ANALYSIS

Green accounting and the welfare gap

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Abstract

Although gross domestic product (GDP) is not intended to be a measure of societal welfare, it is often used as such. One shortcoming as a welfare measure is that it fails to account for the non-marketed value of natural resource flows. The difference between societal welfare and GDP is labelled the ‘welfare gap’. A model that accounts for both market and non-market income flows from natural capital is used to examine this gap. Societal welfare depends on private goods and the stock of natural capital. The latter is subject to a logistic growth relationship common to many non-human species. Private goods are produced using human capital and flows of natural capital. An exogenously growing human population either harvests the natural resource, produces human capital or produces the private good. Optimal control theory and dynamic simulations provide steady-state harvest and human capital growth rates which determine the steady-state natural resource stock, GDP and societal welfare growth rates. The model illustrates the feasibility of explicitly accounting for ecological relationships in economic growth models and shows that, depending on one’s preferences and the growth rate of human population and the intrinsic growth rate of natural resources, GDP may diverge substantially from the growth rate of societal welfare, leaving a large welfare gap.

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1. Introduction

Gross domestic product (GDP) measures the money value of goods and services produced in an economy during a specified time period; neither GDP nor per-capita GDP is intended to be a measure of societal welfare. One reason why GDP is not a measure of welfare is that it does not account for the value of the natural resources upon which all production ultimately depends. The natural resource stock, often referred to as natural capital, includes soil and its vegetative cover, freshwater, breathable air, fisheries, forests,
waste assimilation, erosion and flood control, protection from ultra-violet radiation, and so on. GDP fails to account for the non-marketed value of flows from these natural resources, including many cultural and recreational activities. Nevertheless, GDP mistakenly is used often as an all-encompassing measure of societal welfare by government policy makers and private enterprise.\(^1\)

Because there is a difference between societal welfare and GDP, which we label the ‘welfare gap’, the gap is examined by introducing a model that offers a more inclusive welfare measure, one that accounts for both market and non-market income flows from natural capital.

In the model societal welfare depends on private goods and the stock of natural capital. Private goods are produced using human capital (i.e. educated workers) and flows of natural capital. The annual growth rates of per-capita income (as measured by GDP) and societal welfare are compared along a balanced-growth path by employing simulation modeling software (STELLA II).\(^2\)

Simulations help identify the steady-state equilibrium of the system. The objectives are to: (1) depict the feasibility of explicitly accounting for ecological relationships in economic growth models; (2) illustrate the value of computer simulation exercises in extending the insights of theoretical and empirical growth models; (3) formally show that, depending on one’s preferences and the growth rate of human population and the intrinsic growth rate of renewable resources, per-capita income growth may diverge substantially from the growth rate in per-capita welfare; and (4) use comparative statistics to examine system response to parametric changes.

This work is motivated by the debate between mainstream economists and natural scientists\(^3\)

\(^1\) For example: “Today the two political parties differ somewhat in regard to means, but neither disputes that the ultimate goal of national policy is to make the big gauge—the gross domestic product—climb steadily upward.” (Cobb et al., 1995)

\(^2\) STELLA II is a simulation modeling software package used to model complex systems including ecosystems and economic systems. See Hannon and Ruth (1994).

\(^3\) We refer to those natural scientists, including biologists, ecologists, and environmental scientists, who have traditionally criticized economists for their failure to account for the value of ecosystem functions, as ‘ecologists’.

over the importance of natural capital to human welfare. Each group contends that the other does not understand its pedagogical perspective. Hoping to address the most ardent concerns of each discipline, we construct a model based on the economists’ endogenous technological growth theory and constrained by the biologists’ logistic growth function for a renewable resource. It is assumed, as reflected in the theoretical model, that natural capital is essential to human welfare: “zero natural capital implies zero human welfare because it is not feasible to substitute, in total, purely ‘non-natural’ capital for natural capital” (Costanza et al. 1997).

Section 2 provides motivation for the problem, and Section 3 is a discussion of endogeneous growth theory. The difference between economic growth and economic development in the model is covered in Section 4, and the model is introduced in Section 5. Section 6 presents the model results and a conclusion follows in Section 7. Appendix A presents the model in detail.

2. Motivation

Disciplinary investigation of environmental problems by both economists and ecologists is at least two-centuries old.\(^4\) There is ideologic friction between the two groups as witnessed by the well-publicized dispute between Paul Ehrlich and the late Julian Simon.\(^5\)

\(^4\) The writings of classical economist Thomas Malthus focused on the effect of population growth on the availability of resources necessary to sustain human existence (Malthus, 1960 [1798]). Charles Darwin developed his theory of evolution around the same basic idea: species have the tendency to overproduce, with the result that only those most ‘fit’ to their environmental circumstances survive and reproduce (Darwin, 1958 [1859]).

\(^5\) The well-publicized wagers between Ehrlich (1981, 1982) and Simon (1981b) on the direction of future welfare measures highlight the philosophical rift between ecologists and mainstream economists (Shen, 1996). Simon’s exuberance to bet on “...anything pertaining to material human welfare...” refers almost exclusively to income-based welfare indices neglecting the value of the natural capital on which all economic output is ultimately based (Shen, 1996).
interdisciplinary research between ecologists and economists will play an important role in addressing social problems concurrent with an increasing human population whose consumption hastens the rate of ecosystem degradation.

