shortwave” constituents. This must be even more severe in bin-microphysics models, in which the water is partitioned into a large number of components. Therefore, because those shortwave features are a problem for most numerical schemes, no matter what resolution is used, cloud models in general and bin-microphysics models in particular must have uncertainties with the representation of the water fields, at least at the cloud boundaries. In particular, low-order advection schemes, for which the representation of the phase and amplitude of narrow modes is poor, are probably not suitable for cloud simulations in which cloud-edge processes, such as entrainment, are being investigated, especially if a bin-microphysics is present. Because OS used a very simplified bulk-microphysics parameterization, they could not discuss this issue properly.

In order to check this hypothesis, the role of the order of the advective scheme is investigated in the context of a non-linear model (cloud model described in Sections 2–4). In the next section, results from sensitivity experiments using different advection schemes are presented. The schemes are the standard forward-upstream (FWD), and three other versions of the area-preserving flux-form advection scheme (APF).

FWD advection is widely described in the literature. It uses forward-in-time, upwind-in-space differencing, and produces a very strong damping, particularly for short wavelengths. Due to its severe limitations, FWD was practically abandoned in cloud modeling, however, because it can be treated as the “zero-th order” APF algorithm (see Bott, 1989a; Easter, 1993), it is going to be used in this paper to illustrate the extreme low-order case of area-preserving positive-definite advection. The APF scheme, proposed by Bott (1989a,b), is positive-definite, and presents both small phase and amplitude errors. Like many other advection algorithms (e.g., Crowley, 1968; Tremback et al., 1987), the APF scheme uses polynomial fitting to represent a given variable within a grid box. A detailed description of the APF scheme can be found in Bott (1989a,b) or Chlond (1994). In this study, second-order (APF2), modified (using the terminology by Easter, 1993) third-order (APF3), and modified fifth-order (APF5) versions of the scheme are going to be used. Coefficients for APF2, APF3, and APF5 are given according to Bott (1989b), Easter (1993) and Costa and Sampaio (1997), respectively. It can be shown that FWD (or “APF0”), APF2, APF3 and APF5 provide first-, third-, fourth- and sixth-order accuracy in space, respectively. The shortcomings due to the first-order accuracy in time for a time-varying wind field are not important because of the small timestep used in this study.

7. Sensitivity tests regarding the advection scheme

Sensitivity tests regarding the advection scheme were performed, using the cloud model described in previous sections. In order to compare the performance of the different advection schemes in the context of the cloud model, the ambient and initial conditions, as well as all numerical settings used in the control simulation, were preserved, with the only exception of the advection algorithm. In three sensitivity tests, the order of advection was simply degraded from sixth (control/APF5 simulation) to
Table 2
Comparison among some physical characteristics of the simulated clouds, using different advection schemes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>APF5</th>
<th>APF3</th>
<th>APF2</th>
<th>FWD</th>
<th>MIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum updraft (m s⁻¹)</td>
<td>4.95</td>
<td>4.92</td>
<td>4.84</td>
<td>5.19</td>
<td>4.99</td>
</tr>
<tr>
<td>Maximum downdraft (m s⁻¹)</td>
<td>2.40</td>
<td>2.33</td>
<td>2.19</td>
<td>1.83</td>
<td>2.46</td>
</tr>
<tr>
<td>Maximum cloud water mixing ratio (g kg⁻¹)</td>
<td>3.99</td>
<td>3.99</td>
<td>3.95</td>
<td>4.34</td>
<td>4.04</td>
</tr>
<tr>
<td>Maximum rainwater mixing ratio (g kg⁻¹)</td>
<td>6.54</td>
<td>6.60</td>
<td>6.65</td>
<td>8.10</td>
<td>7.64</td>
</tr>
<tr>
<td>Maximum droplet number concentration (cm⁻³)</td>
<td>563</td>
<td>553</td>
<td>536</td>
<td>674</td>
<td>503</td>
</tr>
<tr>
<td>Maximum cloud top height (mb)</td>
<td>623</td>
<td>623</td>
<td>623</td>
<td>627</td>
<td>623</td>
</tr>
<tr>
<td>Minimum cloud base height (mb)</td>
<td>904</td>
<td>904</td>
<td>904</td>
<td>917</td>
<td>904</td>
</tr>
<tr>
<td>Maximum precipitation rate (mm h⁻¹)</td>
<td>10.2</td>
<td>10.4</td>
<td>10.6</td>
<td>7.1</td>
<td>5.5</td>
</tr>
</tbody>
</table>

fourth (APF3), third (APF2) and first (FWD). In the last sensitivity run (MIX), APF5 advection was used for all fields, except for the distribution-function of drops, advected according to the forward-upstream scheme. The purpose of MIX was to investigate the effect of low-order advection on the liquid water field only, providing a more direct comparison to APF5.

