Monopolistic competition with a mail order business

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Abstract

I analyse free-entry competition between stores and mail order businesses. Consumers purchasing at stores entail distance related transportation costs. Purchase from a mail order business (MOB) involves a fixed cost. Compared to Salop’s model [Salop, S.C., 1979. Monopolistic competition with outside goods. Bell Journal of Economics 10, 141–156], fewer firms are active with free-entry. At most one MOB enters. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

This paper studies free-entry competition when firms can choose between two alternatives to sell a homogeneous good at mill prices. The first alternative consists of opening a retail store, which consumers can visit by paying a linear transportation cost. The second alternative involves setting up a mail order business (MOB), where consumers receive the good by paying an exogenous fixed cost, irrespective of their location. The MOB serves its consumers using this fixed cost technology – e.g. a (electronic) postal service. Its location, therefore, becomes completely irrelevant. Total expenditures from buying at the retail store equal the price at retail plus the transportation cost to the retail store. In contrast, all consumers buying from the MOB have the same total expenditure.

The analysis adds a MOB to Salop’s (1979) circle model. If the fixed cost technology is too expensive and the set-up cost is large relative to the marginal transportation cost, no MOB appears. Otherwise, at most one MOB pops up in equilibrium. The MOB competes in a non-localized fashion with all stores. The retail stores, however, compete in a localized way with the MOB. The
introduction of a MOB implies that a smaller number of firms are active in equilibrium compared to Salop’s original model.  

2. The model

Consider a market for a homogeneous product. Marginal cost of production is constant and normalized at zero. Each firm chooses one of two possible strategies to market the product. The first is the traditional way of opening a store where consumers are charged a uniform mill price $p_i \geq 0$. A purchase at this store for a consumer located at distance $z$ implies a linear transportation cost $tz \geq 0$. I use Salop’s circle model, where firms are located equidistantly from each other. The second strategy consists of a MOB where consumers can order the product (by mail) at mill price $q_i \geq 0$ plus a fixed cost $w_i \geq 0$ (e.g. the price of the stamp or the electronic ordering costs) for sending the product to the consumer’s location. This exogenous fixed cost $w$ is independent of one’s location. One interpretation is that the MOB is located at the center of the circle. The radius of the circle then represents the fixed cost $w$. There is a unit mass of consumers located uniformly on the circle. Consumers have the same reservation price $r$ and unit demand. They buy from the firm offering the lowest full price, i.e. mill price plus linear/ fixed (transportation) cost. Let the number of firms in the market be $N \geq 2$, indexed by $i = 2, \ldots, N$. Consider the following three-stage game. In stage one, each firm decides to enter the market or not (Section 4). In stage two, having observed the number of firms that entered the market, they choose to become a traditional store or a MOB. In stage three, having observed each other’s decision in the second stage, firms compete in prices (Section 3). I solve the game for its Subgame Perfect Nash Equilibria in pure strategies by the method of backward induction.

3. Pricing

Consider first the case in which more than one firm operates a MOB. A standard Bertrand result appears for these firms, since they are not differentiated at all with respect to each other. Price competition results in charging a zero price. Since set-up costs are strictly positive, at most one firm will open a MOB. This results in two possible cases: (i) no firm operates a MOB, or (ii) only one firm sells through the mail. The first case is identical to Salop’s circle model. If firm $i$ sets price $p_i$, and $\bar{p}$ is the price charged by the other firms, a consumer located at distance $x$ from firm $i$, with $x \in [0,1/N]$, is indifferent between buying from firm $i$ or its neighbour if $p_i + tx = \bar{p} + t(1/N - x)$. Define profits as demand at both sides times price, and firm $i$’s profit equals $\pi_i(p_i, \bar{p}) = 2xp_i = [(\bar{p} - p_i + t/N)/t]p_i$. In

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1In Thisse and Vives (1988) firms make strategic choices in terms of spatial price policy; uniform FOB pricing or discriminatory pricing. They remark: ‘[L]et us emphasize (…) that (…) uniform pricing is different from uniform delivered pricing as defined in postage stamp systems.’ I take those two variants of uniform pricing as firms’ available strategies. Other related papers are Heal (1980), Spiegel (1982), Furlong and Slotsve (1983), and Henriet and Rochet (1991).

2The model assumes that price discrimination based on the consumer’s address is illegal. This seems reasonable if the analysis concentrates on competition within one country.