Mainstream neoclassical economists, slow to appreciate the importance of maintaining a healthy and productive environmental resource base, are beginning to accept ecologists' assertions. Nobel laureate economists Robert Solow and Kenneth Arrow recognize the inextricable link between biological and economic systems, and urge a more careful consideration of accounting for biological degradation in economic growth statistics (Solow, 1994; Arrow et al., 1995). While difficult, fraught with uncertainties, and opposed by many as immoral, valuing ecosystems is necessary when making choices. Ecologists, economists and geographers have estimated the value of nature's long list of ecosystem goods and services⁶ and argue that such an exercise is important in order to "make the range of potential values of the services of ecosystems more apparent" (Costanza et al., 1997).

While some economists are heeding the biological wake-up call, others, such as Simon (1981a, 1995a), continue to downplay the economic and sociological importance of maintaining a healthy and viable natural resource base. According to Simon, society's ability to advance technology as necessary should obviate concern for the genius to concoct, devise, and invent ways around barriers that threaten production (and consumption). Because humans have acquired the technology for nuclear fission and space travel, they now have all the tools necessary to...

"...go on increasing our population forever, while improving our standard of living and control over our environment". (Simon, 1995b)

Simon's forecasts are optimistic: the environmental degradation associated with continued population growth and increasing per-capita consumption will test society's collective intellect.⁷ Presuming that past trends of technological progress can be relied on to overcome the environmental stresses concurrent with growing populations is a risky strategy. Humankind currently appropriates approximately 40% of total terrestrial primary production of the biosphere (Vitousek et al., 1986), 25–35% of coastal shelf primary production (Pauly and Christensen, 1995), and half of the available global supply of fresh water (Postel et al., 1996). There is ample evidence that the carrying capacity⁸ of many ecosystems is being severely tested (Kirchner et al., 1985; Rees, 1990; Hardin, 1991; Brown, 1994, 1995). Mapping a course through this uncharted ecological territory should not be left solely to unfettered markets.⁹

Because more people means more land devoted to human habitat and less land devoted to habitat for the millions of other species, Simon's forecasts also downplay the amenity value of nature. People place positive value on many environmental amenities (and negative value on disamenities) which are not easily traded in markets. The tricky part is placing an actual dollar value on these amenities. Economists interested in quantifying use and non-use values for nature's goods employ a variety of measurement techniques such as travel-cost method, hedonics and contingent valuation method. The validity and applicability of

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⁶ Recognizing this pedagogical difference between economists and natural scientists, Costanza et al. (1997) quantify the monetary value of the services of ecological systems and the natural capital stocks that produce them. The current economic value of 17 ecosystem services for 16 biomes is estimated to be in the range of $16–54 trillion, with an average of $33 trillion/year. This compares with a world gross national product of $18 trillion per year (1997 $).

⁷ The precipitation of violent civil or international conflict caused by renewable resource scarcity is another concern for researchers of environmental issues (Homer-Dixon, 1994).

⁸ Carrying capacity is often defined as "the maximum population of a given species that can be supported indefinitely in a defined habitat without permanently impairing the productivity of that habitat" (Rees, 1994).

⁹ The history of Easter Island provides a good example of the catastrophic social effects of exhausting a renewable resource when the availability of adequate substitutes is severely restricted (Ponting, 1991).
the economic values obtained using these valuation techniques are sometimes suspect but the premise holds: people value environmental resources, especially when few satisfactory substitutes exist.

3. Endogenous growth theory and population

Starting with the seminal work of Solow (1957), substantial economics literature, catalogued under the title of “endogenous technological-growth theory”, has sought to explain economic growth. In these models, physical (or human-made) capital, human capital (education) and the latest production technology are combined to generate output which can be re-invested in human-capital (saved) or consumed. While conventional endogenous growth models track investment in physical and human capital stock, few models explicitly incorporate the biological growth/stock relationship of a renewable resource: the supply of natural resources is considered limitless. This assumption is challenged and the role of natural capital in the production process elevated while discounting the explicit importance of human-made capital. Requiring the production process to adhere to the behavior of a natural system such as the fishery or forest subjects the economy to a biological constraint.

Essentially all endogenous growth models of knowledge predict that technological progress is an increasing function of population size. (Romer, 1996). The larger the population, the more people there are to make discoveries and, thus, the more rapidly knowledge accumulates. Alternative interpretations hold that over almost all of human history, technological progress has led mainly to increases in population rather than increases in output per person (Kremer, 1993). Neglected in these stories is the impact of growing populations on natural resource stocks. More efficient harvesting of natural capital along with a growing human population has had a substantial effect on reducing the resource stock of the world’s major fisheries and rainforests (Dietz and Rosa, 1994; Brown, 1995; Resources, 1996). Economists who believe population growth is conducive to economic growth cite data showing that in most parts of the world, food production and per-capita gross income have generally grown since the end of World War II (Dasgupta, 1995a). These figures ignore the depletion of the natural resource base.

