ANALYSIS

Non-renewability in forest rotations: implications for economic and ecosystem sustainability☆

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Abstract

The forest rotations problem has been considered by generations of economists (Fisher, 1930; Boulding, 1966; Samuelson, 1976). Traditionally, the forest resource across all future harvest periods is assumed to grow without memory of past harvest periods. This paper integrates economic theory and intertemporal ecological mechanics, linking current harvest decisions with future forest growth, financial value, and ecosystem health. Results and implications of a non-renewable forest resource and the influence of rotation length and number on forest recovery are reported. Cost estimates of moving from short-term economic rotations to long-term ecological rotations suggest the level of incentive required for one aspect of ecosystem management. A net private cost of maintaining ecosystem health emerges and, for public policy purposes, can be compared with measures of non-timber amenity values and social benefits exhibiting increasing returns to rotation length. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Ecological economics has distinguished itself from traditional economic analysis of the environ-

ment by stressing the essential role of ecosystem services and the maintenance of ecosystem processes. Traditional analysis of renewable resources such as forests, fisheries, and agriculture has long stressed the conditions for steady-state systems of management. The economic study of these natural resources, however, has often overlooked critical intertemporal ecological mechanics related to the timing and impact of disturbing natural systems.

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The study of the regeneration of forests under centuries long harvest sequences is beginning to redefine our understanding of resource renewability. Traditional financial models of the forest resource assume perfect renewability in forest growth following infinite optimal rotations of constant length. Study of forest ecology, however, suggests that rotations affect future growth, product quality, and forest health. For instance, alteration of successional sequences, nutrient cycles, and other components of ecosystem function are influenced by rotation length, harvest intensity, and cutting frequency. These cross-harvest interactions suggest a non-renewable forest growth specification, leading to the addition of a marginal benefit of recovery in the traditional optimal rotation decision rule.

In this paper, an integrated forest succession, product, and price model for the northern hardwood forest ecosystem is developed to evaluate the impact of increasing density of pioneer species following disturbance on rotation length and timber profits. For the ecosystem type examined, the success of early successional species in disturbance–recovery cycles due to short, repetitive rotations have the effect of delaying forest development and entrance into late successional, higher quality, higher return species. Accordingly, an overlooked financial benefit to forest recovery is specified and estimated for a discrete horizon rotations problem. A non-renewable growth specification has the effect over traditional models of lengthening forest rotations, adjusting profits downwards, and valuing the long-term maintenance of ecosystem processes. By incorporating ecosystem modeling into an economic framework, a clearer management picture results.

2. The marginal benefit of recovery

For the commercial forest manager, the principal economic question centers on harvest timing. The majority of the economic literature on this question is grounded in the model developed in the 19th century by the German tax collector Martin Faustmann (Faustmann, 1849). Faustmann was concerned with estimating the bare-land expected profits\(^1\) of a forthcoming forest. Assuming land is to remain in forestry, the problem is to solve for the rotation length \((T)\) over an infinite stream of future profits from harvesting a perfectly renewable resource.

Assuming a continuous-time discount factor \((e^{-\alpha t})\) and a continuously twice differentiable stand profit function \((\pi(t))\), the infinite horizon profit maximization problem converges to:

\[
\text{Max } \Pi = \frac{\pi(t)}{e^{\alpha t} - 1}
\]

where:

\[
\pi(t) = P Q(t).
\]

Stumpage price \((P)\) equals net price per unit volume. Natural regeneration is assumed, so replanting costs are assumed zero. In the most general case of the multispecies, multiquality problem, \(P\) represents a matrix of stumpage prices and, likewise, \(Q(t)\) models a matrix of timber volumes across species and quality classes.

Solving Eq. (1) produces the following first-order condition, known as the Faustmann formula:

\[
\pi'(t) = \delta \pi(t) + \frac{\delta \pi(t)}{e^{\alpha t} - 1}
\]

From Eq. (3), a single optimal rotation length \((T)\) maximizes net present value \((\Pi)\) by equating the marginal benefit of waiting to the marginal opportunity cost of delaying the harvest of the current stand (i.e. interest forgone on current profit) plus the marginal opportunity cost of delaying the harvest of all future stands (i.e. interest forgone on all future profits, often called site value).\(^2\)

---

\(^1\) The term ‘value’ has been used to represent forest profits (e.g. Clark, 1990) in economics. Here, ‘value’ is reserved for problems incorporating non-forest amenities and other positive externalities. For example, forest profits include only income from the sale of timber, where forest value would include non-market goods such as aesthetics, biodiversity, or recreation.

\(^2\) If real stumpage prices are assumed to grow at a rate \(r\), then the Faustmann formula simply becomes: \(\pi'(t) = (\delta - r)\pi(t) + [(\delta - r)\pi(t)]/[e^{\alpha t} - 1]\). Eqs. (29)–(33) in the empirical analysis introduces price growth.
Adaptations and expansions to this model include modeling non-timber benefits (e.g. Hartman, 1976; Calish et al., 1978; Berck, 1981), multiple-use forestry (e.g. Bowes and Krutilla, 1989; Snyder and Bhattacharyya, 1990; Swallow and Wear, 1993), stochastic price paths (e.g. Clarke and Reed, 1989; Forboseh et al., 1996), market structure (e.g. Crabbe and Van Long, 1989), and uneven aged forestry (e.g. Montgomery and Adams, 1995).

