Search theory and the wage curve

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Abstract

This paper investigates whether a search theoretic model can explain the wage curve, that is, a negative relationship between regional wages and regional unemployment rates, even if the workers are mobile between regions. We show that a simple search model if combined with regional variations in productivity and with a monocentric city structure produces the wage curve. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

Blanchflower and Oswald (1994) found that regional wages are negatively related to regional unemployment rates in many countries, such as the USA, UK, etc., and called that correlation the wage curve.¹ This result contradicts the traditional neoclassical view, which was formalized by Harris and Todaro (1970), that a positive relationship exists between regional wages and regional unemployment rates because of compensating differentials.

If we assume that the workers are not mobile between regions, one explanation for the wage curve is given by the search models with regional variations in productivity. By the wage bargaining assumptions normally made, in the region with higher productivity, the wage is higher and the unemployment rate is lower because the firms supply more vacancies.

However, once we assume that there is labor mobility, the above-mentioned argument does not

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¹For a survey of this literature, see Card (1995).
hold. If the workers are mobile, all workers concentrate on the region with the highest productivity and only one combination of the wage and the unemployment rate exists.²

In reality, the wage curves are observed under the circumstances that the workers move between the regions seeking better labor market conditions. This suggests that we should take some factors that deter the concentration of workers on one region, i.e., congestion costs into consideration. Using the monocentric city structure is one simple way to represent congestion costs. In the monocentric city structure, all workers commute to the Central Business District (CBD) and work or seek for jobs there. They live around the CBD so as to save their commuting cost. As the workers in the city increase, the living area of the workers spreads out to make the commuting cost and the land rent higher. These congestion costs deter the concentration of workers on one region even if there is difference in productivity among regions and the equilibrium in which plural regions with workers exist at the same time in spite of labor mobility exists.

In this paper, we show that a simple search model if combined with regional variations in productivity and with a monocentric city structure produces the wage curve even if the workers are mobile between regions with no costs.

2. The model

The model we use here is a simplified version of a search model that is presented in Pissarides (1985) and contains a monocentric city structure presented in Alonso (1964). There are a fixed number of regions (which is denoted by \( K \)) in the economy. Each region \( i (i = 1, \ldots, K) \) has one CBD in it and consists of a variable number of workers and of jobs. At any point in time, in each region \( i \), there are \( e_i \) employed workers and \( u_i \) unemployed workers, summing to \( n_i \) total workers.

We assume that the workers are identical and can move between regions costlessly when they are unemployed, but that it is too costly for the workers employed in one region to move to another region. We also assume that the unemployed workers can search for jobs in only one region at a time. The number of workers in each region is determined to equate the unemployed's asset value.

The rate at which vacant jobs and unemployed workers meet is determined by the homogeneous-of-degree-one matching function \( m(u_i, v_i) \), where \( v_i \) represents the number of vacancies. We assume that, if the number of unemployed workers \( u_i \) rise with the number of vacancies \( v_i \) fixed, the matching function \( m \) increases. Similarly, we assume that a rise in \( v_i \) with \( u_i \) fixed increases \( m \). For simplicity, assume that the value of leisure or unemployment income is zero, and that wages are perfectly flexible and are determined by Nash bargains between the meeting firm and worker. Then, as we see later, with positive output, wages are greater than zero and no job seeker turns down a vacancy, so the transition rate for unemployed workers is \( m(u_i, v_i)/u_i \), and that for vacancies is \( m(u_i, v_i)/v_i \).

