ANALYSIS

Dynamics of soil acidification: an economic analysis

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Abstract

This paper studies the dynamic aspects related to the problem of acidification. It shows how accumulation of acidification in ecosystems can be studied in economic modelling by incorporating dynamic aspects of soil acidification. In contrast to the often applied critical loads approach which only focuses on the final state of a soil, the dynamic approach applied in this study gives information about the temporal development of the quality of a soil. Using an economic optimal control model containing these dynamic aspects the paper compares abatement policies based on static critical loads to abatement policies based on a dynamic analysis. It shows that soil dynamics play an essential role in identifying optimal policies. Based on numerical simulations cost-effective abatement strategies for combined reduction of sulphur dioxide ($SO_2$) and nitrogen oxides ($NO_x$) are determined. It is also shown that abatement cost savings may be realised when intertemporal cost efficiency is taken into account. The results indicate that current European policies which are based on a critical loads approach instead of dynamic analysis of soil quality, are non-optimal from both an ecological and an economic point of view. © 1999 Elsevier Science B.V. All rights reserved.

Keywords: Acid rain; Critical loads; Dynamics of acidification; Soil acidification; Optimal control

1. Introduction

Acidification affects water, soil and ecosystems. Acidifying compounds such as sulphur dioxide ($SO_2$) and nitrogen oxides ($NO_x$) are often produced together, most typically by the burning of fossil fuels. Ammonia ($NH_3$), causing acidification and eutrophication, is usually emitted by agricultural activities. Currently, many ecosystems in the United States, Europe, and the emerging economies in Asia face acid deposition far in excess of their specified critical loads. The excess

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acid depositions and their transboundary nature make it necessary to reduce acidification on an international level.

By introducing the critical loads concept in the 1980s an indicator was created that could be used in policy plans aiming to reduce the problem of acidification. The relative simplicity of the concept has led to many applications in the environmental economic literature. Many studies have calculated cost-effective reduction of sulphur dioxide, nitrogen oxides and ammonia based on deposition calculations and critical loads. Initially the studies contained a limited number of critical loads. Later, however, after increased knowledge of ecosystems and their response to acid deposition and detailed mapping of critical loads, it became possible to develop models that calculate cost-effective reduction strategies based on critical loads for various ecosystems and for different pollutants.

Abatement policies for acidification presently in place in the European Commission and the United Nations Economic Commission of Europe (UN-ECE) are founded on the use of critical loads. In the resulting protocols under the convention on long-range air pollution, deposition targets are formulated in ‘gap closure’ scenarios. The ‘gap’ is the difference between the actual deposition and the critical load, and the target is to reduce this gap by a certain percentage at some point in time. However, in the period between signing the agreements and the time the targets will be met, depositions are exceeding critical loads, and soils will deteriorate. It is unknown to what extent, and how long it will take for the soil to fully recover. Information about the temporal development of the soil quality (as being analysed in this study by explicitly modelling the base saturation) can therefore be useful to redefine policy targets. This would avoid damage to ecosystems in the period during which critical loads are exceeded and would take into account the time it takes the soil to recover. We will show how one can overcome the shortcomings of a critical load as an indicator to protect ecosystems from acidification.

Natural scientists stress that it is not sufficient to focus solely on acid deposition and critical loads: the dynamic aspects of acidification should be taken into account as well. Rennings and Wiggering (1997) point out that the critical load concept is only an initial indicator for the environmental quality of complex ecosystems. In a study on critical loads and recovery of forest soils, Hettelingh and Posch (1994) show the importance of knowing the time a soil or ecosystem needs to recover from excess acid deposition. With knowledge of the recovery period, which can be longer than 70 years (Hettelingh and Posch, 1994), one can determine the rate at which emission abatement should be implemented. The use of dynamic soil acidification models also can provide important additional information for regional steady-state critical load calculations (Forsius et al. 1997). De Vries (1993) states that critical loads give information about the acceptable levels of acid deposition in the long term. Dynamic soil acidification models can be used to predict the time period before a critical ion concentration or ion ratio will be reached. Other natural science studies stressing the importance of analysing abatement policy options using dynamic soil acidification models include Holmberg (1989), De Vries and Kros (1991) and Bull (1995).