All these improvements to the basic Faustmann formula, however, share a strong assumption of perfect growth renewability—a constant growth function \( Q(T) \) across all future planning periods. In contrast, evidence from the study of forest ecology and management indicates a strong relationship between rotation length, rotation frequency, and harvest magnitude in a current management period, with the growth and maintenance of the forest in future periods (e.g. Borman and Likens, 1979; Kimmins, 1987). This is particularly the case where natural regeneration seeds the new forest, or soil renewability is compromised. In the Faustmann framework, this ecological knowledge implies a forest stand profit function dependent on rotation length \( T \) and rotation number \( i \), given constant technology and harvest magnitude.

Growth in merchantable timber volume is typically modeled using a cubic or exponential form, consistent with stages for rapid growth, biological maturity, and disease and decay (Clark, 1990). Consider a cubic functional form for undiscounted profit at constant prices:

\[
\pi(t) = \beta_1 t + \beta_2 t^2 + \beta_3 t^3. \tag{4}
\]

Fig. 1 illustrates three plots of Eq. (4) following a harvest at \( T_0 \) assuming different parameter values for \( \beta_1 \), \( \beta_2 \), and \( \beta_3 \). Suppose \( T_{1A} \) is an optimal Faustmann rotation in the first harvest cycle \( i = 1 \). Therefore, a longer rotation in this first cycle (for instance, \( T_{1B} \)) would be sub-optimal as it would decrease the marginal value of waiting below the sum of first harvest and future harvest opportunity costs.

However, there may be an additional marginal variable to consider in the first rotation decision. Suppose rotation length in the first harvest cycle influences the form of the functional stand profit function in subsequent cycles. For instance, suppose the choice of \( T_{1A} \) in cycle \( i = 1 \) results in the profit function \( \pi(T_{i=2} | T_{1A}) \) in cycle \( i = 2 \). A longer rotation such as \( T_{1B} \), however, results in a higher profit function \( \pi(T_{i=2} | T_{1B}) \) in cycle \( i = 2 \). In this case, a longer first rotation has the benefit of allowing the forest more time to recover from the initial cut at \( T_0 \). Now, waiting until \( T_{1B} \) to harvest during the first cycle has the benefit of shifting the second cycle curve upwards to \( \pi(T_{i=2} | T_{1B}) \). A sufficiently long first rotation would
result in an identical second rotation profit function. Without taking into account this cross-harvest impact, the Faustmann solution of $T_{i,A}$ would lead to a sub-optimal decision.

To incorporate this interaction between current harvest length and subsequent profit functions consider Eq. (5). The function $f(T_{i-1}, i - 1)$ is added as a variable to the period $i$ profit function. The level of $f(T_{i-1}, i - 1)$, or ecological impact, depends on the length of the last period’s rotation ($T_{i-1}$), and the number of rotations since the first cut at $T_0$ to take into account any cumulative impacts. It influences the cubic function parameters ($\beta_1$, $\beta_2$, and $\beta_3$) of the stand profit function through an ecological impact represented by the parameters $x_1$, $x_2$, and $x_3$:

$$\pi(t_i, f(T_{i-1}, i - 1)) = (\beta_1 + x_1 f(T_{i-1}, i - 1)) t_i^1 + (\beta_2 + x_2 f(T_{i-1}, i - 1)) t_i^2 + (\beta_3 + x_3 f(T_{i-1}, i - 1)) t_i^3$$

where:

$$f(T_0, 0) = \Omega$$

$$f(T_{i-1}, i - 1) \geq \Omega$$

$$\frac{\partial f(T_{i-1}, i - 1)}{\partial T_{i-1}} < 0$$

$$\frac{\partial^2 f(T_{i-1}, i - 1)}{\partial T_{i-1}^2} > 0$$

$$\frac{\partial f(T_{i-1}, i - 1)}{\partial t(i - 1)} > 0$$

$$\frac{\partial^2 f(T_{i-1}, i - 1)}{\partial (i - 1)^2} < 0$$

and:

$$x_1 < 0,$$

$$x_2 < 0,$$

$$x_3 < 0$$

for $i = 1, 2, 3, \ldots$

Stand profit in the current rotation cycle ($i$) now depends on the current rotation length ($T_i$), the previous rotation length ($T_{i-1}$), and the number of rotations ($i - 1$) since the pre-disturbance period ($i - 1 = 0$). The ecological impact function, $f()$, represents a forest recovery relationship based on physical and biological parameters. For example, $f()$ might measure the impact on forest regeneration from pioneer species rebound (stems/acre), from soil nutrient loss (nutrients/m$^3$) or erosion (soil depth), or possibly from a general index of resource renewability.

The first-order conditions for $f()$ imply that as the previous period rotation length ($T_{i-1}$) increases, the negative ecological impact decreases. Also, as the number of rotations since the pre-disturbance period ($i - 1 = 0$) increases, the ecological impact increases. An initial condition ($\Omega$) is assumed which defines the level of $f()$ following the initial harvest at $T_0$. This parameter can be considered a forest health endowment left from the previous land manager. In the case of inheriting a mature forest not previously managed, $\Omega$ could be considered the ecological effect on forest growth from periodic natural disturbances (e.g. wind storms, fires).

Assuming this non-renewable, rotation-length dependent, stand profit specification over an infinite horizon, the profit maximization problem becomes:

$$\text{Max}$$

$$\Pi = \pi(t_1, f(T_0, 0)) e^{-\delta T_1} + \pi(t_2, f(T_1, 1)) e^{-\delta T_2} + \pi(t_3, f(T_2, 2)) e^{-\delta T_3} + \ldots$$

Under an assumption of perfect renewability, $f(T_0, 0) = f(T_1, 1) = \ldots = f(T_{\infty}, \infty) = \Omega$, and the profit maximization problem converges to Eq. (1), from which the usual Faustmann result of a constant rotation length in Eq. (3) is obtained.