The global characteristics of the life cycle of the simulated cloud were not very sensitive to the change in the advection scheme. Table 2 compares some parameters of the five simulated clouds (control/APF5, APF3, APF2, FWD, and MIX). Both the vertical and horizontal dimensions of the simulated clouds were very similar. The most significant departures from the control simulation occurred regarding the maximum rainwater content, which was higher in the FWD cloud, and the maximum compensating downdraft, which decreased as the order of advection decreased. Surprisingly, the FWD run showed the greatest updraft. Both sensitivities can be possibly explained by changes in the transport of the vorticity field. In particular, if a highly diffusive scheme is used, vorticity is diffused radially. Eventually, a tail of positive values in the vorticity field (that generates the downdraft) leaves the effective model domain (determined by the absorbing layers). As one integrates the vorticity inward, to obtain the velocity field, the loss of positive vorticity not only weakens the downdraft, but also may produce a stronger updraft at the center of the domain, even with an increased dissipation of kinetic energy. Hence, one should not expect this result to be general; it is probably geometry- and model-dependent.

Despite the similarities in the gross structure of the simulated clouds, the fine structure of the cloud exhibits important sensitivities. In particular, the sensitivity tests suggest that both dynamical and microphysical processes at cloud edges can be significantly influenced by the choice of the advection scheme.

In fact, the greatest departures among the simulated clouds occur at the cloud edges, as shown in Figs. 7–10. The different frames depict the departures of the cloud water content (a), rainwater content (b), perturbation potential temperature (c) and wind vectors (d) between the control run and the simulations using APF3 (Fig. 7), APF2 (Fig. 8), FWD (Fig. 9) and MIX (Fig. 10).

The differences between the APF5 and APF3 clouds are, in general, small. Close to the cloud top, the departures at the cloud water and rainwater fields are mostly of the order of 0.1 g kg⁻¹ (Fig. 7a and b). The maximum differences at the potential
Fig. 7. Differences in the cloud water mixing ratio (a), rainwater mixing ratio (b), perturbation potential temperature (c) and wind vectors (d) between the APF3 and the APF5/control simulations (18 min of simulation). The thick black line is the 0.01g kg⁻¹ cloud water mixing ratio contour for the APF5/control run.
temperature field occurred at the cloud top and were less than 0.5 K (Fig. 7c). Differences at the eddy structure were also small and were present primarily at the cloud top and lateral edges (Fig. 7d).
The fields in the simulation in which APF2 was used depart more from the control run, as expected. Differences greater than 0.5 g kg$^{-1}$ in the cloud water mixing ratio (Fig. 8a) and greater than 2.0 g kg$^{-1}$ in the rainwater mixing ratio (Fig. 8b) were

Fig. 8. Same as Fig. 7 (APF2 and APF5/control simulations).
Fig. 8 (continued).
Close to the cloud top, temperature departures greater than 1 K (Fig. 8c) and significant discrepancies in the simulated wind field were present. The largest departures from the control run were observed in the simulation using FWD. Large differences in the cloud water field (greater than 1.0 g kg\(^{-1}\)) were

Fig. 9. Same as Fig. 7 (FWD and APF5/control simulations).
observed in several portions of the cloud, about the level where the maximum rainwater was attained at the control run (Fig. 9a). In addition, the FWD cloud showed an excess of more than 3 g kg$^{-1}$ at the central axis and a deficit of more than 1 g kg$^{-1}$ off the central axis in the rainwater field, close to the same region (Fig. 9b). It suggests that the
Fig. 10. Same as Fig. 7 (MIX and APF5/control simulations).
use of a highly diffusive scheme has significant influence on the process of precipitation development. Very large departures occurred in both the temperature (Fig. 9c) and wind (Fig. 9d) fields. It must be pointed out that, in contrast with the APF3 and APF2
simulations, for which the differences from the control run were primarily found at the cloud borders, in the FWD simulation they were present throughout the entire cloud.

The structure of the water field in MIX resembled the one in the FWD run, with differences from the control simulation greater than 2 g kg\(^{-1}\) in the cloud water mixing ratio (Fig. 10a) and greater than 3 g kg\(^{-1}\) in the rainwater mixing ratio (Fig. 10b). The major differences in the temperature and wind fields between MIX and APF5 occurred at the cloud edges (mostly top and lateral boundaries), due to the influence of the advective schemes on the transport of droplets in those regions.