Important economic and political reasons also exist for incorporating these dynamic processes into economic models. The timing of emission reduction influences abatement costs, technological development and economic growth. Postponement of emission reductions may be efficient if the marginal productivity of capital is positive. Future burdens are relatively cheaper because fewer current resources are set aside for future reduction. In addition, the costs of reduction are expected to decline over time (Wigley et al., 1996). Earlier reductions, however, may be efficient because earlier emission reductions will stimulate learning (Grübler and Messner, 1998) and emis-

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Acidification as a transboundary air pollution problem has often been analysed in game theory (either static or dynamic) because acidification affects a common property resource in an asymmetric way, involves many countries, all with different incentives, and the countries have not agreed upon a set of rules (Mäler, 1989). In a dynamic acid rain game, damage is determined not by annual depositions, but rather by accumulated depositions (Mäler, 1994). However, not being able to incorporate accumulation of acid depositions, Mäler focuses on dynamics induced by an acid rain game that is played repeatedly in the future. Recently, Mäler and De Zeeuw (1998) introduced in ‘the acid rain differential game’ a way to account for accumulation of acidification. They assume that when critical loads are exceeded the buffer stock for acidification decreases, and when depositions are below critical loads the buffer increases. However, the behaviour of the buffer stock is not related to chemical processes in the soil or to the quality of the soil in a way that describes ‘real’ soil acidification. The studies have in common a focus on steady state outcomes within a game theory framework. Kaitala et al. (1992) include an equation that describes dynamic soil acidification based on results from a dynamic soil acidification model. This equation calculates the temporal development of a chemical soil parameter (the base saturation) that can be considered to be a soil quality indicator. They relate this indicator to reduced forest growth and hence to economic damages caused by acidification. Applying game theory, Kaitala et al. (1992) derive both co-operative and non co-operative solutions to an acid rain game between Finland and the USSR.

The aims of our paper are three-fold; first, to investigate how dynamic aspects of acidification can be included in economic modelling used to calculate cost-effective time paths for reducing acidifying pollutants; second, to show how gap closure scenarios based on critical loads can function in a model that accounts for dynamic processes in acidification; and third, to perform intertemporal cost-effectiveness analysis.

Our study enhances the literature by integrating improved ecological modelling that incorporates dynamic aspects of acidification with economic modelling, and by contrasting the results obtained from a dynamic model with those of more traditional comparative static models based on critical loads. By doing so, this study presents a method for combining dynamic economic modelling with the modelling of dynamic soil acidification processes. The study also provides insights into the temporal development of soil quality and presents intertemporal cost-effective strategies that take into account both environmental and economic factors.

The structure of the paper is as follows, the second section identifies the dynamic behaviour of the quality of soils with respect to acidification. The third section develops a theoretical model suitable for analytical purposes by deriving optimal time paths for the reduction of SO$_2$ and NO$_x$. In the fourth section we illustrate mechanisms using a numerical example, and the fifth section deals with spatial and intertemporal cost-effectiveness. We draw conclusions in the last section.

2. Dynamic aspects of soil acidification

In this section we describe a simplified soil acidification model that includes the dynamics of soil acidification.$^4$ Dynamic aspects of soil acidification can be indicated by the temporal development of the base saturation of a soil.$^5$ The base

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$^3$ From an economic viewpoint it would be desirable to apply cost-benefit instead of cost-effectiveness analyses. However, valuing damages is a difficult task (if not impossible). To overcome these difficulties we apply cost-effectiveness analysis.

$^4$ Acidification models with many equations describing in detail soil and water chemistry are too complex to integrate with economic models; therefore we use a simplified and condensed model.

$^5$ Other indicators that could be used are, among others, the pH of a soil or the aluminium concentration in a soil.
saturation is defined as the fraction of exchangeable base cations in the solid phase of a soil. It is a chemical soil parameter that can be considered as a soil quality indicator for acidification. The fraction of exchangeable base cation indicates the availability of nutrients in the soil and this availability of nutrients influences growth of forests and other vegetation.