Under the assumption of partial non-renewability, however, the selection of the optimal rotation length set ($T_i$ for $i = 1, 2, 3, \ldots$) now considers the impact on each subsequent period’s profits through the addition of a marginal benefit of recovery (MBR). The marginal benefit of recovery in period $i$ from a rotation length in the previous period $i - 1$ is represented as:

$$\text{MBR}_i = \frac{\partial f(T_{i-1}, i - 1)}{\partial T_{i-1}}(x_1 T_i + x_2 T_i^2 + x_3 T_i^3) > 0$$

Thus, balancing the benefits to recovery from longer rotations against the opportunity costs of delaying current and future harvests will determine the optimal rotation set.
In the forest ecology literature, Kimmins (1987) outlines the distinction between a Faustmann-type rotation where net present value is maximized, and an ecological rotation, the time required for a site managed with a given technology to return to the pre-disturbance ecological condition. Fig. 2 demonstrates the concept of an ecological rotation, and the hypothetical case of rotating before a successional sequence is completed. Succession is defined as the orderly replacement over time of one species or community of species by another, resulting from competitive interactions between them for limited site resources (Marchand, 1987). The vertical axis of Fig. 2 delineates a range from early successional species (pioneer) to late successional species (climax).

Under a moderate disturbance regime (for instance, stem harvesting or selective cutting), $T$ and $2T$ represent two Faustmann rotations. The declining path of 'backwards' succession is referred to as successional retrogression. For a moderate disturbance, an ecological rotation is represented by $T^E$, the time when the forest recovers to the original successional condition. A more severe disturbance regime (for instance, whole-tree harvesting or clear-cutting) is also represented where a longer ecological rotation ($T^{E^2}$) would necessarily be required for successional rebound. Ecological observations also suggest the possibility that severe or repeated disturbance could shift the biotic community into a different domain in which the mature (climax) phase of succession is very different than the pre-disturbance condition (Perry et al., 1989). For instance, a clear-cut of a mature forest resulting in the permanent replacement by grasslands might be represented in Fig. 2 as a path that never rebounds.

While Fig. 2 focuses on a potential decay in successional pathways due to short forest rotations, a similar diagram could model other ecosystem retrogressions. For example, Federer et al. (1989) describe the effects of intensive harvest on the long-term soil depletion of calcium and other nutrients, and the potential limiting effect on forest growth.

In the next section, a model is developed to investigate the ecological mechanisms and economic consequences behind a rotation-dependent profit function in the spirit of the Kimmins' successional retrogression hypothesis. Knowledge of the relationship between rotation length and future profit functions may influence rotation deci-
sions, with both economic and ecological benefits. Furthermore, valuing ecosystem recovery may benefit non-timber amenities exhibiting increasing returns in \( T \) as described elsewhere (often referred to as the Hartman model after Hartman, 1976). Lastly, the cost and benefits of moving from economic rotations to ecological rotations can be obtained and used for public policy extensions.

3. An ecological economic model of the northern hardwood forest

The northern hardwood forest ecosystem is the dominant hardwood component of the larger northern forest of the United States, stretching west to northern Minnesota, east through New England, south into parts of the Pennsylvania Appalachians, and north into Canada. It is characterized by sugar maple (\( Acer saccharum \)), American beech (\( Fagus grandifolia \)), and yellow birch (\( Betula alleghaniensis \)) predominance, with varying admixtures of other hardwoods and softwoods. A model was developed to account for forest growth, pioneer species introduction, conversion from biomass to merchantable timber and pulpwood, and stumpage price growth. Development and details of these four components are described in detail in Erickson et al. (1997).

3.1. Growth simulation

The stochastic forest growth simulator JABOWA developed by Christ et al. (1995) was used to model succession and growth following a clear-cut in the northern hardwood forest. The JABOWA model simulates growth of individual trees on small plots at the forest gap level, built on silvical data for the species of the Hubbard Brook Experimental Forest in the White Mountains of New Hampshire. ‘Gap’ refers to a hole in the forest canopy created by the felling of a tree, naturally or otherwise. Christ et al. (1995) developed a version of the model in PASCAL to test the accuracy of the original Botkin et al. (1972) model predictions against forest inventory data. Model development, parameters, and forest species characteristics are described in Erickson et al. (1997). In general, growth algorithms for each species consist of the following components (adapted from Botkin et al., 1972):

\[
\Delta d = G(\sigma, L, d_{\text{max}}, h_{\text{max}}) \cdot r(L(I, Z)) \cdot \eta(D, D_{\text{min}}, D_{\text{max}}) \cdot S(A, \theta)
\]

\[
G() = \sigma L \{1 - [(d \cdot h)/(d_{\text{max}} \cdot h_{\text{max}})]\}
\]

\[
r() = 1 - e^{-4.64(L - 0.05)} \ \{\text{shade-tolerant}\}
\]

\[
r() = 2.24 (1 - e^{-1.136(L - 0.08)}) \ \{\text{shade-intolerant}\}
\]

where:

\[
L = I e^{-kZ}
\]

\[
\eta() = \frac{4(D - D_{\text{min}})(D_{\text{max}} - D)}{(D_{\text{max}} - D_{\text{min}})^2}
\]

\[
S() = 1 - A/\theta
\]

Eq. (17) represents the annual change in species diameter at breast height (\( d \)). Only growth in diameter is modeled because it will be used to predict merchantable volume (\( Q \)) by species and product class for estimating the stand profit function in Eq. (1). The function \( G \) represents a growth rate equation for each species under optimal conditions, depending on a solar energy utilization factor (\( \sigma \)), leaf area (\( L \)), and maximum values for diameter (\( d_{\text{max}} \)) and height (\( h_{\text{max}} \)).