Because the difference in the accuracy of the advective schemes has more influence in the cloud edges, it induces important changes in the cloud microstructure. One important example is the change in the occurrences of bi- and multimodal spectra. As shown in Fig. 11, as the order of the advection scheme increases, there is a general tendency for the bi- and multimodal droplet spectra to become more frequent. This is probably due to the improved representation of the sharp discontinuities in the dynamic, thermodynamic and water fields at the cloud borders by higher-order schemes. The differences are very significant, even considering the APF5 and APF3 runs. In Fig. 11, it is also noticeable that when the FWD scheme is used for the advection of hydrometeors, the number of bi- and multimodal distributions reaches its maximum at a later time.

Fig. 12 shows the evolution of the rainwater mixing ratio at the gridbox closest to the surface and to the domain’s central axis for the five runs. The results from APF5, APF3

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Fig. 11. Total number of bi- and multimodal spectra as a function of time for the control and sensitivity runs.
and APF2 are very similar this time. Despite the difference in the amplitude, they show very peaked rain events, with a maximum at 25 min. In the FWD and MIX runs, the rainwater mixing ratio peaks 1 min later and the precipitation event lasts longer, probably due to a spurious diffusive transport of drops in the vertical. Supposedly, the different types of precipitation evolution are also associated with the different patterns in spectrum broadening shown next.

In order to illustrate how the choice of the order of advection can alter the statistical properties of the droplet spectra, Fig. 13 depicts the standard-deviation of the diameter of the cloud droplets (normalized by the mean) after 18 min of simulation, along with the corresponding observed field. In all runs, immediately above the cloud base, the normalized standard deviation tends to decrease, suggesting that droplet growth by condensation dominates in that region. Between 0.5 and 1.0 km above the cloud base, this tendency reverses, and the spectra tend to become broader, possibly due to significant growth via collision–coalescence. Above 3 km, the normalized diameter standard deviation increases sharply (at least in simulations APF5, APF3 and APF2). As shown in Fig. 6, this is the region corresponding to the dominance of bimodal spectra.

The most significant differences from the APF5 simulation in Fig. 13 are associated with the model runs in which the FWD scheme was used (FWD and MIX). The droplet
Fig. 13. Lines: normalized diameter standard deviation at the cloud’s central axis as a function of height for the control and sensitivity runs (18 min of simulation). Dots: Observed normalized diameter standard deviation in an isolated cumulus clouds studied during the Ceará experiment (borders excluded).
spectra in those two simulations were broader than the ones in APF5 at the lower portion of the cloud. On the other hand, the normalized diameter standard deviation in FWD and MIX does not present the sharp peak found in the other three simulations. Although there is some influence of the different “timing” in the microphysical evolution of those two clouds, in comparison to the other three, this is mostly associated with the smaller number of bi- and multimodal spectra in FWD and MIX. Due to the same reason, the peak relative to the APF2 simulation is less pronounced. It is clear that the agreement between the observed and modeled normalized diameter standard deviation is greater when higher-order advection schemes were used.

Finally, in order to illustrate how the use of more diffusive schemes alter the actual shape of the drop spectra, the distribution-function of drops for the MIX simulation is shown in Fig. 14. The comparison with Fig. 6 shows some clear differences.
(1) Due to the numerical diffusion of drops, at the outflow region (above 2.5 km), the boundaries of the MIX cloud often extend beyond the contour of the APF5 cloud, reducing the gradients in those regions. Because the thermodynamic field, particularly at the cloud top, depends strongly on the evaporation of the water particles, the most pronounced temperature differences between MIX and APF5 occurred in that region. As a consequence, the most striking differences in the wind field were also located by the cloud top.

(2) There are less grid boxes containing multiple modes in Fig. 14 than in Fig. 6, in agreement with what was stated previously.

(3) There is a less dramatic mode separation in the upper grid boxes in Fig. 14 than in Fig. 6, causing the reduction in the normalized diameter standard deviation at the upper portion of the cloud, from APF5 to MIX, shown in Fig. 13.

(4) The MIX cloud (Fig. 14) exhibits relatively broad spectra in its mid and lower portions, in contrast with the narrower spectra found in the APF5 cloud (Fig. 6). Again, this is in agreement with what was shown in Fig. 13.