The base saturation at time \( t \), \( B(t) \) is defined as the stock quantity and is expressed as a fraction with \( 0 < B(t) \leq 1 \). The dynamic behaviour of the base saturation in a soil can be described by the following state equation (Eq. (1))\(^6\):

\[
\dot{B}(t) = \frac{dB(t)}{dt} = -\beta B(t) \ln(B(t)) - \gamma D(t) B(t) \tag{1}
\]

with \( D(t) \) being a deposition function, and \( \beta > 0 \) and \( \gamma > 0 \) as coefficients which can be estimated using dynamic soil acidification models. By a state transformation of \( \phi(t) = \ln(B(t)) \) (and thus \( B(t) = e^{\phi(t)} \)) the state equation transforms into (omitting subscript \( t \) with respect to \( \phi \)):

\[
\dot{\phi} = -\beta \phi - \gamma D(t) \tag{2}
\]

Parameter \( \beta \) can be interpreted as a recovery rate, while \( \gamma \) identifies how fast a soil responds to (changes in) acid deposition. A lower base saturation will result in a more acidified and less fertile soil. From Eq. (2) we can derive that the base saturation recovers faster at lower levels (that is, the more a soil is degraded by acidification the faster the soil recovers) and that the process of acidification is always reversible. This is a suitable description of the behaviour of the base saturation (De Vries and Reinds, 1998). Below a certain value of the base saturation, what we call the critical value, vegetation and ecosystems that depend on the soil are assumed to face increasing risks of damage and harmful effects, due to long-term acidification (Posch, 1998).

The critical loads for acid deposition are derived from the critical base saturation \( B_c \), and are derived for steady state conditions that is for \( \dot{\phi} = 0 \). Using Eq. (2) and \( B_c \), which transforms to \( \phi_c = \ln(B_c) \), and, assuming an infinite time horizon, we can derive the critical load \( (D_c) \) for acid deposition from the following expression:

\[
D_c = -\frac{\beta \phi_c}{\gamma} \tag{3}
\]

If we solve Eq. (3) for the critical base saturation \( \phi_c \), it can be seen that \( \phi_c \) is independent of the initial condition of the soil, and independent of depositions before the time period under consideration (before \( t = 0 \)), and the analysis does not have to start before the process of acidification began (before industrialisation). For our calculations we set the parameter values at \( \beta = 0.015 \), \( \gamma = 0.0001 \) and the critical limit for the base saturation \( B_c = 0.10 \). The values for \( \beta \) and \( \gamma \) are chosen arbitrarily, but magnitudes are based on parameters estimated for a poor Finnish forest soil as applied in Kaitala et al. (1992). Using soil scientific research, we know that a reasonable critical limit for the base saturation is either 0.05 or 0.10 (Posch, 1998).

Using Eq. (3) we can compute the critical load, \( D_c \approx 345 \) acid equivalents per hectare per year (eq/ha per year). The resulting soil is quite sensitive to acidification since actual critical loads range from less than 200 acid eq/ha per year in some places in Scandinavia and Central Europe to over 1000 acid eq/ha per year in most of Southern Europe. From Eq. (3) and Fig. 1 we also see that for a constant deposition below the critical load \( (D_{\text{con}} = 100 \text{ acid eq/ha per year}) \), the base saturation will converge to \( B \approx 0.51 \). For a constant deposition higher than the critical load \( (D_{\text{con}} = 900 \text{ acid eq/ha per year}) \), \( B \) will converge to \( B \approx 0.002 \).\(^7\) See Fig. 1 for the temporal develop-

\(^6\) The only relation we could find in the literature that describes soil acidification dynamics in one simple equation reads \( \dot{B}(t) = \frac{dB(t)}{dt} = \alpha B(t) - \beta B(t) \ln(B(t)) - \gamma D(t) B(t) \). The equation is taken from Kaitala et al. (1992) and is derived by these authors using a model developed by Holmberg (1990). To further simplify the equation we set \( \alpha \) equal to zero by assuming that the term is captured by \( \gamma D(t) \) as a type of background deposition. Further, we express the base saturation as a fraction and not as a percentage.

\(^7\) Solving Eq. (3) for the ln-transformed base saturation gives \( \phi = -\frac{D_c}{\beta} \). For known \( \beta \) and \( \gamma \) and given constant deposition \( D_c \) and by applying the exponential transformation the base saturation \( B_c \) can be determined.
ment of the base saturation for different initial conditions of the soil \((B_0)\) and different levels of deposition.

By formulating a model that incorporates soil dynamics one can determine when the critical base saturation \((B_c)\) will be reached given initial conditions of a soil and assuming a known deposition function \((D(t))\). By using critical loads only, as is being done in many studies, one cannot analyse the temporal development of the condition of a soil itself.

