The expectations hypothesis of the term structure of interest rates, open interest rate parity and central bank policy reaction

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Received 23 February 1999; accepted 9 September 1999

Abstract

A rational expectations model with endogenous monetary policy reacting to the exchange rate and the term spread shows that the empirical performance of the expectations hypothesis of the term structure and the uncovered interest rate parity hypothesis improves with the strength of the policy reaction to the exchange rate and the term spread, respectively.

Keywords: Expectations hypothesis of the term structure; Policy reaction to the exchange rate and the term spread; Uncovered interest rate parity

JEL classification: E43; E58

1. Introduction

Several papers have been published since 1988 which show that the expectations hypothesis of the term structure of interest rates (EHTS) works much better in European countries than in the US (among others Kugler, 1988; Hardouvelis, 1994; Gerlach and Smets, 1998). The recent work of Gerlach and Smets suggests that the EHTS works best for the cases where monetary policy is restricted by an intermediate exchange rate target as in the European exchange rate mechanism (ERM). They argue that the central bank reacts to exchange market pressure by temporarily increasing short-term interest rates. This systematic policy response makes short rates relatively well predictable and leads, therefore, to a good performance of the EHTS. The primary aim of this paper is to provide a theoretical analysis of the performance of the EHTS under conditions of policy reaction of the central bank to the exchange rate. A natural starting point for our analysis is McCallum’s (1994a,b) two policy reaction model for the explanation of the failure of the unbiasedness hypothesis of the

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forward rate or the open interest rate parity (UIP) and the EHTS, respectively. The first model shows that interest rate smoothing and leaning against the wind with respect to the exchange rate by the central bank, leads to a negative relationship of the spot rate change and the lagged forward premium and, therefore, a rejection of UIP. The second model provides the result that the performance of the EHTS crucially depends on the reaction of the central bank to the short long spread and the persistence of the term premium. The model considered in this paper blends the two models to a framework with a central bank reacting to the exchange rate and to the term spread. This model allows the theoretical analysis of the performance of the EHTS under conditions of policy reaction to the exchange rate. Moreover, it suggests considering a second and closely related question, namely the influence of policy reaction to the term spread on empirical tests of UIP. These two issues are analyzed in section two and section three offers some conclusions.

2. The model

In order to keep our model simple we consider directly international differentials in interest rates and the exchange rate. Denote the exchange rate by \( s_t \), the difference between the home and foreign one period short rate by \( x_t = r_t - r_t^* \) and the international difference in the long short spread \( z_t = R_t - r_t - (R_t^* - r_t^*) \). Moreover, assume that the long rate is a two period rate for the sake of simplicity. This model does not explicitly deal with forward rate, but covered interest rate parity, which clearly holds in order to prevent arbitrage opportunities, implies \( x_t = f_t - s_t \).

Given this notation we can write our model as follows: first we introduce the EHTS in its form for the short rate change:

\[
\Delta x_{t+1}^e = 2(z_t - \xi_t)
\]

(1)

Upper case \( e \) denotes expected values given information available in period \( t \) and \( \xi_t \) is an exogenous term premium which follows a stable AR(1) scheme as in McCallum’s (1994b) model.

The second ingredient of our model is the UIP hypothesis:

\[
\Delta s_{t+1}^e = x_t - \xi_t
\]

(2)

where \( \xi_t \) is an exogenous risk premium which follows a stable AR(1) scheme, again as in McCallum’s (1994a) model.

Eqs. (1) and (2) are used to test the expectations and the unbiasedness hypothesis, respectively: realized values for the future change in the short rate and the spot rate are regressed on the spread and the interest rate differential (forward premium) and the hypothesis that the slope coefficient is 2 or 1 is tested, respectively. Of course, this estimate may be strongly biased when the variances of the premia are large compared to that of the expectations errors and when the correlations between \( x \) and \( z \) with the premia are strong.

This bias will be analyzed using the policy reaction framework suggested by McCallum. In fact, we consider joint policy reaction to the exchange rate and to the long short spread:

\[
x_t = \lambda \Delta s_t = \theta z_t + \sigma x_{t-1} + \zeta_t
\]

(3)

\( \zeta_t \) is a white noise error term representing exogenous short rate shocks, whereas the first two terms in
the RHS with coefficient positive coefficients account for the endogeneity of monetary policy: the central bank increases the short rate in response to a depreciating exchange rate and when a widening spread signals higher expected future inflation and correspondingly higher future short rates and the lagged short rate term indicates interest rate smoothing. That is, Eq. (3) mimics policy of the central bank in a highly stylized way ignoring other indicators for monetary policy. For more motivation of this approach the reader is referred to the two papers by McCallum (1994a,b), who discusses the rationale for this specification of policy behavior in more detail.