4. Economic growth versus economic development

“The general proposition that economic growth is good for the environment has been justified by the claim that there exists an empirical relationship between per-capita income and some measures of environmental quality. …as income goes up there is increasing environmental degradation up to a point, after which environmental quality improves (The relation has an ‘inverted-U’ shape)… in the earlier stages of economic development, increased pollution is regarded as an acceptable side effect of economic growth. However, when a country has attained a sufficiently high standard of living, people give greater attention to environmental amenities.” (Arrow et al., 1995)

The inverted U-shaped curve has been shown to apply to a select set of pollutants (as quoted in Arrow et al. (1995)). Some economists have sug-

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11 This quadratic relationship between a biological stock and its instantaneous growth rate is often represented with the Schaefer curve (Gordon, 1954; Schaefer, 1957; Clark, 1990).


13 Kremer (1993) concludes that endogenous technological models predict that over most of human history, the rate of population growth should have been rising.
As suggested that the notion of people spending proportionately more of rising incomes on environmental quality also applies to the natural resource base. While economic growth is associated with improvements in some environmental indicators, it is not a sufficient condition for general environmental improvements nor is it a sufficient condition for the earth supporting indefinite economic growth. In fact, if this base were severely degraded, economic growth itself could be jeopardized (Arrow et al., 1995).

Although GDP is commonly used as a measure of economic health and even personal well being, it is far from adequate as a measure of true economic performance. Numerous authors point out the need for a more comprehensive index that includes the flow of environmental services as well as the value of net changes in the stocks of natural capital so that the true social costs of growth are internalized. A better indicator would ameliorate the conflict between growth and the environment by subtracting environmental depletion from the index. The United Nations has recommended accounting for depletion in ‘satellite accounts’ as opposed to including depletion in the main tables. Repetto et al. (1989) have argued forcefully against this approach as it continues the use of misleading economic indicators, and these same authors have offered methodologies to improve national accounting.

Net domestic product (NDP), calculated by deducting environmental degradation of the resource base from GDP, is a more inclusive measure of personal well-being today and tomorrow (as quoted by Dasgupta (1995b)). That NDP is a better measure of the improvement in societal well-being is similar to the claim of Daly and Cobb (1994) that economic development is a better measure of well being than economic growth. (These authors also highlight the importance of sustainability, but sustainable issues are not directly addressed in this paper. Repetto et al. (1989) indicate that only by including environmental depletion into the national accounts will “…economic policy be influenced toward sustainability”. (p.12)) The analysis is concerned with an optimal growth path that includes the unharvested natural resource stocks in the welfare function, and it is, therefore, consistent with using some form of NDP.

The model herein allows the social planner to make perfectly timed, precise adjustments to the harvest and human capital investment rates that keep the bionomic system at, or returns it to, a steady-state equilibrium along the balanced growth path. Assuming the social planner has perfect information in such decisions is unrealistic. As pointed out by Stiglitz (1996), informational uncertainty pervades economies making identification of and adherence to a balanced growth path and overly optimistic outcome. Including a time lag to represent the reaction and adjustment process associated with data accumulation and analysis, or an uncertainty parameter to represent the inherent lack of analytical precision would add realism to the model. Either modification would almost certainly provide results different from those presented here. Time lags and uncertainty may prevent adherence to the balanced growth path and possibly even preclude maintenance of a steady-state equilibrium. If sufficiently severe, the bionomic system could tend toward a socially undesirably corner solution with complete exhaustion of the natural resource. Exploration of these issues is left for further research.

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14 The Bush Administration acknowledged this in the Council on Environmental Quality Annual Report (1992). “Accounting systems used to estimate GDP do not reflect depletion or degradation of the natural resources used to produce goods and services”. Consequently, the more the nation depletes its natural resources, the more the GDP goes up.

15 El Serafy (1997) also recognizes the importance of greening the national accounts (particularly of developing countries) to include the flows from natural capital: “…it seems rather elementary that economists should pay attention in their analysis to the sustainability of natural resources”. (1997 p. 219)

16 The model captures some notions of ‘weak sustainability’ in that the welfare growth rate depends on the natural capital stock as well as private good production.
5. Model

The social planner is responsible for maximizing welfare of a representative agent from the population of identical individuals. Welfare is a function of the representative agent’s shares of the renewable natural resource stock and of the private good. The planner solves an optimal control problem with two state variables and two control variables. She chooses (i) a harvest effort rate that determines the size of the renewable natural resource stock; (ii) the number of people devoted to human capital production (education) that determines the human capital stock, and the number of people employed in private good production.

The private good, produced by combining the current production technology with human capital augmented labor and natural capital, is shared equally among the population. Preferences for natural resources and private good consumption are captured by the relative size of the coefficients in a Cobb–Douglas welfare function. Natural capital, therefore, enters the welfare function directly as a stock and indirectly as an input used in the production of the private good.

