Optimal consumption when capital markets are imperfect

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Abstract

Capital market imperfections are widely believed to explain why consumption deviates from the martingale implications of the REPIH. However, empirical modelling and theoretical analysis of consumption under capital market imperfections is hindered by the absence of any tractable theoretical model. By modelling capital market imperfections as consumers facing an upward sloping interest rate schedule (i.e. they can borrow more funds but only at higher rates) we outline just such a model. We derive an analytical Euler equation in observable variables which nests the standard REPIH as well as a model of binding credit constraints. We show via simulation the properties of this Euler equation and its potential at explaining previous rejections of the REPIH. We also consider the wider impact of consumption under capital market imperfections on economic fluctuations. Examining how consumption responds to cyclical and permanent changes in loan supply we conclude that more radical alterations to the REPIH are required if consumption and capital market imperfections are to play a substantive role in accounting for business cycle fluctuations.

Keywords: Capital market imperfections; REPIH; Optimal consumption

JEL classification: E21

1. Introduction

The inability of the Rational Expectations Permanent Income Hypothesis (REPIH) model of Hall (1978) to account fully for the stochastic properties of consumption has been extensively documented. An oft cited reason for this failure is the existence of borrowing constraints (see Flavin, 1981; Hall and Mishkin, 1982; Campbell and Mankiw, 1989; Zeldes, 1989). However, empirical tests of this hypothesis and theoretical attempts at modelling credit constraints are complicated by the absence of analytical solutions for consumption in the presence of borrowing constraints. The aim of this paper is...
to propose a tractable model of borrowing imperfections which has some empirical support and which produces straightforward expressions in observable variables for non-durable consumption growth.

2. Optimal consumption when capital markets are imperfect

In the spirit of Milde and Riley (1988) we assume consumers rarely face binding credit constraints but are able to borrow funds at ever higher interest rates. Table 1 shows the interest payable on different forms of debt in the UK and reveals (i) loan rates are substantially above deposit rates and (ii) there is substantial diversity in loan rates. One explanation for these findings is that different types of loan provide different services to the borrower. However we simply assume, with support from Canner and Fergus (1987) and Ausubel (1991), that even if these different interest rates reflect extra services consumers may simply use these sources to borrow more funds.

2.1. Maximisation problem

We assume utility is defined over current consumption of goods and services, \( \{c_t\} \). Agents choose their asset holdings to maximise lifetime utility subject to an intertemporal budget constraint:

\[
\max E \sum_{t=0}^{\infty} B^t u(c_{t+j})s \cdot ta_{t+j+1} = a_{t+j} \phi(a_{t+j}) + y_{t+j} - c_{t+j}
\]

where \( \phi(.) \) denotes \( 1 + i \), and \( i \) is the interest rate the consumer receives/pays on their asset holdings. Because our focus is on the loan market we assume interest rates do not vary with positive asset holdings (\( \phi' = 0 \), \( a > 0 \)). A key issue for our analysis is the properties of \( \phi(.) \) over the range of negative assets. If there exists a wedge between the deposit and borrowing rate or if loan rates jump at particular levels of debt then \( \phi(.) \) is not continuously differentiable. In this case, as in King (1986), we can only focus on the first-order conditions characterising each segment of the loan schedule. Because our emphasis is on offering a tractable model we wish to avoid such complications. We therefore make the strong assumption that \( \phi(.) \) is everywhere continuously differentiable and that \( \phi'(.) \leq 0 \) for \( 0 \leq a \leq a^* \), where \( \phi'(a^*) = -\infty \). In other words \( a^* \) denotes the level of assets beyond which the consumer cannot borrow and the interest rate schedule becomes vertical.