8. Summary, concluding remarks, and future work

A two-dimensional cloud model, conceived to simulate deep, isolated convective cells is presented, in which a warm, bin-microphysics representing multiple processes (nucleation, condensation/evaporation, coalescence, collisional and spontaneous breakup and sedimentation) and an accurate, positive-definite advection scheme are used. Despite the limitations regarding the lack of the third degree of freedom and the assumption of axisymmetry, the model is capable to simulate realistically several aspects of the evolution of a convective cloud.

The model was able to capture the generation of bi- and multimodal spectra at the cloud boundaries during its mature stage, particularly at the cloud top. The significant variability in cloud spectra, as well as the occurrence of multimodes, suggests that most of the microphysics bulk-parameterizations, based, for instance, on exponential or generalized gamma distributions, cannot represent some important aspects of the microstructure and life cycle of convective clouds.

Sensitivities with respect to the use of different advection schemes were explored. The results show that the gross aspects of the life cycle of the different simulated clouds were similar in all cases, even when a very poor scheme, such as the forward-upstream, was used. It suggests that the ambient and initial conditions are far more important to determine the bulk properties of the simulated clouds.

One should point out, however, that important differences occurred among the simulated clouds, regarding their dynamics, thermodynamics and microphysics, particularly at the cloud boundaries. In fact, the use of low-accuracy advection produced important changes in the entire cloud, while the use of intermediate-accuracy advection caused deviations mostly restricted to the cloud edges.

The results suggest that, in fact, low-order advection schemes can be used to simulate clouds or cloud systems if one is only interested in their global characteristics (gross dynamics, bulk condensate fields, etc.). However, they become inappropriate when one
is interested in looking at their fine structure, particularly at the microphysics. In the latter case, intermediate-order schemes also have shortcomings, because they still produce poor results close to sharp discontinuities. In this case, although far from being a “perfect solution” for the problem, high-order schemes (such as APF3 or even APF5) are certainly useful. In terms of the statistics of droplet spectra, again a reasonable agreement between observed and modeled fields could only be found when intermediate to high-order advection was used. Those conclusions are expected, since the bulk characteristics of the cloud are dominated by the coarse modes in the model grid. On the other hand, cloud-edge processes, which are very important to the evolution of the cloud microphysics (and other phenomena, such as the ones associated with radiation, which were not explored in this paper) are very much influenced by narrow modes, such as $2\Delta$ or $4\Delta$ features.

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Appendix A

List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>radial velocity</td>
</tr>
<tr>
<td>$w$</td>
<td>vertical velocity</td>
</tr>
<tr>
<td>$\psi$</td>
<td>streamfunction</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>air density</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$r$</td>
<td>radial coordinate</td>
</tr>
<tr>
<td>$z$</td>
<td>height</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>azimuthal vorticity</td>
</tr>
<tr>
<td>$g$</td>
<td>gravity</td>
</tr>
<tr>
<td>$\theta$</td>
<td>perturbation potential temperature</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>basic state potential temperature</td>
</tr>
<tr>
<td>$L$</td>
<td>latent heat of evaporation</td>
</tr>
<tr>
<td>$q'_e$</td>
<td>perturbation water vapor mixing ratio</td>
</tr>
<tr>
<td>$q_{e,0}$</td>
<td>basic state water vapor mixing ratio</td>
</tr>
<tr>
<td>$q_l$</td>
<td>liquid water mixing ratio</td>
</tr>
<tr>
<td>$n_j$</td>
<td>category of cloud condensation nuclei</td>
</tr>
<tr>
<td>$f_i$</td>
<td>category of liquid-phase hydrometeor</td>
</tr>
<tr>
<td>$\tau_t$</td>
<td>turbulent mixing terms</td>
</tr>
<tr>
<td>$C$</td>
<td>condensation rate</td>
</tr>
</tbody>
</table>
\( E \) evaporation rate

\[
\begin{align*}
(\partial n_i / \partial t)_{\text{nuc}} & \text{ change in } n_i \text{ due to nucleation} \\
(\partial f_j / \partial t)_{\text{nuc}} & \text{ change in } f_j \text{ due to nucleation} \\
(\partial f_j / \partial t)_{\text{con}} & \text{ change in } f_j \text{ due to condensation/evaporation} \\
(\partial f_j / \partial t)_{\text{coa}} & \text{ change in } f_j \text{ due to coalescence} \\
(\partial f_j / \partial t)_{\text{cbrk}} & \text{ change in } f_j \text{ due to collisional breakup} \\
(\partial f_j / \partial t)_{\text{sbrk}} & \text{ change in } f_j \text{ due to spontaneous breakup}
\end{align*}
\]

References