The model contains both rival and non-rival knowledge. The choice of how many people to devote to human capital production determines the growth rate of non-rival technology. That is, devoting a greater proportion of labor to education can increase the human capital stock. Placing labor in human capital production and applying its skills to the existing technology creates this human capital. Patenting activity is used to represent the growth rate in rival knowledge. Both the human capital and private good production function are generalized Cobb–Douglas functions. The renewable natural resource is constrained by a biologic logistic growth function. The resource might be a forest that has value left standing including both use and non-use values. The natural resource is characterized as a contestable public good in that the allocation to any individual is the total stock divided by the population.

Finally, the welfare gap is used to determine how well GDP tracks societal welfare. Defining \( r_y \) as economic growth measured by GDP growth and \( r_u \) as economic development measured by welfare growth, the welfare gap is \( \Delta W = r_u - r_y. \)

The optimality conditions in Appendix A provide the basis for the STELLA simulations. A comparative static analysis using STELLA is carried out, and the simulations show how per-capita income and welfare growth rates, the welfare gap and resource stock levels vary over time as the social planner adjusts the harvest rate and human capital investment rate in order to keep the bionomic system on a balanced growth path following a transient, parametric perturbation. The full model is presented in the Appendix A.

6. Results

The simulations require parameter values for the social discount rate (\( \rho \)), the population growth rate (\( \mu \)), and the growth rate of proprietary technology (\( \gamma \)). Selection of a social discount rate is contentious (Lind, 1982): here \( \rho \) is the long-run real rate of growth in GDP in the US which is about 3% (see Kahn, 1998 p. 113). Initially, it is assumed that the population grows at the current average growth rate of world population (\( \mu \)) estimated at 1.5% (Reddy, 1996). The 3% growth rate of Fagerberg (1987) in patenting activity among OECD countries between 1974 and 1983 is used as a proxy for technological growth (\( \gamma \)).

The importance people place on consumption relative to natural amenities and the importance of human capital relative to natural capital in the production function are captured by the coefficients \( \phi \) and \( \alpha \), respectively. Initially, the relative importance of the two goods are weighted equally in the welfare function (\( \phi = 0.5 \)), as are the relative importance of two inputs in the production function (\( \alpha = 0.5 \)). The natural resource carrying capacity (\( S \)), is unitized which allows all steady-state harvest and resource stock values to be expressed as a percentage of \( S \). The intrinsic biological growth rate was calculated using the time–growth–volume relationship for a stand of Douglas Fir (Hartwick and Olewiler, 1986, p. 353). For a forest that is less than 70 years of age, the average biological growth rate (\( g \)) is 25%.

Steady-state equilibrium values are determined using STELLA. These steady-state values include
per-capita harvest effort ($v^*$), total harvest effort ($V^*$), total harvest ($h^*$), resource stock ($X^*$) and human capital growth rate ($rA^*$). The asterisk denotes steady-state equilibrium values. The variables $v$, $V$, $h$, and $X$ are measured in terms of ‘$S$’ and described by the ‘catch-per-unit effort hypothesis’ (Hartwick and Olewiler, 1986).

Modeling Appendix A Eqs. (A.11), (A.13), (A.14), (A.16), (A.17), (A.18) and (A.19) in STELLA allows investigation of the behavior of the rates of human capital growth, economic growth, economic development and the welfare gap along the balanced growth path in both steady-state and transient periods. Fig. 1, which is generated by STELLA, establishes the base case scenario where the system is initially in a steady-state equilibrium. Curves 1, 2, 3, 4 and 5 in Fig. 1 show that if the initial resource stock equals the steady-state stock, economic growth or GDP ($r_y$), economic development ($r_u$), the welfare gap ($\Delta W$), the natural resource stock ($X$) and the human capital growth rate ($rA$) start and remain at their steady-state levels. The resource stock initially starts at its steady-state level, $X^*$ and the social planner makes the requisite adjustments to keep the bionomic system in a steady-state equilibrium along the balanced growth path. To obtain Fig. 1, the social planner maximizes intertemporal welfare by choosing optimal steady-state harvest rates and human capital accumulation rates that result in an optimal steady-state stock of 80.875%\(^\text{17}\) and a human capital accumulation rate of 4.50%. Consequently, per-capita GDP increases at an annual rate of 3.75% with per-capita welfare growing at 0.5625% annually. A negative welfare gap ($\Delta W^* = -3.1875\%$) implies that given the model’s parameters, per-capita GDP overstates the rate at which people’s lot in life is improving by 3.1875%.

Simulations identify the steady-state equilibrium, and they are also helpful in examining how parametric changes from the status quo affect the transient and steady-state behavior of the system. Consider two cases in which: (1) preferences for preserving the natural resource stock

\[\text{This refers to the percentage of the carrying capacity, } S, \text{ of the biological resource. This steady-state stock is right of maximum sustained yield, which is the ‘stable’ half of the biological growth function.}\]
and (2) population growth change over a discrete time period. In the first case, the relative weight placed on the private good in the welfare function \((1 - \phi)\) is decreased while the weight placed on the natural resource stock \((\phi)\) is increased. Specifically, \(\phi\) is increased at an annual rate of 1% over the time period \(t = 10–40\). The increasing weight on the natural resource stock reflects the ‘inverted-U’ hypothesis discussed above which maintains that as a society enjoys economic prosperity, it will place more emphasis on the environment. In the second case, the current population growth rate \((\mu)\) is increased at an annual rate of 1% over the time period \(t = 10–40\), after which it remains at its new level for the remaining analysis period.