In the next section we develop an analytical model that accounts for the dynamic behaviour of soils, and we derive optimal cost-efficient time paths for emission reduction.

3. Dynamic cost-effective reduction of acidification

In our model we want to reduce emissions of pollutants that lead to acidification of soils. Unabated emissions are represented by \(E_{n,t}^k\), and abatement is given by \(A_{n,t}^k\) for acidifying pollutant \(k\), with \(n\) representing an emitting country for any time \(t\) and both emission and abatement are expressed in kilotons (1 kiloton is 1 000 000 kg). The abatement cost functions are given by \(C_k(A_{n,t}^k)\) and are assumed to be twice differentiable, convex and increasing in \(A_{n,t}^k\) (that is \(C''_{n,k} > 0, C'''_{n,k} > 0\)) at any time \(t\). Total abatement cannot exceed total emissions \((E_{n,t}^k \geq A_{n,t}^k \ \forall t)\). For now we as
sume no technical progress, resulting in constant abatement cost functions over time. For reasons of simplicity we assume further that unabated emissions (\(E_n^k\)) are given and are constant over time. This can be explained by assuming no economic growth and no growth in energy consumption. Emissions in country \(n\) are transported to receptor countries \(m\) by air, and depositions in receptor country \(m\) can be calculated using transport matrices \(M^k\). The matrix \(M^k\) is assumed to be constant over time.

Total reduction costs (\(TC\)) are calculated as the sum of both the discounted reduction costs in period \(t\) to the time that the problem reaches the terminal time \(T\), and, the discounted reduction costs beyond time \(T\) to maintain reduction levels that avoid further decrease of the base saturation.

\[
TC = \sum_n \sum_k \left( \int_0^T e^{-rt}C_{n,k}(A_{n,k})dt + \int_T^\infty e^{-rt}C_{n,k}(A_{n,k})d\tau \right) \tag{4}
\]

We can rewrite this expression resulting in Eq. (5) yielding a maximisation problem formulated as maximising the negative costs, with \(V\) being the present or discounted value of the sum of total abatement costs over the whole period, \(A_{n,k}\) being the variable to control, and \(\theta(\phi_m, A_{T', r, T})\) expressing a scrap value function depending on the lower bound of the base saturation, as well as the abatement level in period \(T(A_T)\), the discount rate \(r\) and \(T\). Accordingly, we want to find the abatement path \(A_{n,k}\) that maximises (omitting subscript \(t\)):

\[
\text{Max} \; V = \sum_n \sum_k \left( \int_0^T -e^{-rt}C_{n,k}(A_{n,k})dt - \theta(\phi_m, A_{T', r, T}) \right) \tag{5}
\]

subject to the soil equations for every country \(m\) (we assume that every country consists of one homogeneous soil):

\[
\dot{\phi}_m = -\beta_m \phi_m - \gamma_m \sum_k m_{n,k}(F^k - A^k) \tag{6}
\]

with \(\beta_m > 0, \gamma_m > 0\). Row vector \(m_{n,k}\) is the \(m\)th row of matrix \(M^k\). \(F^k\) and \(A^k\) are, respectively, the emission and abatement vectors of pollutant \(k\). Further, abatement cannot exceed emissions (\(E_n^k - A_n^k \geq 0\)). The ln-transformed base saturation is required to be greater than a critical value \(\phi_m^0\) in any country \(m\) at any \(t(\phi_m - \phi_m^0 \geq 0)\) and the initial condition for a soil is given by the initial \(\phi_m^0\) value and is assumed to be known \((\phi_m(0) = \phi_m^0)\).

The current value Lagrangian \(L\) reads as follows:

\[
L = H = \sum_n \sum_k (-C_{n,k}(A_{n,k}))
\]

\[
+ \sum_m \eta_m \left( -\beta_m \phi_m - \gamma_m \sum_k m_{n,k}(F^k - A^k) \right)
\]

\[
+ \sum_m \mu_m (E^k_n - A^k_n) + \sum_m (\phi_m - \phi_m^0) \tag{7}
\]

This results in a control problem with \(m\) state equations and \(k\) pollutants in every emitting country. As long as the constraints are not binding (and hence the Kuhn Tucker multipliers \(\mu_m\) and \(\mu_k\) are zero) the optimal marginal costs of reduction are given by (the details of deriving Eq. (8) are given in the Appendix).