Now we introduce the monocentric city structure. Each region has one CBD and all workers in the region commute to the CBD to work when employed, or to find a job when unemployed. The workers

²If we assume the wage determination differently, we can obtain the positive relationship between regional wages and regional unemployment rates. For instance, assume that the wages are determined so that the labor share is larger in the region with higher productivity. Then, in the equilibrium with labor mobility, higher productivity means higher wage and larger labor share, which discourages the firms' supply of vacancies. Consequently, the unemployment rate is higher in the region with higher productivity. See Moen (1997) for the example of this type of equilibrium.
live, occupying a certain amount of land \( h \), around the CBD so as to make the commuting cost as small as possible. We assume that the amount of the residential land that is some distant from the CBD is limited. From this assumption, the more workers live in the region, the broader the residential district is. This produces higher commuting cost and higher incentives to live near the CBD and raises the rent of the land near the CBD. The land rent at the edge of the monocentric city is determined to be equal to the rural rent. The equilibrium in the monocentric city is achieved when no worker has an incentive to relocate, i.e., when commuting cost plus residential land rent is the same everywhere in the city.

Let \( W^e \) be the worker’s asset value when he/she is employed in region \( i \) and \( W^u \) be his/her asset value when he/she is unemployed in region \( i \). The distance from the CBD to the worker’s location is denoted by \( x \), and the residential land rent per unit time is denoted by \( R(x) \). When the worker’s wage per unit time is \( w \), we obtain

\[
\begin{align*}
    rW^e_i &= w_i - R(x)h - tx + s(W^u_i - W^e_i), \\
    rW^u_i &= - R(x)h - tx + \frac{m(u_i, v_i)}{u_i}(W^e_i - W^u_i),
\end{align*}
\]

where \( r, t, s \) are, respectively, the discount rate, the per-distance commuting cost per unit time and the rate at which each filled job-worker pair breaks up. Let us assume all these variables are exogenous.

Let \( V^f_i \) be the asset value of a filled job in region \( i \) and \( V^v_i \) be the asset value of a vacancy in region \( i \). We assume that a filled job produces a certain amount of output \( y_i \), which is exogenous and varies from region to region, and that the machines for production bear the fixed costs \( c \) regardless of whether jobs are filled or not. Then, we obtain

\[
\begin{align*}
    rV^f_i &= y_i - w_i - c + s(V^v_i - V^f_i), \\
    rV^v_i &= - c + \frac{m(u_i, v_i)}{v_i}(V^f_i - V^v_i).
\end{align*}
\]

From Eqs. (1) and (2), we get

\[
\begin{align*}
    W^e_i - W^u_i &= \frac{w_i}{r + s + m(u_i, v_i) / v_i}, \\
    V^f_i - V^v_i &= \frac{y_i - w_i}{r + s + m(u_i, v_i) / v_i}.
\end{align*}
\]

Since wages are assumed to be determined by Nash bargains, \( w \) is the solution to the following problem,

\[
\max_{\{w_i\}} (W^e_i - W^u_i)^\beta(V^f_i - V^v_i)^{1-\beta}, \quad 0 < \beta < 1,
\]

where \( \beta \) and \( 1 - \beta \) have the interpretation of measures of the bargaining power of workers and firms with filled jobs. Substituting (3) into (4) and solving this problem, we obtain

\[
w_i = \beta y_i.
\]

Since the workers are identical, in equilibrium the cost of living for workers must be the same
everywhere in each city. Otherwise, the workers at a place of higher costs have incentives to relocate, and the allocation cannot be a market equilibrium. Thus, the residential land rent must satisfy

\[ R(x)h + tx = F_i, \]  

(6)

where \( F_i \) is constant in each region but is different among the regions.

For simplicity, we assume that the quality of land is homogeneous, that the density of land equals \( H \) at any location \( x \) and that the rural rent is zero. The land constraint is \( \int_0^x H \, dx = H\bar{x} = Hn_i \), where \( \bar{x} \) represents the distance between the CBD and the edge of the city. Since at the edge of the city, where \( x = \bar{x} \), the residential land rent must equal the rural rent \( 0: R(\bar{x}) = 0 \), taking the land constraint into consideration, the cost of living for workers in region \( i \) is

\[ F_i = \frac{hn_i}{H}, \]  

(7)

which is an increasing function of \( n_i \).