Fig. 2 pertains to case (1). The system is initially at the steady-state equilibrium shown in Fig. 1. At \(t = 10\), the preference for natural resources begins to increase at an annual rate of 1%. As expected, the natural resource stock begins to climb owing to lower harvests. To compensate for the lower harvest rate, the social planner increases the rate of human capital accumulation \((r_A)\) rises). This trend continues until the point where, given the decreased flow from the natural capital stock, the marginal returns from additional human capital investment begin to decline at approximately \(t = 25\) \((r_y\) falls). At this point, lower harvest levels encourage a decrease in the human capital investment rate. At \(t = 40\), the preference for natural resources stops increasing and holds steady at approximately \(\phi = 0.67\). The resource stock increases to a new steady-state level of 90.73% with the human capital growth rate eventually returning to its initial steady-state level of 4.5%.

During this time when the preference for natural resources is increasing, the social planner adjusts the harvest rate and human capital accumulation rate to remain on the balanced-growth path. In turn, this adjustment process affects GDP and welfare growth rates \((r_y\) and \(r_u\)). Initially in a steady-state equilibrium, GDP growth rate begins to climb in conjunction with the rise in the human capital growth rate.\(^{18}\) However, as the growth rate in human capital falls, the GDP growth rate eventually returns to its pre-perturbation steady-state level.\(^{19}\) The transient and

\(^{18}\) See Eq. (A.13).
\(^{19}\) See Eq. (A.14).
post-perturbation behavior of the growth rate in welfare differs significantly from the growth rate in GDP. Initially in steady-state at approximately 0.56%, the welfare growth rate first climbs in response to the increase in the resource stock and GDP growth rate reaching a maximum of about 0.63%. As the rate of increase in GDP growth rate begins to diminish, the welfare growth rate begins to fall in conjunction with the increase in preferences for the natural resource. This is attributable to natural resource preferences increasing faster than the stock of natural resources, so that even though the resource stock and its relative weight in the welfare function is increasing, the net effect cannot overcome an increasing GDP growth rate whose relative weight is declining. As seen in Eq. (A.14), because the steady-state GDP growth rate returns to its pre-perturbation level but is now weighted less in the welfare function, the new steady-state value for the welfare growth rate is less than its pre-perturbation level. Thus, even though the natural resource stock and its relative weight in the welfare function are larger, the net effect is a lower steady-state welfare growth rate.

The welfare gap, as measured by $\Delta W$, initially at $-3.1875\%$, continues to widen as the preference for natural resources increases. When the welfare weight placed on natural resources reaches a new steady-state at 0.675 at $t = 40$, the widening trend ceases and the steady-state welfare gap increases slightly to $-3.65\%$. The new steady-state welfare gap is wider than its pre-perturbation level. These changes capture the notion that as preference for the natural resource increases relative to GDP, the welfare gap widens.

In Fig. 3, system sensitivity is examined to an annual 1% increase in the population growth rate from its original steady-state value of 1.5% at $t = 10$ to a new-steady-state value of 2.02% at $t = 40$. When the population growth rate begins to increase, the added population results in an increase in the number of people involved in human capital production, private good production and resource harvesting. The increase in the steady-state human capital growth rate from 4.50 to 5.02% is wholly a result of the increased population growth rate with none of the increase due to a change in the percentage of labor working in

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$^{20}$ See Eq. (A.16).
private good production and that devoted to human capital production.\(^{21}\) Even after the population growth rate levels off at 2.02% at \(t = 40\), the human capital growth rate does not reach its new steady-state equilibrium until \(t = 70\). It takes the system time to accommodate the changes instigated by the parameter change.

The natural resource stock sees its steady-state stock decline from 80.875 to 77.857%. Reflective of the increased demand for private good concurrent with a larger population, total output increases as the increased human capital stock requires additional natural resources. The added population each period results in an increase in total consumption, which requires an increase in the steady-state harvest rate and a decline in the natural resource stock to a lower steady-state equilibrium.\(^{22}\)

The decline in resource stock that begins at \(t = 10\) results in the income growth rate initially declining until the human capital growth rate increases to compensate. The rapid accumulation of human capital stock leads to an increase in the income growth rate until \(t = 40\). As the population growth rate stabilizes at 2.02%, the income growth rate continues to increase but at a decreasing rate. In a sense, the social planner, recognizing that the perturbation has ended, is no longer required to take the drastic steps necessary to keep the system on the balanced growth path. The new steady-state per-capita income growth rate is now at a higher level as a result of the higher steady-state population growth rate.\(^{23}\)

As a result of the initial decline in both the resource stock and income growth rate, and the need to now share the stock and income with more people, the welfare growth rate witnesses a precipitous decline at \(t = 10\). Even after the income growth rate recovers and begins to increase, the protracted decline in the per-capita share of resource stock results in a continued decline in welfare growth reaching a low point of 0.428% at \(t = 40\). Heretofore, the stock, declining at an increasing rate, continues to decline after \(t = 40\) but at a decreasing rate.\(^{24}\) The net result of this change is a partial recovery for the welfare growth rate from its minimum of 0.428% to a new steady-state equilibrium of 0.497%. The new steady-state welfare growth rate of 0.497% is lower than its pre-perturbation level of 0.5625%.