The remaining right-hand side functions act as multipliers to the optimal growth function to take into account shading, climate, and soil quality. The shading function, \( r \), is modeled separately for shade-tolerant and -intolerant species and depends on available light to the tree (a function of annual insolation (\( I \)) and shading leaf area (\( Z \))). The function \( \eta \) accounts for the effect of temperature on photosynthetic rates, and depends on the number of growing degree-days (\( D \)) and species specific minimum and maximum values of \( D \) for which growth is possible. Lastly, \( S \) is a dynamic soil quality index dependent on total basal area (\( A \)) on the plot and maximum basal area (\( \theta \)) under optimal growing conditions.

Stochastic dynamics of stand growth enter the model through stem birth and death subroutines. Each year, individual trees competing for light become established on the forest floor, grow, or
Species characteristics and chance determine the dynamics of these birth−growth−death cycles. New saplings randomly enter the plot within limits imposed by their relative shade tolerance and degree-day and soil moisture requirements. As taller trees shade smaller ones, the amount of shading is dependent on the species’ characteristic leaf number and area, and survival under shaded conditions depends on the shade tolerance of a species. Given this stochasticity, simulation data vary widely with each model run.3

3.2. Successional retrogression

Building on the JABOWA model, the challenge is to incorporate an ecological mechanism to capture Kimmins’ hypothesis of rotation-dependent succession and growth. Such a mechanism is evident in the early succession rebound of pioneer species. A possible succession of dominant species is represented by Fig. 3, adapted from Marks (1974).

During the first 15−20 years following a clear-cut, the recovering forest is dominated by pioneer species such as raspberry bushes, birches, and pin cherry. These fast growing, opportunistic species play a critical role in ecosystem recovery from a clear-cut by reducing run-off and limiting soil and nutrient loss (Marks, 1974). However, their initial density will also influence stand biomass accumulation and growth of commercial species (Wilson and Jenson, 1954; Marquis, 1969; Mou et al., 1993; Heitzman and Nyland, 1994).

In this application to the northern hardwood forest, pin cherry (Prunus pensylvanica) is assumed to be the dominant pioneer species. As a particularly fast growing, short-lived, shade-intolerant species with no commercial value, the effect of its growth following a clear-cut on forest succession and future harvest profits can be significant. Tierney and Fahey (1998) demonstrate the influence of short rotations on the survival of its seeds, and its subsequent germination and growth at very high density in young stands. This forest ecology research indicates that pioneer species densities may stabilize at low levels following a 120-year or more rotation regime (comparable with a Kimmins’ ecological rotation). Rotations at 60-year intervals (closer to a Faustmann economic rotation) result in increasing pioneer species densities toward a carrying capacity asymptote.

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3 Data specific to defining equations in the remainder of this section can be obtained from the authors, and are based on ten runs (ten 100 m² plots). This builds an approximately 1/4 acre plot, which is subsequently expanded to a full acre by assuming each tree represents four trees per acre.
The dependence of the initial density of a pioneer species (PS) on the previous harvest rotation length \((T_{i-1})\) and the number of previous harvests \((i - 1)\) is used to represent the more general case of sucessional retrogression from Fig. 2. The following ordinary least squares model was estimated to capture the hypothesis of a rotation-dependent ecological impact function proposed in Eq. (5). Estimation is based on data from the seed bank dynamic modeling results of Tierney and Fahey (1998). Ecological assumptions and research results are reported in Erickson et al. (1997).

\[
PS = f(T_{i-1}, i - 1) = 100 = \Omega
\]

for \(T_i > 140 \text{ years}\) \hspace{1cm} (24)

\[
PS = 7342.27 - 89.18 T_{i-1} + 0.25 T_{i-1}^2 + 550 (i - 1) \quad \text{for} \quad T_i \leq 140 \text{ years}^4
\]

\hspace{1cm} (25)

3.3. Multiproduct, stochastic quality model

The third model component converts biomass output from JABOWA into economic output. The financial value of standing timber depends on age, size, species, and quality distributions. A typical northern hardwood stand can provide sawtimber, pulpwood, and firewood. Depending on the market and the land owners motivations, any combination of these three product classes may be managed. Stand profit is represented as:

\[
\Pi(t, PS) = \left\{ \sum_{s=1}^{9} \sum_{c=1}^{6} Q_{S,C}[d, M, PS] \right\} \cdot P_t
\]

Profit \([\pi(t, PS)]\) is defined at a year following a clear-cut and before the next \((t = 0, 1, 2, \ldots, T)\), given initial pioneer species density (PS). Pioneer species density influences profitability through introducing significant competition for light and other resources in the JABOWA model during early stand development. As in Eq. (1), total stand profit (US$/acre) is the product of a price matrix \((P_t)\) and merchantable volume \((Q)\) for eight commercial species \((S = 1–8)\) and a non-commercial species group \((S = 9)\) in each product category \((C = 1–6)\). Product categories are comprised of grade 1–3 timber \((C = 1–3)\), below grade sawtimber \((C = 4)\), and hardwood \((C = 5)\) and softwood \((C = 6)\) pulp. To assign quality classes, a random number is generated and assigned to each stem and compared with class probability limits as estimated by a generalized logistic regression (GLR) model developed by Yaussy (1993). The GLR procedure, parameters, and an example are described in Erickson et al. (1997). Firewood output was not considered.