This model can be conveniently solved under the rational expectations hypothesis using the method of undetermined coefficients. First of all, we eliminate \( z \) by combining (1) in (3). The resulting system can be written in matrix notation as

\[
G_1 y_t = G_2 y_{t+1} + h_1 x_{t-1} + H_1 e_t,
\]

\[
y_t' = (\Delta s, x_t, e_t),
\]

\[
G_1 = \begin{bmatrix}
-\lambda & 1 + \theta/2 \\
0 & 1
\end{bmatrix}, \quad G_2 = \begin{bmatrix}
0 & \theta/2 \\
1 & 0
\end{bmatrix}, \quad h_1 = \begin{bmatrix}
\sigma_x' \\
0
\end{bmatrix}, \quad H_1 = \begin{bmatrix}
1 & 0 & \theta
\end{bmatrix}
\]

In addition, our assumption about the error terms is written compactly as a VAR(1) model:

\[
e_t = B e_{t-1} + w_t,
\]

\[
B = \begin{bmatrix}
0 & 0 & 0 \\
0 & \rho & 0 \\
0 & 0 & \tau
\end{bmatrix}, \quad w_t = (\zeta_t, u_t, v_t), \quad E(w_t'w_t) = \begin{bmatrix}
\sigma_x^2 & 0 & 0 \\
0 & \sigma_u^2 & 0 \\
0 & 0 & \sigma_v^2
\end{bmatrix}.
\]

The minimum state variable (MSV) solution can be written as

\[
y_t = \Phi_1 x_{t-1} + \Phi_2 e_t,
\]

\[
\Phi_1 = \begin{bmatrix}
\phi_{11} \\
\phi_{21}
\end{bmatrix}, \quad \Phi_2 = \begin{bmatrix}
\phi_{12} & \phi_{13} & \phi_{14} \\
\phi_{22} & \phi_{23} & \phi_{24}
\end{bmatrix}.
\]

From Eqs. (5) and (6) we get the following expression for the expected future values

\[
y_{t+1}' = \Phi_1 x_t + \Phi_2 Be_t.
\]

Inserting (7) in (4) results in

\[
G_1 y_t = G_2 \{ \Phi_1 x_t + \Phi_2 Be_t \} + h_1 x_{t-1} + H_1 e_t.
\]

Collecting all current \( x_t \) in (8) and inserting the MSV solution finally provides

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1Eq. (3) has an immediate appeal when the central bank uses the short rate as its instrument. However, I would suggest that (3) remains valid description when the central bank uses other instruments as these actions are immediately transmitted to the short rate, as a rule.
\[ G_i^*(\Phi_1 x_{i-1} + \Phi_2 e_i) = G_2 \Phi_2 B e_i + h_1 x_{i-1} + H_1 e_i \]
\[ G_i^* = \begin{bmatrix} -\lambda & 1 + \theta(1 - \phi_{21})/2 \\ 0 & 1 - \phi_{11} \end{bmatrix} \]  \hspace{1cm} (9)

First, equating the coefficients of \( x_i \) and \( e_i \) on both sides of (9) results in

\[ G_i^* \Phi_1 = h_1 \]
\[ G_i^* \Phi_2 = G_2 \Phi_2 B + H_1 \]

Solving these equations for the undetermined coefficients\(^2\) and inserting the MSV solution for the current and expected future value of \( x_i \) in Eq. (1) results in the following solution:

\[ \Delta s_i = -\frac{\sigma}{\lambda} x_i - \frac{1}{\lambda} \xi_t - \frac{1 + \theta(1 - \rho)2}{a} \xi_i + \frac{\theta(\lambda + \sigma)}{\lambda b} e_i \]
\[ x_i = -\frac{\lambda}{a} \xi_i + \frac{\theta \tau}{b} e_i \]
\[ z_i = \frac{\lambda(1 - \rho)2}{a} \xi_i + \frac{\tau - (\lambda + \sigma)}{b} e_i \]
\[ a = \rho(1 + \theta(1 - \rho)/2) - (\lambda + \sigma), \quad b = \tau(1 + \theta(1 - \tau)/2) - (\lambda + \sigma) \]  \hspace{1cm} (10)