The gap between the growth rates in GDP and welfare widens as the growth rate in population increases. That is, the faster population multiplies, the greater the divergence between per-capita GDP and welfare growth rates. The reason for this divergence is primarily the reduced natural resource stock. As mentioned before, the stock declines from 80.875 to 77.857% while concurrently the number of people who share it increases. The net effect is less public good per-capita, which leads to a decline in the welfare growth rate and an increase in the welfare gap.

### 7. Conclusion

Standard methods employed to solve dynamic problems such as those seen in the endogenous growth literature are often unable to accommodate modifications such as the parabolic behavior of renewable resources. The clever method of Romer (1990) of identifying the balanced growth path was not, by itself, sufficient to solve our problem of accounting for the behavior of a nonlinear, biological growth function. The simulation software allowed the retention of the most interesting characteristics of the problem while also providing a tractable solution. It is felt that simulations can also be employed to provide insights and possibly help solve problems that heretofore have been beyond the scope of traditional solution methods.

Using simulations, the growth rate expressions for GDP and welfare are used to examine why a

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\(^{21}\) This can be seen by comparing the percentage changes between \(\mu\) and \(rA\).

\(^{22}\) This is different from the situation depicted in Fig. 2 where the increase in \(\phi\) eventually led to diminishing returns to additional human capital investment which resulted in \(ry\) returning to its original steady-state level.

\(^{23}\) See Eq. (A.14).

\(^{24}\) See curve 4 in Fig. 3.
commonly used indicator such as per-capita GDP is a poor proxy for per-capita welfare owing to the welfare gap. In steady-state, the per-capita income growth rate exceeds the per-capita welfare growth rate.

Simulations generated with STELLA were employed to examine how the variables in the model are affected by a slight but persistent parametric perturbation. This exercise helps to illustrate how the social planner adjusts the harvest rate and human capital accumulation rate in response to the parametric perturbation. The parameter that changes influences the size of $\Delta W^*$. However, as shown above, the effect on $r_i$ and $r_o$ is not always permanent, illustrating that some exogenous parameter shifts cause only transient changes. Once the perturbation ceases, the income or welfare growth rate may return to its original steady-state levels.

These results have important policy implications. For example, a policy that encourages higher population growth does increase per-capita income; however, the policy also diminishes welfare growth resulting in an increase in the welfare gap. As expected, a policy designed to improve technology will have the intended effect of narrowing the welfare gap. And policymakers in developing countries may question the ‘inverted-U’ relationship between income growth and environmental quality; while an increase in preference for the natural resource does increase the natural resource stock, it leads to a decline in the steady-state welfare rate.

Growth theoretic models are highly abstract, extremely aggregated representations of the real world. Nevertheless, empirical applications of growth models are used by economists in attempts to explain the wide divergences in living standards across nations, and why some countries seem to be making the transition from less to more developed, while other countries remain in poverty. Absent from these models is any recognition that economies function in and depend on a biological world that places limits on physical growth. When Solow first introduced his growth model in the 1950s, these biological constraints may have seemed more remote because the world’s population was half of what it is today. The results suggest that models which continue to ignore the biological constraints are desperately incomplete; they will yield misleading guidance owing to their focus on income growth instead of a more complete welfare measure.

Appendix A

The per-capita growth expressions for income and welfare are derived from a one-region endogenous technological growth model that is constrained by a natural capital production function. Welfare for a representative individual is based on her consumption of a public and private good. All people in the region $(m+n)$ share a private good, $Y$, produced using the harvested renewable resource. The individual also derives welfare from that portion of renewable resource that remains unharvested, $X$. The renewable resource represents a public good such as a forest or fishery and is shared among everyone in the region $(m+n)$.

In the instantaneous welfare function, (A.1), $\eta$ determines the consumer’s attitude toward intertemporal consumption. The smaller is $\eta$, the more slowly marginal welfare falls with a rise in consumption. It is assumed that $\eta = 0.5$ in the model. The relative weight that one places on the unharvested resource stock (private good) is represented by $\phi (1 - \phi)$.

$$U\left(\frac{X}{m+n}, \frac{Y}{m+n}\right) = \frac{1}{1-\eta} \left[ \left( \frac{X}{m+n} \right)^\phi \left( \frac{Y}{m+n} \right)^{1-\phi} \right]^{1-\eta} \quad (A.1)$$

The goal is to maximize intertemporal welfare by identifying the optimal per-capita consumption path for the resource and private good. Welfare is constrained by a biological growth function and human capital stock accumulation relationship.