Let us assume that new vacancies can be created and eliminated costlessly. Then, the asset value of vacancies must be driven to be zero: \( V^v_i = 0 \). \( V^v_i = 0 \) and (2) imply

\[ c(r + s)\frac{v_i}{m(u_i, v_i)} = (1 - \beta)y_i - c. \]  

(8)

Finally, we assume that the economy is in the steady state. The flow into the stock of employed workers is \( m(u_i, v_i) \) and the flow out of the stock of employed workers is \( s e_i \). The steady-state condition requires that the stock of employed workers is unchanged given the stock of workers in region \( i \):

\[ m(\bar{u}_i, \bar{v}_i) = s\bar{e}_i = s(n_i - \bar{u}_i), \]  

(9)

where bars over the variables represent that they are steady-state variables.

3. Differentials in productivity and the wage curve

Eqs. (8) and (9) determine \( \bar{u}_i \) and \( \bar{v}_i \) given \( n_i \). Differentiating (8) and (9), we obtain

\[ \frac{\partial \bar{u}_i}{\partial n_i} = \frac{\bar{u}_i}{n_i} > 0, \quad \frac{\partial \bar{v}_i}{\partial y_i} = -\frac{(1 - \beta)m^2(\partial m/\partial u_i)}{(m + s)(\partial m/\partial u_i)} < 0, \]  

(10)

\[ \frac{\partial \bar{v}_i}{\partial n_i} = \frac{\bar{v}_i}{n_i} > 0, \quad \frac{\partial \bar{v}_i}{\partial y_i} = \frac{(1 - \beta)m^2(\bar{u}_i(\partial m/\partial u_i) + s)}{\bar{u}_i(\partial m/\partial u_i)(m + s)} > 0. \]

Using (10), we can see that an increase in the number of workers does not affect unemployment rate \( U_i(= \bar{u}_i / n_i) \) and that a rise in output \( y_i \) lowers \( U_i \):

\[ \frac{\partial U_i}{\partial n_i} = \frac{1}{n_i} \frac{\partial \bar{u}_i}{\partial n_i} - \frac{\bar{u}_i}{n_i} = 0, \quad \frac{\partial U_i}{\partial y_i} = \frac{1}{n_i} \frac{\partial \bar{v}_i}{\partial y_i} < 0. \]  

(11)
When the workers can move between regions with zero costs, in the long-run equilibrium, it is required that no unemployed worker has an incentive to move between regions. This condition implies that the asset value of the unemployed workers is the same in all regions where there are any workers and is a decreasing function of \( n_i \). Let \( k (\leq K) \) denote the number of regions with workers in the long run equilibrium and the long run equilibrium values be distinguished by an asterisk. Then, the preceding condition becomes

\[
W_i'' (n_i^*) = \text{const.} \tag{12}
\]

Differentiating (12), it is shown that in the long run equilibrium the region with higher productivity must have a larger number of workers:

\[
\frac{\partial n_i^*}{\partial y_i} > 0. \tag{13}
\]

From (5), (11) and (13), for all \( i, j \) \((i, j = 1, 2, \ldots, k - 1, k)\) such that \( y_i \geq y_j \),

\[
n_i \geq n_j, \quad w_i \geq w_j, \quad U_i \leq U_j. \tag{14}
\]

In the region with higher productivity the wage is higher and the unemployment rate is lower. This means that the wage curve exists in this model in spite of the workers being mobile between regions with no costs. If there is not the monocentric city structure, all workers go to one region with the highest productivity and only one combination of the wage and the unemployment rate exists. The effect of differentials in congestion, which is caused by population accumulation in the cities and consists of commuting cost and residential land rent, eliminates the effect of differentials in productivity and the long run equilibrium which has the wage curve is established.

4. Conclusions

This study tried to explain the wage curve when the workers move between regions freely, using a simple search model with regional variations in productivity and with a monocentric city structure. We found that the monocentric city structure enables the existence of variations in combination with the regional wage and the regional unemployment rate to produce the wage curve.

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\(^1\)Using (10), we can see \((\partial W_i^*)/(\partial n_i) < 0\), so in the steady state the stability is always satisfied.
References