\[
C_{nk} = \frac{C'_{nk}}{r + \beta_m}
\]

This indicates that the marginal abatement cost of a pollutant \(k\) is equal to the change in the marginal cost multiplied by the change in abatement divided by the discount rate plus the recovery rate \(\dot{\beta}_m\). In other words, according to Hotelling’s efficiency rule, the relative change in marginal abatement costs should be equal to the sum of the discount rate and the recovery rate

\[
\left( \frac{C'_{nk}}{C_{nk}} \right) = r + \beta_m \tag{9}
\]

It should be noted that the relative change is independent of both how the soil responds to acid deposition (parameter \(\gamma_m\)) and the transport matrix \(M^k\). Both the discount and the recovery rate have
the same effect, a higher value shifts abatement effort to the future and a lower value shifts the efforts to the present. The results are very similar to optimal paths of abatement of greenhouse gases (see, for example, Falk and Mendelsohn (1993)). The optimal time path of the base saturation can be determined by substituting the optimal paths of emission reduction in Eq. (6). Together with the initial and transversality condition the abatement levels at \( t = 0 \) can be calculated. The optimal time path of reduction is also determined by parameter \( \gamma_m \) and transport matrix \( \mathbf{M}^k \). Moreover, the asymmetric transport matrix partially determines the level of the base saturation and may lead to different developments in different countries, even if the soils are assumed to be identical. By using proper functional forms of the cost functions together with the transversality conditions the optimal path for the base saturation can be determined (see Appendix).

From this section we conclude that a high discount rate \( r \), a high recovery rate \( b \) as well as high marginal abatement cost \( C'(A) \) tend to postpone emission reduction, though for different reasons. A high recovery rate has a volume effect and makes it less necessary to abate because from a natural science point of view the soil can absorb a greater acid deposition. A high discount rate or high marginal costs have a price effect and lead to postponement of emission reduction because of a higher present value of abatement costs. The slope of the marginal abatement cost curve \( (C'(A)) \) also determines the distribution over time, a steeper marginal cost curve results in smaller changes in abatement over time. Each is an important factor for obtaining cost-effective solutions.

4. Accumulation of acidification, critical loads and damage delay time

To further investigate factors that influence cost-effective solutions we analyse the results of a numerical optimisation model that contains dynamic aspects of soil acidification. Two abatement scenarios are of particular importance and we compare them to show the main mechanisms. First, we examine the ‘optimal abatement’ scenario in which the model calculates cost effective reduction of \( \text{SO}_2 \) and \( \text{NO}_x \) given a minimum standard for the quality of the soil (a lower bound on the base saturation). Second, we investigate ‘the protocol implementation’ scenario in which we assume that emission reductions are implemented analogous to the current European policies based on the sulphur and nitrogen oxides protocols (United Nations, 1985, 1988). In this scenario it is assumed that no emission reduction is realised in the first period 1980–1984. In 1985 the target reduction is minus 40% of 1980 emission levels to be realised in the year 2000 for both \( \text{SO}_2 \) and \( \text{NO}_x \). Between 2001 and 2010 a further reduction is implemented to minus 80% of 1980 levels, and beyond 2010 emission levels are assumed to be constant. In addition, we assume that emission reduction measures are linearly implemented between the year that the target becomes effective and the year the target is to be met. The data used in the numerical model are given in Table 1.

4.1. Accumulation of acidification and critical loads

Fig. 2 shows the typical development of deposition patterns for the cost effective abatement path and the protocol implementation scenario. Emissions show similar patterns because they are linearly transformed by the transport matrix to