Merchantable volume \((Q)\) is modeled on stem diameter \((d)\), provided for each tree by a growth simulation, and merchantable length \((M)\), which is also modeled on \(d\). The level of initial pioneer species density (PS) is predicted from Eq. (25) based on the previous period’s rotation length \((T_{i-1})\) and number \((i - 1)\). PS influences diameter growth through the dynamics of the forest growth simulator, as well as influencing merchantable volume calculations through impacting forest site quality. The procedures for converting diameter estimates to merchantable volume by species and product class are described in detail in Erickson et al. (1997).

3.4. Parameterization

Integrating the first three components of the model outlined above, merchantable stand volumes were generated at 10-year intervals from year 20 to 250, at initial pioneer species densities of 0, 10, 20, 50, 100, 200, 500, 1000, 2000, and 5000 stems per 100 m\(^2\). Volume within each species, product class, and year was then converted to profit by multiplying a net price matrix of 1995 stumpage prices. The initial distribution of net prices \((P_0)\) across product classes and species is summarized in Table 1. Stand profit for each year was then summarized across all products and species to generate data for \(\pi(t, PS)\) at each PS value run.
Table 1
Initial sawtimber stumpage and pulpwood prices ($P_0$)*

<table>
<thead>
<tr>
<th>Species</th>
<th>Below grade (US$/thousand board feet)</th>
<th>Grade 3</th>
<th>Grade 2</th>
<th>Grade 1</th>
<th>Pulp (US$/cord)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sugar maple</td>
<td>125</td>
<td>298.30</td>
<td>471.5</td>
<td>650</td>
<td>7</td>
</tr>
<tr>
<td>Beech</td>
<td>20</td>
<td>38.15</td>
<td>56.3</td>
<td>75</td>
<td>7</td>
</tr>
<tr>
<td>Yellow birch</td>
<td>50</td>
<td>99.50</td>
<td>149.0</td>
<td>200</td>
<td>7</td>
</tr>
<tr>
<td>White ash</td>
<td>75</td>
<td>182.30</td>
<td>289.5</td>
<td>400</td>
<td>7</td>
</tr>
<tr>
<td>Balsam fir</td>
<td>30</td>
<td>53.10</td>
<td>76.2</td>
<td>100</td>
<td>12</td>
</tr>
<tr>
<td>Red spruce</td>
<td>30</td>
<td>53.10</td>
<td>76.2</td>
<td>100</td>
<td>12</td>
</tr>
<tr>
<td>Paper birch</td>
<td>45</td>
<td>56.55</td>
<td>68.1</td>
<td>80</td>
<td>7</td>
</tr>
<tr>
<td>Red maple</td>
<td>50</td>
<td>83.00</td>
<td>116.0</td>
<td>150</td>
<td>7</td>
</tr>
<tr>
<td>Non-commercial</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

* Note: Sawtimber prices in each quality class were calculated from ranges of stumpage prices reported in NYDEC (1995) for the Adirondack region. Within each range: min = below grade price, 33rd percentile = grade 3 price, 66th percentile = grade 2 price, and max = grade 1 price.

This specification results in a $264 \times 6$ explanatory variable matrix. The following cubic model was fitted:

$$
\pi(t, PS) = (\beta_1 + x_1PS)t + (\beta_2 + x_2PS^2)t^2 + (\beta_3 + x_3PS)t^3 \tag{27}
$$

$$
\pi(t, PS) = (7.718 - 0.0025PS)t
+ (0.219 + 1.52 \times 10^{-9}PS^2)t^2
- (0.00082 + 1.40 \times 10^{-8}PS)t^3 \tag{28}
$$

Fig. 4 plots $\pi(t)$ at some illustrative PS values. Here $\pi(t)$ represents stand profit at 1995 prices. Price growth is taken up separately in Section 3.5.

3.5. Price growth ($P_t$)

The influences on stumpage prices at the forest stand level are complex. They might include: timber quality, volume to be cut per acre, logging terrain, market demand, distance to market, season of year, distance to public roads, woods labor costs, size of the average tree to be cut, type of logging equipment, percentage of timber species in the area, end product of manufacture, landowner requirements, landowner knowledge of market value, property taxes, performance bond requirements, and insurance costs (NYDEC, 1995). At the macroeconomic level, exports, mill stocks, and aggregate demand are typically explanatory variables (Luppold and Jacobsen, 1985). Emerging effects on northeast stumpage prices include increasing substitution of recycled fibers in paper making, board feet restrictions on removals in the Northwestern United States, and continued growth in global wood demand.

For the purposes of this model, the $P_t$ matrix will depend on an initial price distribution at $t = 0$ (see Table 1), and algorithms for growth in three product classes. As a stand matures, it is assumed to enter three stages of product development: (1) pulpwood, (2) low quality sawtimber, and (3) high quality sawtimber. To illustrate, Fig. 5 plots a representative model run. Here prices are assumed...
Fig. 5. Sugar maple stumpage and hardwood pulp value, PS = 0, 1995 US$/acre.