What are the consequences of this solution for the tests of the EHTS? In order to show these effects as simply as possible let us assume \( \tau = 0 \), which means that the term premium is white noise, whereas the risk premium follows an AR(1) process. This case is of some additional interest in our context as it implies no predictive power of the term spread for the short rate change and, therefore, a bad performance of the EHTS in McCallum’s term structure model. The hypothesis is tested by regressing \( \Delta x \) on the lagged \( z \). According to our model solution with \( \tau = 0 \) we can write \( \Delta x = -\lambda(\xi - \xi_{i-1})/a \) and the plim of the OLS slope coefficient estimate of the test regression is, therefore,

\[ \text{cov}(\Delta x, z_{i-1}) = \frac{\lambda^2 (1 - \rho)^2 \sigma^2 \xi / 2a^2}{\lambda^2 (1 - \rho)^2 \sigma^2 \xi / 4a^2 + \sigma^2_{x_i}} = 2 \frac{\lambda^2 (1 - \rho)^2}{\lambda^2 (1 - \rho)^2 + 4a^2 \sigma^2_{x_i} / \sigma^2_{\xi}} \]

\[ \sigma^2_{\xi} = \text{var}(\xi), \quad \sigma^2_{x_i} = \text{var}(e_i) \]

If \( \lambda \) is equal to zero then the plim of the regression coefficient is zero, a value strongly at odds with the EHTS. By contrast, if \( \lambda \) is larger than zero we have a non-zero slope coefficient. It increases with \( \lambda \) (note that \( a \) depends negatively on \( \lambda \)) and approaches the value implied by the EHTS, namely 2, if the variance ratio of the term premium and the risk premium goes to zero. Thus, we have the result that the strength of the central bank reaction to the exchange rate has a positive effect on the empirical performance of the EHTS. The intuition of these results is easily understood: the systematic reaction

\(^2\)There are two solutions for \( \phi_{11} \) and \( \phi_{21} \). The bubble-free stable solutions \( \phi_{11} = -\sigma / \lambda \) and \( \phi_{21} = 0 \) are adopted. A detailed solution is available from the author on request.
of the central bank to the exchange rate leads to predictable variations of the short rate triggered by risk premium changes and produces results in favor of the EHTS.

Is there a similar result for tests of the UIP with respect to the strength of the central bank’s reaction to the term spread? Under our assumption we get a very simple reduced form equation for the spot rate change: inserting the AR(1) scheme for the risk premium in the first equation of the model solution and replacing the lagged \( \xi \) value, according to the second equation, by \(-ax_{t-1}/\lambda\) results in the UIP test regression with a white noise error term:

\[
\Delta s_t = \frac{\rho(1 + \theta(1 - \rho)/2) - \sigma}{\lambda} x_{t-1} - \frac{1}{\lambda} \xi_t - \frac{1 + \theta(1 - \rho)/2}{a} u_t + \frac{\theta(\lambda + \sigma)}{b\lambda} \varepsilon_t \tag{11}
\]

Thus, an increase in \( \theta \) leads to an increase in the slope coefficient of the regression of the change in the spot rate on the lagged interest rate differential. In order to get an impression of this effect let us assume that \( \theta = 0, \lambda = 0.1, \sigma = 0.9, \rho = 0.5 \). These values give a slope coefficient of \(-4\), which characterizes many UIP regressions for USS exchange rates and which is strongly at odds with UIP. If we set \( \theta = 2 \) we obtain a value of \( 1 \), which is perfectly in line with UIP. These calculations nicely illustrate that the strength of the reaction to long short spread may have a crucial influence on tests of the UIP. This analytical result can be interpreted as follows: the interest rate reaction of the central bank to spot rate changes caused by risk premium changes leads to the negative correlation between regressor and error term in the UIP test regression. The size of this correlation is decreased when the corresponding increase in the term spread acts against the exchange rate induced interest rate reaction of the central bank. If this effect is sufficiently strong, as in our example, regressor and error term are even uncorrelated and there is no bias in standard UIP regressions.

3. Summary and conclusion

This paper extends the endogenous policy reaction model developed by McCallum (1994b) and Kugler (1997) for the explanation of the results of empirical test of the EHTS, by taking into account policy reaction to the exchange rate in addition to the term spread. The two period model consists of the EHTS, UIP with a stochastic term and risk premium as well as short rate policy reaction equation. The rational expectations solution shows that a strong reaction of the central bank to the exchange rate improves the empirical performance of the EHTS. When the variance of the risk premium is clearly larger than that of the term premium we get results in line with the EHTS. This theoretical result is in line with those empirical studies showing a much better performance of the EHTS for ERM currencies than for flexible exchange rate currencies. Moreover, the striking change at the short end of the maturity of the US term structure in 1987 reported by Hsu and Kugler (1997), which was attributed to policy reaction to the spread in that paper, may be, in addition, caused by a greater role of the exchange rate in the conduct of US monetary policy since the mid-1980s. As a second result, our model shows that the empirical performance of the UIP hypothesis improves with the strength of the reaction of the central bank to the term spread. This reaction pattern decreases the correlation between the term premium and the interest rate differential which leads to the negative bias of the slope coefficient in the UIP test regression in McCallum’s (1994a) model.
Acknowledgements

Helpful comments of an anonymous referee are gratefully acknowledged.

References