<table>
<thead>
<tr>
<th>Table 1</th>
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<tbody>
<tr>
<td>The most important data used in the one country model run*</td>
</tr>
<tr>
<td><strong>Value</strong></td>
</tr>
<tr>
<td>Constant emissions (for all ( t ) before abatement)</td>
</tr>
<tr>
<td>( \text{SO}_2 ) (Kton/year)</td>
</tr>
<tr>
<td>( \text{NO}_x ) (Kton/year)</td>
</tr>
<tr>
<td>Deposition before abatement (acid eq/ha per year)</td>
</tr>
<tr>
<td>Initial value base saturation, ( \phi_m^0 ) (fraction)</td>
</tr>
<tr>
<td>Critical value base saturation, ( \phi_m^c ) (fraction)</td>
</tr>
<tr>
<td>Critical load (acid eq/ha per year)</td>
</tr>
<tr>
<td>Initial exceedance of critical load (times critical load)</td>
</tr>
</tbody>
</table>
| Discount rate (%) | *Data are hypothetical but are selected according to European magnitude. Numbers between brackets are used in sensitivity analysis.
depositions. The reduction is higher in the optimal reduction strategy relative to reduction in the protocol scenario, due to the extra restriction that we impose, and the quality of the soil is not allowed to decrease under a certain level. Efficiency gains are realised by a trade-off between SO$_2$ and NO$_x$ reduction depending on their marginal abatement costs. This is comparable to the ‘ordinary’ cost effective solution in a multiple pollutant setting. Due to technical limitations, the maximum reduction for both pollutants is assumed to be 90% of the unabated emission level. This explains the kink in the optimal deposition path (at $t = 2002$). At that point sulphur reduction is limited to 90% and further reduction is realised by NO$_x$ abatement with higher marginal costs. The discount rate and the marginal abatement costs determine the distribution of abatement over time and hence the reduction time path. For a lower (higher) discount rate, the constraint on the soil quality becomes binding later (earlier) in time.

Fig. 2 shows that the depositions reach the critical load in both countries after the year 2010. With the deposition at 330 acid eq/ha per year and the critical load at 345 acid eq/ha per year, depositions are strictly below the critical load because a uniform cut back of 80% for both NO$_x$ and SO$_2$ is more than sufficient to reach the critical load. If one were to compare the deposition to the critical loads, one would conclude that ecosystems are protected and will not encounter an increased risk of damage due to acidification after the year 2011. Although many studies draw this conclusion it is not always correct (e.g. some cost-effective acid reduction scenarios calculated with the RAINS model that are based on a target of x%-percent ecosystems protection (Amann et al., 1998b). Below we explain why.

4.2. Accumulation of acidification and the damage delay time

Fig. 3 shows the development of the base saturation in both the optimal reduction strategy and the protocol implementation scenario. Under the optimal reduction strategy the lower bound on the base saturation is reached at about $t = 2011$ and then remains constant.\textsuperscript{10} From Eq. (8) we have

\textsuperscript{10}It should be noted that different soils characterised by different values for the parameters $\beta_m$ and $\gamma_m$ respond differently to excess acid loads. Therefore, for some soils a temporal exceedance of critical loads might not be a problem because the soils respond so slowly that they would never reach their critical values. However, some soils may respond so quickly that critical values are soon surpassed and the quality of the soil will deteriorate even more then in the previous analysis. In that case, stringent reduction policies may be required.
already concluded that the recovery rate $\beta$ and the discount rate $r$ both have the same effect. An increased time preference delays the time at which abatement costs are incurred, causing the critical value for the base saturation to be reached earlier in time.

Recalling Fig. 2, one would believe, that according to the deposition levels, ecosystems are protected from increased risk of damage due to acidification in both scenarios after 2010. Fig. 3, however, leads to a different conclusion. In the protocol implementation scenario, the quality of the soil in the country indicated by the base saturation falls under the critical value in 1990 and stays under for more than 90 years after the full implementation of the protocols becomes effective (provided that depositions remain constant after 2010). Accordingly, the soil needs a long period to recover from the excess acid loads in the first three decades of the period under consideration. In this recovery period, also denoted as damage time lag (DTL) (Hettelingh and Posch, 1994), ecosystems face increased risk to acidification damage. This may result in reduced forest growth, a decrease in biodiversity, or other negative effects. This mechanism shows exactly what could happen when politicians agree on deposition targets based solely on critical loads to be reached at some point in time. Damage may occur, because the critical loads approach does not take into account the extent that soils are degraded and how long recovery may take. Insights into the temporal development of the quality of a soil can thus be useful for redefining emission and deposition policy targets. It may even be the case that early reduction or more stringent policy targets at an earlier date may be cost-efficient when the monetary value of damage is taken into account. Moreover, a cost benefit analysis including dynamics of soil acidification and potential damage caused by low soil quality may result in shifting emission reduction closer to the present.