...to remain constant over a 250-year horizon, no additional pioneer species are added, and only sugar maple and total hardwood pulpwood values are plotted. Initially the stand generates mostly hardwood pulpwood. Below grade sugar maple sawtimber rises steadily over time, surpassed first by grade 3 lumber, and eventually by grade 2 and 1 as the stand matures.

To capture these dynamics, an exponential model for stand profit growth with a shifting growth rate is assumed. In the northeastern US, from 1961–1991, Sendak (1994) reports average real hardwood stumpage prices for sawtimber rose 4.3% per year, and for pulpwood rose 1.3% per year. As these rates are an average across all quality classes and species, the following price growth model is assumed to apply to the entire price matrix:

\[ P_t = P_0 e^{rt} \]  

where:

\[ r(t) = 1\% \quad \text{if} \quad t \leq t_L + \Delta_L \]

\[ r(t) = 3\% \quad \text{if} \quad t_L + \Delta_L < t \leq t_H + \Delta_L \]

\[ r(t) = 4\% \quad \text{if} \quad t > t_H + \Delta_L \]

and

\[ \Delta_L = \text{PS}/250 \]

The parameters \( t_L \) and \( t_H \) represent the number of years since harvest when the growing forest stand shifts into higher quality product classes. Following a clear-cut, the recovering forest stand can only produce pulpwood, a product class where prices are growing slowly at an exponential growth rate of \( r(t) = 1\% \). At \( t_L \), the stand shifts into a low quality sawtimber phase (below grade and grade 3), and the exponential growth rate increases to 3%. As the stand continues to mature, high quality timber becomes more prevalent until a time \( t_H \) is reached when timber prices grow at a maximum rate more characteristic of high quality timber.

As continued short rotations enhance pioneer species abundance, species competition pushes commercial species development further into the future, thus delaying the entrance into higher quality product classes. To capture this successional retrogression hypothesis, a shift variable \( \Delta_i \) is assumed to add years to \( t_L \) and \( t_H \) depending on pioneer species density at the beginning of each rotation.

This model is applied by mapping three exponential growth functions over the planning horizon at each rate. The function is applied as a multiplier to the initial species by product price matrix \( (P_0) \), with \( r \) depending on \( t \).

4. Rotation analysis

With the non-renewable stand value specification of Eq. (28) and the price growth model assumed in Eqs. (29)–(33), the analysis turns to estimating and comparing rotation lengths. Specifically, the question of whether the benefits from recovery in future harvest periods influence the harvest timing decision in current periods is addressed. Four harvest cycles are modeled. A positive discount rate causes profits from harvest cycles beyond four periods to have a negligible effect on the choice of rotation lengths in earlier periods.

The applied problem is to choose the rotation set that maximizes the present value of profits over four harvest cycles. Again, timber cutting costs are reflected in the stumpage price paid to the forest owner. High labor costs are also assumed to prohibit thinning pioneer species from young dense stands. This can be solved as a four-stage dynamic programming problem.
Max
\[ \Pi = \frac{e^{rT_1} \pi(T_1, PS_0)}{e^{rT_1}} + \frac{e^{r(T_2 + T_2)} \pi(T_2, f(T_1, 1))}{e^{r(T_1 + T_2)}} + \ldots + \frac{e^{r(T_4 + T_2 + T_2 + T_4)} \pi(T_4, f(T_3, 3))}{e^{r(T_1 + T_2 + T_3 + T_4)}} \] (34)

Max
\[ \Pi = e^{(T_1) - \delta T_1} \pi(T_1, PS_0) \] (35)

4.1. Risk and choosing an economic optimum

The difficulty in solving Eq. (35) over four periods is that as \( r \) varies within each rotation cycle (from 1 to 3 to 4%), the possibility of multiple optima arises. To illustrate, take the case of four harvest period solution with 2% long-run risk factor. The first is the successional retrogression hypothesis with \( \pi(T_f, f(T_{i=1}) \). The second is the traditional perfectly renewable growth hypothesis with

<table>
<thead>
<tr>
<th>Rotation</th>
<th>Rotation-dependent specification ( \sigma(T_N, f(T_{N-1}, N)) )</th>
<th>Renewable growth mis-specification ( \sigma(T_N, PS_N = 100) - 1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T_1 )</td>
<td>58</td>
<td>40</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>68</td>
<td>44</td>
</tr>
<tr>
<td>( T_3 )</td>
<td>70</td>
<td>51</td>
</tr>
<tr>
<td>( T_4 )</td>
<td>83</td>
<td>70</td>
</tr>
</tbody>
</table>

Net present value US$400.3/acre US$470.2/acre

\(^7\) If stochastic growth was carried through, or stochastic price growth introduced, risk could be modeled with option value methodology by including growth or price variance. Clarke and Reed (1989) found an optimal stopping frontier assuming brownian motion for age-dependent growth and geometric brownian motion for price evolution, and an optimal stopping rule under deterministic growth.
esis with $\pi(T, PS_i = 100)$. The sum of present value over four periods reveals a 17.5% overestimate of stand profits in the traditional specification. Rotation lengths differ by as much as 24 years in the second cycle, and become longer in future cycles as prices continue to grow exponentially and profit from future rotations goes to zero. The rotation length for $T_4$ simply maximizes profits in this cycle.

Compare the first cycle rotation lengths with that of the single rotation problem, where $T$ equaled 70 years. The effect of considering profits in cycles 2, 3 and 4 at considerably higher prices and identical growth conditions reduces $T_1$ from 70 to 40 years. This is the result of considering three period future profits. When successional re-gression is assumed, the shift from 40 to 58 years is the result of including a marginal benefit of recovery.