Maximize $\int_0^\infty \frac{1}{1-\eta} \left[ \left( \frac{X}{m+n} \right)^\phi \left( \frac{Y}{m+n} \right)^{1-\phi} \right]^{1-\eta} e^{-\rho t} dt$

subject to $\dot{X} = gX(1 - X/S) - vmX$

$A = \sigma AH_A$
\[ Y = (A(n - H_A))^\gamma (vmX)^{1 - \gamma} (\Gamma)^{1 - \gamma} \]

and \( A(0) = A_0 \), \( vmX(0) = (vmX)_0 \)

The relative importance of the productive inputs is captured in the Cobb–Douglas production coefficient \( z \). The larger is \( z \), the more important is labor in the production process and the less important is natural capital. The subjective discount weight on consumption is \( \rho \). The constraint on the renewable resource stock is represented by the biological growth relationship \( F(X) = gX(1 - X/S) \) where \( g \) depicts the intrinsic biological growth rate and \( S \) the carrying capacity of the resource (Hartwick and Olewiler, 1986). The human capital constraint, \( H \), is a function of educational success, \( \sigma \), the current stock of human capital, \( A \), and the number of people involved in human capital production, \( H_A \).

A frequently used assumption in endogenous growth models is to have population growing at a constant rate (Romer, 1996). All population terms are assumed to grow at the constant rate \( \mu (m = m e^{\mu t}, n = n e^{\mu t}, m + n = (m + n) e^{\mu t}) \), and \( H_A = H_A e^{\sigma t} \). Also, proprietary production technology that converts natural capital to a durable good of any design grows at a constant rate, \( \gamma (\Gamma = \Gamma e^{\gamma t}) \), where \( \Gamma \) represents the current state of technology by denoting the amount of natural capital necessary to produce one unit of private good. Because the exponent on \( \Gamma \) in the production function for \( Y \) is negative, \( \gamma \) must be negative in order to represent improving production technology with large negative values representing relatively better technology. An increase in \( \gamma \) represents a reduction in (or worsening of) the growth rate of production technology; it takes more natural capital to produce one unit of private good. Making these substitutions, \( m, n, H_A \), and \( \Gamma \) represent initial condition levels and the problem becomes

\[
\text{Maximize } \int_0^\infty \frac{1}{1 - \eta} \left[ \frac{X}{m+n} \phi \left( \frac{Y}{m+n} \right)^{1 - \phi} \right]^{1 - \eta} e^{-\Psi t} dt
\]

subject to \( \dot{X} = F(X) - vm e^{\mu t} \)
\( \dot{A} = \sigma AH_A \ e^{\sigma t} \)
and \( A(0) = A_0 \), \( vmX(0) = (vmX)_0 \)

where \( \Psi = \rho - (1 - \eta)(1 - z)(1 - \phi); \)
\[ + (1 - \gamma) \phi \mu. \]

The current-value Hamiltonian can be expressed as:

\[
H_e = \frac{1}{1 - \eta} \left[ \left( \frac{X}{m+n} \right)^{\phi} \left( \frac{Y}{m+n} \right)^{1 - \phi} \right]^{1 - \eta} - \lambda_X (gX(1 - X/S) - vm e^{\mu t}X) - \lambda_A (\sigma AH_A e^{\sigma t}) \]

(A.2)

There are four first-order conditions: one for the control variable, \( v, (\dot{c}H_c/cv) = 0 \), and its corresponding state variable, \( X, \dot{X} = -(\ddot{c}H_c/cX) + \Psi \lambda_X \); and one for the control variable, \( H_A, (\ddot{c}H_c/cH_A) = 0 \), and its state variable, \( A, \dot{A} = -(\ddot{c}H_c/cA) + \Psi \lambda_A \). Solving the first-order condition for the control variable \( v \) in terms of \( \lambda_X \), the shadow price of the resource, gives

\[
\dot{X} = \frac{(1 - \phi)(1 - z)}{vm e^{\mu t}X} \left[ \frac{X}{m+n} \phi \left( \frac{Y}{m+n} \right)^{1 - \phi} \right]^{1 - \eta} \]

(A.3)

The first-order condition for \( X \) is

\[
\dot{X} = -\left[ \left( \frac{X}{m+n} \right)^{\phi} \left( \frac{Y}{m+n} \right)^{1 - \phi} \right] \]
\[ - \phi \frac{(1 - z)(1 - \phi)}{X} \]
\[ + \lambda_A (\Psi - F' - vm e^{\mu t}) \]

(A.4)

Dividing (A.4) by (A.3) provides

\[
\frac{\dot{X}}{\lambda_X} = \Psi - F' - \frac{\phi}{(1 - z)(1 - \phi)} vm e^{\mu t} \]

(A.5)

Solving the first-order condition for the control variable \( H_A \) in terms of \( \lambda_A \), the shadow price of human capital investment, provides

\[
\dot{A} = \frac{f(1 - \phi)}{\sigma AH_A e^{\sigma t}} \left[ \frac{X}{m+n} \phi \left( \frac{Y}{m+n} \right)^{1 - \phi} \right]^{1 - \eta} \]

(A.6)