5. Spatial and intertemporal cost-effectiveness in a transboundary context

In this section we show how allowing for intertemporal cost-effectiveness can lead to better co-operative cost-effective solutions relative to static cost-effective solutions. The model in this
To analyse potential cost savings of flexible timing we follow the analyses of Manne and Richels (1995) with respect to greenhouse gas reduction. We distinguish three situations. In Case I, the time path of emission reduction is fixed and the countries both reduce a fixed percentage of their unabated emissions. From this ‘uniform cut back’ scenario (similar to implementation of the European SO$_2$ and NO$_x$ protocols) the model calculates depositions and the resulting quality of the soil indicated by the base saturation (a similar scenario has been investigated in the prior section).

In Case II we allow for spatial effectiveness, which in fact is comparable to the cost-effective solution that can be calculated with IIASA’s RAINS model (Alcamo et al., 1990). In this scenario we set maximum depositions equal to the calculated depositions of Case I for each corresponding period. Therefore, Cases I and II have identical paths for depositions (see Fig. 4) and the base saturations (see Fig. 5) in both countries. Results for Country A and Country B are comparable, therefore, we only show the results for Country B because its soil is binding.

Fixing maximum deposition loads is similar to what is often called a ‘gap closure scenario’. The difference between a uniform cut back (Case I) and a scenario allowing for spatial cost-effectiveness (Case II) occurs in the allocation of the required emission reduction between countries. Countries are co-operative in finding the cost-effective solutions by reducing emissions in the country facing lowest marginal costs either for SO$_2$ or NO$_x$ reduction.

Case III allows for intertemporal and spatial cost-effective considerations by setting a minimum level for the base saturation rather than by fixing emission or deposition levels. From Cases I and II we know the lowest reached base saturation at any point in time (i.e. $B_{m_0} = 0.043$), and therefore we require that $B_{m_0}(t) \geq B_{m_0}$ for all $t$. To make Case III comparable to Cases I and II we add one more restriction. The base saturation in Case III should have at least the level of the base saturation in Cases I and II for every corresponding year after 2020. The latter restriction implies that emission reduction cannot be postponed further into the future relative to Case I and II.

### Table 2
The most important data used in the numerical model runs

<table>
<thead>
<tr>
<th></th>
<th>Country A</th>
<th>Country B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant emissions (for all $t$ before abatement)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SO$_2$ (Kton/year)</td>
<td>500</td>
<td>2000</td>
</tr>
<tr>
<td>NO$_x$ (Kton/year)</td>
<td>300</td>
<td>800</td>
</tr>
<tr>
<td>Initial value base saturation $\phi_{m_0}$ (fraction)</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>Critical value base saturation $\phi_{m_c}$ (fraction)</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Critical load (eq/ha per year)</td>
<td>345</td>
<td>345</td>
</tr>
<tr>
<td>Discount rate (%)</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

* Data are hypothetical but are selected according to European magnitude.
If we allow for spatial and intertemporal trade-offs (in the way described under Case III) we find that emissions are lower (and abatement costs are higher) for the first years compared to Cases I and II (see Fig. 4). The quality of the soil, indicated by the base saturation, in Country B stays at least as good over the whole period under consideration (see Fig. 5). In country A the base saturation decreases below the level of Cases I and II after 2010 and remains slightly below.

In this numerical illustration, international cooperation will lead to 26% cost saving compared to a uniform emission reduction. If the countries co-operate in their reduction strategies and if they allow for intertemporal cost-effectiveness, then they can realise an additional 6% cost savings relative to a uniform emission reduction. Additionally, the quality of the soil in Country B stays higher for the first 33 years compared to Case I and Case II (see Fig. 5). Different discount rates do not change the percentage cost savings much.

From this analysis we can conclude that cost savings, along with the same or less acidified soil, are possible if countries were to distribute their emission reductions cost-effectively over time, taking into account dynamic aspects of soil acidification.

6. Conclusions

We have performed a dynamic optimisation of cost-effective abatement strategies for a combined reduction of the major acidifying compounds SO₂ and NOₓ. The study provides a way to analyse optimal time paths of emission reduction for SO₂ and NOₓ by showing how dynamic aspects of soil acidification can be incorporated into economic modelling. Based on dynamic soil acidification models we can conclude that the stock behaviour of acidification cannot be described empirically using linear functional forms. Using a numerical model we show the main features of the model. The results indicate qualitatively that an analysis of acidification in a dynamic setting takes into account hard ecological constraints better than the traditional critical loads approach.