Differentiating Eq. (35) by $T_1$ yields the first-order condition for $T_1$. The terms can be arranged so that the marginal benefit of waiting another period equals the marginal cost of delaying first cycle profits plus the marginal cost of delaying all future profits (site value), as was the case in the traditional Faustmann formula, and the addition of a marginal benefit of recovery in the second cycle:

$$e^{\frac{R}{T_1}} \frac{\partial \pi(T, PS_0)}{\partial T_1} = R e^{\frac{R}{T_1}} \pi(T, PS_0)$$

$$+ R \phi + e^{R(T_1 + T_2)} \times \frac{\partial \pi(T_2, f(T_1, 1))}{\partial f(T_1, 1)}, (36)$$

where:

$$r(T) = 0 = r(T_1) = r(T_2) = r(T_4) = r,$$ (37)

$$R = r - \delta,$$ (38)

$$\phi = e^{R(T_1 + T_2)} \pi(T_2, f(T_1, 1))$$

$$+ e^{R(T_2 + T_3)} \pi(T_3, f(T_2, 2))$$

$$+ e^{R(T_1 + T_2 + T_3 + T_4)} \pi(T_4, f(T_3, 3))$$ (39)

At the optimal first cycle rotation ($T_1 = 58$) the marginal benefit of waiting another year until harvest is US$7.50. It equals the marginal cost of delaying first cycle profits of US$6.30, the marginal cost of delaying the next three harvests (site value) of US$1.70, and the marginal benefit of recovery in future cycles of US$0.50. Site value well exceeds MBR because of the effect of exponential price growth.

4.3. Economic and ecological indicators under various discount rates

<table>
<thead>
<tr>
<th>$\delta$ (%)</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Optimal rotation set</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_1^*$</td>
<td>101</td>
<td>31</td>
<td>8</td>
</tr>
<tr>
<td>$T_2^*$</td>
<td>107</td>
<td>51</td>
<td>37</td>
</tr>
<tr>
<td>$T_3^*$</td>
<td>108</td>
<td>48</td>
<td>30</td>
</tr>
<tr>
<td>$T_4^*$</td>
<td>115</td>
<td>51</td>
<td>30</td>
</tr>
<tr>
<td><strong>Economic indicators</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net present value</td>
<td>1122.6</td>
<td>48.6</td>
<td>24.0</td>
</tr>
<tr>
<td>Undiscounted profit @ year 10</td>
<td>5909</td>
<td>1592</td>
<td>500</td>
</tr>
<tr>
<td><strong>Ecological indicators</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(T_3, 3)^c$</td>
<td>2224</td>
<td>5277</td>
<td>6538</td>
</tr>
<tr>
<td>$t_f + \Lambda_f^*$</td>
<td>39</td>
<td>51</td>
<td>56</td>
</tr>
<tr>
<td>$t_d + \Lambda_d^*$</td>
<td>109</td>
<td>121</td>
<td>126</td>
</tr>
</tbody>
</table>

| 

Note: a Unit: years.  

b Unit: US$/acre.  

c Unit: stems/acre.

The discount rate measures the landowner’s opportunity cost. A relatively low opportunity cost of $\delta = 5\%$ may be characteristic of a large landowner with many sources of income. For instance, the highest return for a pulp and paper mill in the northern hardwood forest is in making paper. As long as their mill is fed with a continuous, inexpensive supply of fiber, management can hold onto timber stands for speculation in the higher return sawtimber markets, particularly when land is drawing income between rotations, for instance, through recreational leasing. Medium opportunity cost in the range of $\delta = 10\%$ may be more characteristic of a small primary forest product industry or small woodlot owner. A discount rate of 15% may be characteristic of a landowner not necessarily in the timber industry. In this case it may be more profitable to use the land for an activity with a shorter investment horizon, for instance, housing development.
Table 3 lists the results of the four-cycle optimization when the discount rate is varied, assuming no risk factor. In the case of high opportunity cost ($\delta = 15\%$), four pulpwood rotations are optimal at interior solutions of 8, 37, 30 and 30 years with a total present value of US$24/acre. At $\delta = 10\%$ the optimal rotation set occurs in the low quality sawtimber phase at rotations of 31, 51, 48 and 51 years, all of which are corner solutions since $t_L = 30$, $\Delta_1 = 21$, $\Delta_2 = 18$ and $\Delta_3 = 21$. At $\delta = 5\%$, the solution occurs at the corner of the high quality sawtimber phase.

The sum of present value over four cycles indicates the effect on profit of both shorter rotations with lower quality products and a higher discount rate. A second economic indicator, summarizing stand profit at year 105 (the end of the fourth cycle under $\delta = 15\%$) with no discounting, indicates only the effect of shorter rotations and lower quality products on future profits. Under this second indication, just over one rotation of high quality sawtimber (at $T_1 = 101$ and $T_2 = 4$) produces 2.7 times more undiscounted profits than three and a half rotations under the low quality management case, and 10.8 times more undiscounted profits than four full pulpwood rotations.

The ecological indicators of the three management scenarios reflect the important ecological benefits to longer rotations. At the beginning of the fourth harvest cycle, pioneer species density is 2224 stems under long rotations, 5277 stems under medium length rotations, and 6538 stems under short rotations. In the pulpwood harvesting case, entrance into both sawtimber phases is delayed a full 27 years by the fourth harvest cycle. The cases where $\delta = 10\%$ and $\delta = 15\%$ demonstrate the declining trend in successional integrity as suggested by the Kimmins’ successional retrogression hypothesis, while the case where $\delta = 5\%$ perhaps approaches a set of ecological rotations (i.e. both scenarios outlined in Fig. 2).