3The analysis rules out that only one firm enters the market in equilibrium. See also footnote 6.
the symmetric solution every firm’s market share is $1/N$ and $p_i^* = p_j^* = t/N$ is the equilibrium price.\footnote{This analysis also assumes that the market equilibrium lies in the competitive region of firm $i$’s demand curve. That is, the reservation price $r \approx 3t/2$ (see Salop (1979) for the exposition).}

Every firm’s gross profit, expressed as a function of the number of firms $N$, equals

$$\pi_s^*(N) = \frac{t}{N^2}. \quad (1)$$

which will be referred to as the $S$-equilibrium profit. In the second case, exactly one firm operates a mail order business. The other firms are equally spaced around the circle at distance $1/(N - 1)$ from each other. Each of them faces three competitors: its two neighbours on the circle and the MOB. In between every two neighboring firms on the circle, there are two indifferent consumers. One is indifferent between firm $i$ and its neighboring firm on the circle. Given $p$ charged by this competitor on the circle, this indifferent consumer is located at $y$, where $p_i + ty = \tilde{p} + t(1/(N - 1) - y)$ if $y \leq 1/(N - 1)$. The other is indifferent between firm $i$ and the MOB. Given a price $\tilde{q}$ charged by the MOB, this indifferent consumer is located at $z$ such that $p_i + tz = \tilde{q} + \varphi$. If $y > z$, the MOB has no market share and zero profits. If $y \leq z$, the mail order business serves part of the market. Define total demand for firm $i$ on the circle by

$$D_i(p, \tilde{q}, q) = \begin{cases} 2y & \text{if } 0 \leq p_i \leq 2(\tilde{q} + \varphi) - (\tilde{p} + t/(N - 1)) \\ 2z & \text{if } 2(\tilde{q} + \varphi) - (\tilde{p} + t/(N - 1)) \leq p_i \leq \tilde{q} + \varphi \\ 0 & \text{if } \tilde{q} + \varphi \leq p_i. \end{cases} \quad (2)$$

Profits for firm $i$ on the circle are then

$$\pi_i(p, \tilde{p}, \tilde{q}) = D_i(p, \tilde{p}, \tilde{q})p_i. \quad (3)$$

Since the MOB’s location is in the center of the circle, it faces $(N - 1)$ neighbours. For a given price $\tilde{p}$, the MOB competes for the more distant consumers in between every two firms on the circle. The most distant consumer is at distance $1/(2(N - 1))$ from every firm on the circle. The consumer who is indifferent between buying from firm $i$ or from the MOB, is located at $\tilde{z}$ such that $\tilde{p}_i + \tilde{t}\tilde{z} = q + \varphi$. This expression applies for each side of all $(N - 1)$ firms on the circle. Therefore, the MOB’s total demand $D_M$ is defined as

$$D_M(\tilde{p}, q) = \begin{cases} 0 & \text{if } q \geq \tilde{p}_i + t/2(N - 1) - \varphi \\ \left(\frac{2(N - 1)}{t}\right)(\tilde{p}_i - \varphi - q + \frac{t}{2(N - 1)}) & \text{if } \tilde{p}_i - \varphi \leq q \leq \tilde{p}_i + t/2(N - 1) - \varphi \\ 1 & \text{if } q \leq \tilde{p}_i - \varphi. \end{cases} \quad (4)$$

The profit for the MOB equals

$$\pi_M(\tilde{p}, q) = D_M(\tilde{p}, q)q. \quad (5)$$

Eq. (5) is continuous and quasi-concave in $q$. Optimizing (3) with respect to $p_i$, and (5) with respect to $q$, the first-order conditions if all firms have a positive market share, are $p_i = 0.5[\tilde{q} + \varphi]$ and $q = 0.5[\tilde{p}_i - \varphi + t/2(N - 1)]$. Using the assumption of symmetry ($p_i = p_j = p$) and $\tilde{q} = q$ for the
MOB, the Nash-equilibrium of the pricing-game equals \((p^*, q^*)\) where \(p^* = \frac{2\varphi + t/(N - 1)}{6}\) and \(q^* = \frac{t/(N - 1) - \varphi}{3}\) at every store on the circle and the MOB, respectively. Substitution of these prices into (3) and (5) leads to profits expressed as a function of the number of firms \(N\):

\[
\pi^*_c(N) = \frac{1}{18t} \left( t/(N - 1) + 2\varphi \right)^2
\]

for every firm on the circle, and

\[
\pi^*_M(N) = \frac{2(N - 1)}{9t} \left( t/(N - 1) - \varphi \right)^2
\]

for the MOB. Eqs. (6) and (7) will be referred to as the \(M\)-equilibrium profits. Define

\[
h(N) = t \left( \frac{1}{N - 1} - \frac{3}{N\sqrt{2(N - 1)}} \right).
\]

\(h(N)\) is non-negative for all \(N \geq 3\); furthermore, \(h(3) = 0\), \(h(\infty) = 0\) and \(h(2) < 0\).

**Proposition 1.** (a) If \(\varphi \leq h(N)\), we have an \(M\)-equilibrium. (b) Otherwise, the \(S\)-equilibrium is the unique equilibrium in pure strategies.