Table 1

<table>
<thead>
<tr>
<th>Date</th>
<th>Base rate</th>
<th>Deposit</th>
<th>Mortgage</th>
<th>Personal loan</th>
<th>Credit card</th>
</tr>
</thead>
<tbody>
<tr>
<td>10/90</td>
<td>14.0</td>
<td>12.3</td>
<td>15.4</td>
<td>26.9</td>
<td>32.1</td>
</tr>
<tr>
<td>4/91</td>
<td>12.0</td>
<td>9.9</td>
<td>13.8</td>
<td>26.9</td>
<td>28.1</td>
</tr>
<tr>
<td>7/91</td>
<td>11.0</td>
<td>8.9</td>
<td>12.5</td>
<td>23.6</td>
<td>28.1</td>
</tr>
<tr>
<td>5/92</td>
<td>10.0</td>
<td>7.8</td>
<td>11.0</td>
<td>22.6</td>
<td>28.1</td>
</tr>
<tr>
<td>8/92</td>
<td>10.0</td>
<td>7.2</td>
<td>10.7</td>
<td>22.6</td>
<td>26.8</td>
</tr>
<tr>
<td>10/92</td>
<td>8.0</td>
<td>5.0</td>
<td>10.7</td>
<td>22.1</td>
<td>26.8</td>
</tr>
<tr>
<td>12/92</td>
<td>7.0</td>
<td>4.0</td>
<td>9.3</td>
<td>25.3</td>
<td></td>
</tr>
<tr>
<td>3/93</td>
<td>6.0</td>
<td>3.0</td>
<td>8.0</td>
<td>23.1</td>
<td></td>
</tr>
<tr>
<td>1/94</td>
<td>5.5</td>
<td>2.75</td>
<td>7.6</td>
<td>22.3</td>
<td></td>
</tr>
</tbody>
</table>

* Source: Dale and Haldane (1993).
Because interest rates are rising with debt we also have to be careful that a transversality condition holds. For our model this is established in Magill and Quinzii (1994) who show in the context of an infinite horizon model with incomplete insurance and impatient consumers that for every equilibrium which satisfies a transversality condition there is an equilibrium which imposes (potentially time-varying) non-binding sequential borrowing constraints. This is achieved in our model by the assumption that \( \phi'(a^*) = -\infty \) for a finite \( a^* \).

Under these assumptions the first order condition for (1) is

\[
E_t \beta \frac{u_{t+1}'}{u_t} \phi_{t+1} (1 - \eta_{t+1}) = 1, \quad \text{where} \quad \eta_{t+1} = -\frac{\partial \phi_{t+1}}{\partial a_{t+1}} \frac{a_{t+1}}{\phi_{t+1}}
\]

where \( u_{t+1}' \) denotes the marginal utility of consumption at time \( t+1 \). For \( a_t \leq 0, \eta \leq 0 \) and (2) shows (assuming \( u''<0 \)) debtors have faster consumption growth when they are on an upward sloping part of the interest rate schedule. The intertemporal price which determines the slope of the consumption profile consists of (i) a term reflecting the interest rate, \( \phi_{t+1} \) and (ii) the marginal cost of the last unit of a loan, \( \eta_{t+1} \). When \( \eta = 0 \) (2) collapses to the standard Euler equation under perfect capital markets (e.g. Hall, 1978). However, if borrowing rates are higher than deposit rates, then even if the interest rate schedule is flat debtors will have faster consumption growth due to intertemporal substitution.

Using the budget constraint and assuming a CRRA utility function \( u(c_t) = (c_t^{1-\sigma}/(1-\sigma)) \) we can re-write (2) as

\[
E_t \beta \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \left( \phi_{t+1} + \frac{\partial \phi_{t+1}}{\partial a_{t+1}} (\phi_{t+1} a_t + y_t - c_t) \right) = 1
\]

For debtors facing a rising interest rate schedule \( (\phi'<0) \) consumption growth depends negatively upon lagged income (as in Hall and Mishkin, 1982; Zeldes, 1989): higher income/savings last period results in a lower loan rate this period and a slower rate of consumption growth. Eq. (3) suggests that the coefficient on the loan rate should be the same as that for the deposit rate (consistent with Zeldes’ (1989) findings). Earlier we suggested that binding credit constraints can be interpreted as \( \phi' = -\infty \) in which case for (3) to hold consumption must tend to cash in hand. More generally, however, consumption need not be well approximated by Campbell and Mankiw’s (1989) rule of thumb consumers who simply set consumption growth equal to income growth.