Differentiating (A.6) with respect to time provides the following expression.
\[
\frac{\dot{A}}{A} = \frac{z(1-\phi)}{\sigma AH_Y e^\sigma} \left[ \left( \frac{X}{m+n} \right)^\phi \left( \frac{Y}{m+n} \right)^{1-\phi} \right]^{1-\eta} \\
\left\{ \left( 1-\eta \right) \left( \phi + (1-\eta) \left( 1-\phi \right) \right) \frac{\dot{X}}{X} \right. \\
\left. + (\phi(1-\phi)(1-\eta) - 1) \frac{\ddot{A}}{A} - \mu \right\} 
\]

(A.7)

Dividing (A.7) by (A.6) gives
\[
\frac{\dot{A}}{A} = \left( 1-\eta \right) \left( \phi + (1-\eta) \left( 1-\phi \right) \right) \frac{\dot{X}}{X} \\
+ (\phi(1-\phi)(1-\eta) - 1) \frac{\ddot{A}}{A} - \mu 
\]

(A.8)

Solving the system explicitly for its dynamics is complex. To simplify the analysis, the discussion is focused on the properties of the balanced growth equilibrium inherent in the model. Eq. (A.9) illustrates a basic feature of the steady-state equilibrium: along the balanced growth path, the growth rate of the human capital stock \((r_A = \dot{A}/A)\) equals the sum of the growth rates of natural capital, population and proprietary production technology.

\[
r_A = \frac{\dot{X}}{X} + \mu + \gamma 
\]

(A.9)

As Romer shows in his paper (1990), in steady-state, \(\frac{\dot{A}}{A} = \frac{\dot{X}}{X}\). Equating (A.5) and (A.8) and using (A.9) provides the steady-state expressions for the optimal harvest effort level \((V)\) and the optimal human capital growth rate \((r_A)\) as a function of the various system parameters, Cobb–Douglas production and welfare coefficients and resource stock level (see A.10 and A.12).

\[
V = vm e^{\mu t} \left[ \frac{\rho + (1 - (1-\eta)(1-\phi))(\phi - z(1-\phi)))\mu - F' + \eta \frac{F'}{X} \right] 
\]

(A.10)

\[
r_A = \frac{F}{X} + \gamma + \mu - \left[ \frac{\eta + (1-z)(1-\phi)(\phi - z(1-\phi)))\mu - F' + \eta \frac{F'}{X} \right] \\
\eta + \frac{\phi}{(1-z)(1-\phi)} 
\]

(A.11)

With the resource stock variable \(X\) still present, Eq. (A.10) and Eq. (A.11) do not accurately represent the first-order conditions for \(V\) and \(H_A\).

The expression for the biological growth function could be used to eliminate \(X\) from (A.10) and (A.11). However, solving this quadratic equation for \(X\) requires assumptions that detract from the issues of interest.\(^25\) For this reason, dynamic simulations are used to determine the optimal steady-state equilibrium for the bionomic system.

The expressions for per-capita income and welfare growth rates are obtained from (A.1). In Eq. (A.12), \(y\) represents the equal income share to all agents in the region.

\[
y = \frac{Y}{m+n} = \frac{\left( A(n-H_A)^x(umX)^{\frac{1}{2}} - \varepsilon \right)}{m+n} 
\]

(A.12)

Totally differentiating (A.12) with respect to time, (A.13) represents the general expression for the growth rate in per-capita income along the balanced growth path

\[
r = \frac{\dot{y}}{y} = \alpha r_A + (1-\alpha) \left[ \frac{\dot{X}}{X} + \gamma \right] 
\]

(A.13)

with the steady-state growth rate of per-capita income represented by (A.14)

\[
r^* = \alpha \mu + \gamma 
\]

(A.14)

where the ‘*’ represents a steady-state equilibrium value.\(^{26}\)

Knowing that per-capita welfare is represented by

\[\hat{X} = \alpha X + \gamma + \mu - \left[ \frac{\eta + (1-z)(1-\phi)(\phi - z(1-\phi)))\mu - F' + \eta \frac{F'}{X} \right] \\
\eta + \frac{\phi}{(1-z)(1-\phi)} \]

\(25\) For example, assuming that is valid for steady-state conditions but not accurate for transient periods when \(X\) is changing.

\(26\) At steady-state.
the growth rate expression for per-capita welfare is

\[ r_u = (1 - \eta) \left[ \phi \left( \frac{X}{\Lambda} - \mu \right) + (1 - \phi) r_y \right] \quad (A.16) \]

In steady-state, \( (A.16) \) becomes

\[ r_u^* = (1 - \eta) [(1 - \phi) r_x^* - \phi \mu] \quad (A.17) \]

The difference between \( r_u \) and \( r_y \) is the welfare gap, \( \Delta W = r_u - r_y \). Using \( (A.13) \) and \( (A.16) \), the general welfare gap along a balanced growth path can be written as

\[ \Delta W = \phi \left[ \frac{\dot{X}}{X} - r_A - \alpha \mu \right] \quad (A.18) \]

with \( (A.19) \) representing the welfare gap in steady-state.

\[ \Delta W^* = - \phi [r_A^* + \alpha \mu] \quad (A.19) \]

References


Cobb, C., Halstead, T., Rowe, J., 1995. If the GDP is up, why is America down? Atlantic Monthly 276(4) 59–78.