We also show that intertemporal cost-effective reduction strategies may lead to cost savings compared to spatial international co-operative cost-ef-
effective solutions. This can be explained by a more cost-effective distribution of emission reduction over time when intertemporal efficiency is allowed compared to spatial efficiency only. By allowing intertemporal efficiency, reduction measures with low marginal costs are implemented early in time, and measures with high marginal costs are implemented later in time. In addition, we show that these cost-savings can be combined with a better soil quality indicated by a higher base saturation.

Currently European acidification policies are founded on the use of critical loads resulting in acid depositions temporally exceeding the critical loads. Based on critical loads analysis it is unknown to what extent a soil recuperates, and how long it will take for the soil to fully recover. Information about the temporal development of the soil quality by explicitly modelling the base saturation as is being done in this study can therefore be useful to redefine policy targets. This avoids damage to ecosystems in the period during which critical loads are exceeded, and takes into consideration the time necessary for soil recovery. Therefore, the results indicate that current European policies which are based on a critical loads approach instead of dynamic analysis of soil quality, are non-optimal from both an ecological and an economic point of view.

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Appendix A

The first order conditions and the necessary conditions that can be derived from the Lagrangian given in Eq. (7) are (Eqs. (A1), (A2) and (A3)): 

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Fig. 5. The base saturation for Case I: Uniform emission reduction, Case II: Spatial cost-effectiveness emission reduction, and Case III: Intertemporal and spatial cost-effective emission reduction. Case I and II result in the same line and only the result for (binding) Country B are shown.
\[ \frac{\partial L}{\partial A_n^m} = -C_n^k + \sum_m \lambda_m^m \sum_k M^k - \mu_m^m = 0 \forall k, n \tag{A1} \]

\[ \dot{\lambda}_m = r \lambda_m - \frac{\partial L}{\partial \phi_m} = \lambda_m (r + \beta_m) - \sum_m \mu_m^m \tag{A2} \]

\[ \dot{\phi}_m = \frac{\partial L}{\partial \lambda_m} = -\beta_m \phi_m = -\lambda_m \sum_k \mu_m^k (E^k - A^k) \tag{A3} \]

The Kuhn Tucker conditions are shown in Eq. (A4) and Eq. (A5):

\[ \frac{\partial L}{\partial \mu_m} = E_n^k - A_n^k \geq 0, \mu_m \geq 0, \mu_m (E_n^k - A_n^k) = 0 \forall k, n \tag{A4} \]

\[ \frac{\partial L}{\partial \mu_m} = \phi_m - \phi_m^C \geq 0, \mu_m \geq 0, \mu_m (\phi_m - \phi_m^C) = 0 \tag{A5} \]

If the optimal path of abatement is denoted by \( A_n^* \) and the optimal terminal time by \( T^* \) then the transversality conditions are (with \( \pi_m(t) = e^{r T} \dot{\lambda}_m \)) Eq. (A6):

\[ \pi_m(T) - \frac{\partial \theta (\phi_m^*, T)}{\partial T} \geq 0, \phi_m(T) - \phi_m^C \geq 0, \]

\[ \left( \pi_m(T) - \frac{\partial \theta (\phi_m^*, T)}{\partial T} \right)(\phi_m(T) - \phi_m^C) = 0 \tag{A6} \]

\[ L(C_n^k (A_n^*, T^*), A_n^* (T^*), \phi_m^*(T^*), \pi_m^*(T^*), \mu_m^*(T^*), \mu_m^*(T^*)) + \frac{\partial \theta (\phi_m^*, T^*)}{\partial T} = 0 \tag{A7} \]

The problem is formulated using a current value Hamiltonian, the shadow price is given by \( \dot{\lambda}_m \) and the current value shadow price by \( \pi_m = e^{r T} \dot{\lambda}_m \).

In economic interpretation, Eq. (A5) means that if the lower bound \( (\phi_m^C) \) for the base saturation level is not binding, namely, that the stock or buffer capacity of the soil to neutralise acidifying depositions is not totally used up, then the price of abatement must be zero. According to Eq. (A7), a marginal change in the value of the scrap term due to a marginal change of the terminal time \( T^* \) will be equal to an opposite change of the value of the Hamiltonian. The terminal time \( T \) depends on the discount rate, the recovery rate \( \beta_m \), the sensitivity of the soil \( \gamma_m \) to deposition and the initial quality of the soil \( \phi_m^0 \).