2.2. An analytical Euler equation

Assume \( \phi(.) = r \) for \( a > 0 \) and \( = r e^{-\gamma a/s} \) for \( a \leq 0 \), so that the borrowing rate rises with the debt/income ratio but is constant for creditors, and also, in the spirit of Hansen and Singleton (1983), that \( \Delta \ln c_t \) and \( \phi(1-\eta) \) are jointly distributed log normally. Under these assumptions the Euler equation for debtors is\(^1\)

\[
\Delta \ln c_{t+1} = \alpha + \frac{1}{\sigma} \ln(1 + R_{t+1}) + \frac{\gamma D_{t+1}}{\sigma Y_{t+1}} + \epsilon_{t+1} = \alpha + \frac{1}{\sigma} R_{t+1} + \frac{\gamma D_{t+1}}{\sigma Y_{t+1}} + \epsilon_{t+1}
\]

\(^1\)Using the fact \( \ln(\phi(1-\eta))=\ln \phi - \eta \) for small \( \eta \).
where $D$ denotes debt, $\alpha = (\sigma/2) \text{Var}(\Delta \ln c_{t+1} - (\phi_{t+1} - (1 - \eta_{t+1})/\sigma))$ and $R_t$ the borrowing rate. Therefore consumption growth depends positively upon the borrowing rate and the debt to income ratio. Combining (4) with the standard Euler equation for creditors and denoting the proportion of debtors by $\theta$ we have an equation for aggregate consumption

$$
\sum \Delta \ln c'_{t+1} = \alpha^* + \frac{1}{\sigma} r_{t+1} + \frac{\theta}{\sigma} (R_{t+1}^* - r_{t+1}) + \gamma \frac{\theta}{\sigma} D_{t+1}^* + e_{t+1} + \epsilon_{t+1}
$$

where $R^*$, $D^*$ and $Y^*$ denote the average value of borrowing rates, debt and income across debtors and $\alpha^*$ is an average over all creditors and debtors. Aggregate consumption growth therefore depends positively upon the deposit rate, the wedge between the borrowing and deposit rate and the debt to income ratio. The presence of both the wedge and debt to income ratio adds significant extra dynamics to consumption over and above the influence of rates of return. Support for (5) can be found in the cross-country evidence of Bachetta and Gerlatz (1997) who find that various measures of debt have predictive power for consumption growth rather than labour income changes. Eq. (5) suggests that any changes either in the degree of capital market imperfections ($\gamma$) or in the proportion of debtors ($\theta$) will lead to time variation in the parameters of the consumers Euler equation.

2.3. Simulation evidence

Assume a CRRA utility function, choose $\beta = 1/1.06$ and assume $\ln y_i = N(\mu, \sigma^2_y)$, where $y_i$ denotes an individual’s income and $\mu$ is interpreted as aggregate income. To simplify our simulations we assume $\phi(a) = r$ for $a_i \geq 0$ and $\phi(a) = re^{-\gamma a}$ for $a_i < 0$ and we set $r = 0.04$. As $\gamma \to 0$ the model tends to the perfect capital market case while as $\gamma \to \infty$ we come closer to the pure credit rationing model.

Under perfect capital markets the standard case to consider is $\beta r = 1$. If instead $\beta r > 1$, consumption rises continuously, financed by rising capital income, while if $\beta r < 1$ consumers always wish to borrow and roll over infinite amounts of debt violating the no-Ponzi condition. By contrast when $\beta r = 1$ individual consumption is constant and asset markets are used to smooth out purely idiosyncratic shocks so that individual consumption is correlated more with aggregate than individual income (see Mace, 1991; Cochrane, 1991). However, in the case of imperfect capital markets, if $\beta r = 1$ we change dramatically this full insurance property. Even though individuals can borrow they choose never to do so. This is because $\beta Er_{t+1} > 1$ if agents expect to borrow. Therefore they accumulate enough initial assets to finance consumption in low income periods rather than go into debt and in this way avoid higher interest rates. Therefore to examine the case where consumers hold debt we consider $\beta r < 1$ but $\gamma > 0$ such that over some range of debt $\beta \phi(a_i) > 1$. Under these assumptions, consumers will never hold infinite amounts of debt but will sometimes let their net assets go negative.

