A problem with Euclidean preferences in spatial models of politics

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Abstract

Euclidean public sector preferences can not be induced from a strictly quasiconcave primitive utility function and a linear constraint. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

In spatial models of collective choice, individual preferences are often represented as induced preferences in policy space (or ‘public sector preferences’). These induced preferences are typically understood to be derived from an underlying constrained maximization involving a primitive utility function of the conventional sort (e.g. Slutsky, 1975, 1977; Denzau and Parks, 1977, 1979; Diba and Feldman, 1984). Given the usual assumptions of individual utility maximization, it is well-known that the induced preferences over policy exhibit a unique satiation point (corresponding to the constrained maximum of the underlying primitive utility function) and have strictly convex level sets.

In many applications, public sector preferences also are assumed to be Euclidean; that is, the public sector preferences are not just separable, but utility declines monotonically in distance from the ideal point. This assumption drives many important results in collective choice (e.g. McKelvey, 1976, Laver and Shepsle, 1990, Ferejohn and Krehbiel, 1987; Koford, 1989; also, see Milyo, 1999). Below, I demonstrate that well-behaved public sector preferences are never Euclidean.

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2. Public sector preferences

The canonical derivation of public sector preferences involves a collective of individuals in an economy consisting of two publicly provided goods \((z_1, z_2)\) and one composite private good \((x)\). For simplicity, let the publicly provided goods be pure public goods; therefore, any individual citizen in the collective has three arguments in his utility function: \((x, z_1, z_2)\). From the perspective of this citizen, the private good is a free choice variable and the publicly provided goods (i.e. ‘policies’) are strictly rationed at a level chosen by the collective.

Assume a linear budget constraint and normalize prices relative to the price of \(x\). The individual’s utility maximization problem is then:

Maximize \(U(x, z_1, z_2)\) subject to \(B = p_1z_1 + p_2z_2 + x\) \hspace{1cm} (1)

Let \(U\) be strictly quasiconcave and continuously differentiable, so that there exits a unique constrained maximum to this problem and well-behaved demand functions. The solution to Eq. (1) is trivial when both publicly provided goods are rationed:

\[ x^* = B - p_1z_1 - p_2z_2 \] \hspace{1cm} (2)

Finally, define the individual’s ‘ideal point’ to be the constrained maximum of Eq. (1) when no goods are rationed. Denote this as \((x^*, z_1^*, z_2^*)\).

In spatial models of politics, it is the convention to work with the individual’s induced preferences over policy (or ‘public sector preferences’). These may be represented by substituting Eq. (2) into the utility function. Denote the resulting mixed indirect utility function by \(V(z_1, z_2)\). The individual’s preferences over policy are then described by the level sets of \(V(z_1, z_2)\) and the ideal point is simply the maximum of this function.

3. Separable and Euclidean public sector preferences

It is common in spatial models of politics to assume that \(V(z_1, z_2)\) is separable; that is, the individual’s most preferred level of \(z_1\) is independent of any fixed level of \(z_2\), and vice versa. Clearly, this property can not hold for all feasible policies (e.g. when \(p_1z_1 + p_2z_2 = B\)), so throughout this analysis I restrict attention to regular and interior solutions to Eq. (1).

There are several caveats and criticisms in the economics literature concerning the properties of public sector preferences (for a review, see Milyo, 1999). For example, Slutsky (1975) shows that \(V(z_1, z_2)\) is separable when the demands for the publicly provided goods are independent of income. Diba and Feldman (1984) demonstrate for the canonical three-good case presented here, that a quasilinear utility function can generate separable public sector preferences.

Milyo (1999) extends Diba and Feldman’s result to show that given a strictly quasiconcave primitive utility function and a linear constraint, separability can be a general property of the public sector preferences only for the case of one private good. Consequently, the example presented in this exercise is fairly general. This is because any number of publicly provided goods may be imagined to be fixed at their ideal level in this example.

Euclidean preferences are a subset of separable preferences for which utility declines monotonically
This implies that the level sets of $V(z_1, z_2)$ are concentric circles about the ideal point.

4. Public sector preferences are not Euclidean

Suppose an individual’s public sector preferences are Euclidean with an ideal point located at $(z_1^*, z_2^*)$. Consider a particular level set, $V(z_1, z_2) = c$, with radius $r > 0$ (i.e. away from the ideal point). This implies that:

$$V(z_1^* + r \cos \theta, z_2^* + r \sin \theta) = c \quad \text{for all } r > 0 \text{ and } 0 \leq \theta \leq 2\pi$$

Making use of Eq. (2) and the definition of public sector preferences, Eq. (3) may be rewritten as:

$$U(B - p_1 z_1^* - p_1 r \cos \theta - p_2 z_2^* - p_2 r \sin \theta, z_1^* + r \cos \theta, z_2^* + r \sin \theta) = c \quad \text{for all } r > 0 \text{ and } 0 \leq \theta \leq 2\pi$$

Differentiate Eq. (4) with respect to $\theta$ and rearrange terms to show that:

$$r \cos \theta*(p_2 U_x - U_{z_2}) + r \sin \theta*(p_1 U_x - U_{z_1}) = 0$$

It will now be shown that this condition can be satisfied for only four points in any given level set (other than the ideal point).

First, consider the case when only $z_2$ is fixed at some arbitrary $z_2^0$, while $x$ and $z_1$ are left as choice variables. In this case, and for a well-behaved primitive utility function, the first-order conditions for the constrained maximum of $U(x, z_1, z_2)$ imply that $(p_1 U_x - U_{z_1}) = 0$ (i.e. the ‘equimarginal’ condition). Note also that the definition of Euclidean preferences implies that the constrained optimal value of $z_1$ is $z_1^*$. Consequently, $r \cos \theta = 0$ and condition (5) is satisfied at the constrained maximum, $(B - p_1 z_1^* - p_1 r, z_2^0, z_2^*, z_2^*, z_2^0)$. There will be two points on any level set of $V(z_1, z_2)$ which are constrained optima when $z_1$ is a free choice variable (when $z_2^0 = z_2^* \pm r \sin \theta$); at these two points, condition (5) is satisfied. Similarly, when $z_1$ is fixed at an arbitrary level, condition (5) will be satisfied at two analogous points on the given level set (these four points correspond to the four ‘compass points’ on any level set).

Now consider the case when both $z_1$ and $z_2$ are fixed at arbitrary points $(z_1^0, z_2^0)$ not equal to $z_1^*$ or $z_2^*$. It follows that neither $r \cos \theta$ nor $r \sin \theta$ is equal to zero. It is also the case that each of the publicly provided goods is either being under-consumed or over-consumed relative to consumption when either or both of the $z$’s are free variables. Consequently, for a well-behaved primitive utility function, the equimarginal condition does not hold and condition (5) is not satisfied at $(B - p_1 z_1^0 - p_2 z_2^0, z_1^0, z_2^0)$. This demonstrates that not only are well behaved public sector preferences never Euclidean, but that (for $r > 0$) no level set of $V(z_1, z_2)$ can be a circle centered on the ideal point.

References