4.4. Single period management under declining forest health

Another method to solving the multiple rotations problem is to assume the values for PS$_i$ over subsequent rotations are forest health endowments to new generations of owners or managers. In other words, a different owner during each cycle solves a single rotation problem, without consideration of site value or benefits to recovery. Here, the first-order condition within each cycle becomes:

$$\delta - r(T_1) = \frac{\pi'(T_n, \text{PS}_{i-1})}{\pi(T_n, \text{PS}_{i-1})}$$

Again, assuming the landowner will manage either in the low quality sawtimber or pulpwood price growth phases, the four-cycle interior solution for $T$ is 70, 80, 80, and 83. Future landowners must wait longer to maximize profits due to poorer forest health endowments. Profits continue to increase in later cycles due to exponential price growth, however, timber quality and quantity are constrained by a degrading resource. By the fourth cycle, the pulpwood price phase is 70 years long, and initial pioneer species density is 3979 stems/acre.

4.5. Ecological rotations and valuing non-timber amenities

As was evident in the single period problem, given low constant discount rates ecological rotations may be economically optimal. Solving Eq. (35) with a constant discount rate of 5% yielded the rotation length set of 101, 107, 108 and 115 years with a total present value of US$1122.6/acre. Assuming constant initial pioneer species density across all future harvest cycles (i.e. PS = 100 and $\Delta_1 = 0$), the optimal set becomes 101, 101, 101 and 116 with a total present value of US$1200.2/acre. Not accounting for non-renewability results in a negligible 2% overestimate of total present value.

These rotation lengths are approaching what might be considered ecological rotations as described by Kimmins. Ignoring risk consideration and assuming preventively high forest maintenance costs, such lengths may also be economically optimal. Therefore, the question remains under what conditions will landowners rotate forests at 100 + years?
Perhaps including the value of non-timber amenities would justify ecological rotation lengths as socially optimal. Amenity values that exhibit increasing returns to rotation length might include recreation value, provision for certain habitats, and watershed protection. For example, referring to Table 3, consider the low discount rate solution ($\delta = 5\%$) and middle discount rate solution ($\delta = 10\%$) as the social and private optimal rotation sets. Next, evaluating the social rotation set at the private discount rate of 10% results in a total present value of just US$5/acre. If a landowner was forced to rotate at these lengths, this would result in a private loss of US$41/acre. However, if the sum of non-timber amenities exceeds this loss and the landowner experiences these benefits directly (for example, hunting or recreational use), then there may be a private incentive to lengthen rotations.

Alternatively, if the amenity values are of a strictly social nature (for example, watershed protection or biodiversity preservation) then an opportunity may exist for the government or an environmental group to accommodate the landowner’s loss in timber profits through a payment or incentive mechanism (for example, paying for a conservation easement or providing tax relief). In addition, alternative silviculture practices such as selective cutting may strike common ground between the interplay of social and private benefits. Furthermore, assessing the ecological impact of economic decisions contributes to the definition and assessment of ‘new’ or ‘sustainable’ forestry, which embraces management practices derived from ecological principles (Franklin, 1989; Gillis, 1990; Gane, 1992; Fiedler, 1992; Maser, 1994).

5. Concluding remarks

Accounting for the ecological recovery of the northern hardwood forest over a series of harvests was shown to increase rotation lengths over the traditional Faustmann result. A positive marginal benefit of recovery offsets the marginal costs of delaying current and future rotations, creating a benefit to delaying rotations under a non-renewable stand value growth specification. The model presented in this paper is limited to the specific ecological dynamic of pioneer species introduction and interspecies competition for light and other resources. In the context of forest management, this approach can be generalized to many intertemporal dynamics such as other successional sequences, alteration of nutrient cycles, or disturbance from anthropogenic climate change. Knowledge of benefits to ecosystem recovery can help define both ecological and economic rotation lengths under various scenarios of Kimmins’ ecosystem retrogression. At one extreme, given low discount rates and risk, relatively long ecological rotations may be economically optimal. At the other extreme, a site managed with short rotations motivated by short-term profits and a high discount rate may result in degraded forest stands with low value species—a detriment to long-run ecological health and social benefits.

When considering social welfare and the maintenance of ecosystem processes from multiple-use management, many non-timber benefits have increasing returns in rotation length and decreasing returns to harvest intensity. The benefit of recovery was shown to have a market value, and its inclusion more accurately estimates the optimal rotation set. Including this benefit, however, may not completely provide the private incentive to move from ecologically unsustainable to sustainable rotation lengths and practices, particularly when the net private cost of doing so is high. However, this net private cost can be compared with benefits from non-timber amenities and alternative management practices, or to costs of forest maintenance (i.e. thinning undesirable species), providing rationale for social management.

Questions of where to manage along ecological economic dimensions in a forest will ultimately depend on a region’s spatial ownership pattern, land holder motivations, policy variables, management costs, timber markets, and ecosystem characteristics. These modeling results suggest very different economic and ecological outcomes by varying opportunity cost and ecosystem recovery assumptions, and suggest a positive benefit to recovery. Estimating economic benefits across
ecological gradients, rather than through non-market valuation techniques, could capture non-timber amenity value and contribute to stewardship policies aimed at managing multiple, spatial benefits of a forest ecosystem.

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