Our simulation results are shown in Table 2. The simulations are based on 1000 periods but all statistics quoted exclude the first 200 observations so as to remove any initial condition effects. Every column is based on exactly the same income sequence so that differences result from variations in the degree of prudence ($\sigma$) and capital market imperfections ($\gamma$). For $\sigma$, the degree of relative risk

\footnote{To solve this non-linear stochastic Euler equation we use the parameterised expectations algorithm (PEA) of den Haan and Marcet (1990). We use a third-order polynomial in the state vector (the interest rate, asset holdings and the income disturbance) and a convergence criteria of six decimal places for the coefficients of the polynomial.}
Table 2

<table>
<thead>
<tr>
<th>σ, γ</th>
<th>σ_Δ ln c_Δ ln y</th>
<th>Cor(Δ ln c, Δ ln y)</th>
<th>Cor(Δ ln c, Δ ln y_{t-1})</th>
<th>% of periods a_0 &lt; 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0004</td>
<td>0.109</td>
<td>0.788</td>
<td>0.057</td>
<td>0.153</td>
</tr>
<tr>
<td>1.0002</td>
<td>0.098</td>
<td>0.785</td>
<td>0.047</td>
<td>0.238</td>
</tr>
<tr>
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<td>0.087</td>
<td>0.781</td>
<td>0.037</td>
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</tr>
<tr>
<td>2.0004</td>
<td>0.090</td>
<td>0.712</td>
<td>0.142</td>
<td>0.108</td>
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<td>0.079</td>
<td>0.719</td>
<td>0.115</td>
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</tr>
<tr>
<td>2.0001</td>
<td>0.065</td>
<td>0.727</td>
<td>0.079</td>
<td>0.278</td>
</tr>
<tr>
<td>5.0004</td>
<td>0.084</td>
<td>0.571</td>
<td>0.299</td>
<td>0.105</td>
</tr>
<tr>
<td>5.0002</td>
<td>0.067</td>
<td>0.595</td>
<td>0.241</td>
<td>0.132</td>
</tr>
<tr>
<td>5.0001</td>
<td>0.042</td>
<td>0.640</td>
<td>0.138</td>
<td>0.143</td>
</tr>
</tbody>
</table>

* First column lists parameter calibrations (σ first and then γ). The second column shows the standard deviation of consumption growth from our simulations, the third column the correlation between current consumption and income growth, and the fourth column between current consumption growth and lagged income growth (excess sensitivity). The final column reports the proportion of periods in which the consumer is in debt.

aversion, we use the values 1 (the estimate of Attanasio and Weber (1993) using UK micro data), 5 (the estimate of Acemoglu and Scott (1994) using UK aggregate data) and an intermediate value of 2. In the absence of any econometric studies we selected values for γ of 0.001, 0.002 and 0.004. These values resulted in ‘plausible’ variation in the borrowing rate (that is simulated borrowing rates are in the range suggested by Table 1).

Table 2 shows that in the presence of an upward sloping loan schedule individual consumption is very volatile. Focusing first on a given level of risk aversion, the steeper the loan rate schedule the more volatile is consumption growth and the greater the correlation between consumption and current income — the perfect insurance result breaks down. Focusing on a given slope for the interest rate schedule Table 2 shows that increasing risk aversion leads to less volatile consumption changes, a lower correlation between current consumption and income and also fewer periods when the consumer borrows. As shown in Deaton (1991) and Xu (1995) when consumers run the risk of facing a binding credit constraint they accumulate precautionary balances to minimise the risk of hitting the constraint. In the case of an upward sloping interest rate schedule the consumer has even more incentive to accumulate precautionary balances. As well as avoiding the credit constraint the consumer also wishes to avoid finding themselves on a steeply sloped part of the interest rate schedule.

In summary the stochastic properties of consumption in Table 2 are driven by two offsetting influences: the rising interest rate schedule moves consumption away from the full insurance result (and more so than the pure credit constraint model) and increases the volatility of consumption, whereas precautionary behaviour leads consumers to self insure by accumulating positive assets. The more important is precautionary behaviour the less correlated consumption is with current income and the lower the volatility of consumption changes as high levels of precautionary saving helps the consumer self-insure.

3. Conclusion

The aim of this paper has been to provide an extremely tractable model of optimal consumption
when capital markets are imperfect. Assuming an upward sloping interest rate schedule and joint log-normality we show that optimal consumption is characterised by an analytic relationship between consumption growth, the interest rate and the debt to income ratio. As a consequence we can account for many of the observed rejections of the standard REPIH in particular the very different dynamic patterns of consumption and rates of return.

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References