**Proof.** Firm \(i\)'s profit equals \(t/N^2\) if all firms locate on the circle. If only firm \(i\) operates a MOB, its profit is \(2((N - 1)/9t)(t/(N - 1) - \varphi)^2\) by (7). Therefore, firm \(i\) finds it optimal to start up a MOB if \(t/N^2 \leq 2((N - 1)/9t)(t/(N - 1) - \varphi)^2\). This condition is equivalent to \(\varphi \leq h(N)\). From (6) all other firms on the circle have a profit of \(\pi^*_c(N) = (1/18t)(t/(N - 1) + 2\varphi)^2\). The standard Bertrand argument says that switching to the center yields zero profits. Since \(\pi^*_c(N) > 0\), these firms remain on the circle. This proves part (a). If, however, \(t/N^2 > 2((N - 1)/9t)(t/(N - 1) - \varphi)^2\), the opposite inequality holds, i.e. \(\varphi > h(N)\). Firm \(i\) locates on the circle and no other firm switches to the center. This proves part (b).

Proposition 1 tells that in any \(M\)-equilibrium the cost of sending the good through the mail should be small enough. Since \(0 \leq \varphi \leq h(N)\), the lower bound on the number of firms in an \(M\)-equilibrium is \(N \geq 3\). The intuition is that a firm has an incentive to open a MOB only if its profit as a firm on the circle is relatively small. In an \(M\)-equilibrium, the MOB foregoes some market power by a decrease in the equilibrium prices. Therefore, a single firm on the circle has sufficient incentives to become a MOB if the gain in market share is large enough. Calculations show that the MOB has a larger market compared with the \(S\)-equilibrium. Finally, each firm competes only with its two neighbours in the \(S\)-equilibrium. The cross-price elasticities are positive for neighbouring firms, but zero for all other firms. That is, there is localized competition. In the \(M\)-equilibrium, the firm in the center competes with every firm on the circle. Clearly, this generates some form of nonlocalized competition, as the cross-price elasticity \((\partial D_i/\partial q)(q/D_i)\) is positive (and identical) for all \(i\). The MOB shoulders itself in

\[\text{If } t/(4(N - 1)) > \varphi, \text{ the MOB charges a higher price than the firms on the circle. For higher values of } \varphi, \text{ lower prices result. The MOB and the firms on the circle charge always lower prices compared to the situation in which firms operate only on the circle.}\]
between every firm on the circle. The firms on the circle have only one direct competitor; the MOB. A small change in their own price, affects only the MOB’s market share. The cross-price elasticity \((\partial D_i / \partial p_j)(p_i / D_i)\) is positive (and identical) for all \(i\). The cross-price elasticity \((\partial D_i / \partial p_j)(p_i / D_i)\) equals zero for all \(j \neq i\). They are engaged in some form of localized competition. Fig. 1 shows an example with \(N = 7\). The bold lines represent the retail stores’ market share.

**Proposition 2.** Firms on the circle earn higher profits in the S-equilibrium compared to the M-equilibrium: \(\pi_S^*(N) > \pi_M^*(N)\).

**Proof.** (1) strictly exceeds (6) if and only if \(\varphi < t(3/\sqrt{2N} - 1/2(N - 1))\). Compare the right-hand side of this inequality with \(h(N)\) to see that \(t(3/\sqrt{2N} - 1/2(N - 1)) > h(N)\) if and only if \(\sqrt{2} > N/(N - 1) - \sqrt{2}/(N - 1)\). For all \(N \geq 2\), the right-hand side of the latter inequality is an increasing function. By applying l’Hôpital’s rule, it reaches its maximum of 1 for \(N \rightarrow \infty\). Since \(\varphi < h(N)\) in the M-equilibrium, the result follows. □

4. Free-entry

This section adds an entry stage to the above analysis. Thus, each firm decides first whether or not to enter the market. Having observed the number of firms entered the market, the entrants play the two-stage game of the previous section. Those who do not enter receive zero profits. Consider a fixed cost of entry \(F > 0\). Section 3 established that the S- and M-equilibrium are possible candidates satisfying the subgame perfectness condition. Our concept of free-entry equilibrium requires that entering firms earn non-negative profits, and all other firms anticipate non-positive profits when entering (see Anderson et al. (1992)). This motivates the following two definitions:

**Definition 1.** \(N_S^*\) is the number of firms in a free-entry S-equilibrium if (i) \(\pi_S(N_S^*) = F\); and (ii) \(\pi_S(N_S^*) \geq \pi_M(N_S^*)\).

Condition (i) ensures that all firms make zero profits. It implies that \(N_S^* = \sqrt{F}\), by (1). Condition
Lemma. (i) guarantees that with the equilibrium number of firms in the market, no firm wants to switch to a MOB. The condition is equivalent to $\varphi \leq h(N^*_M)$ (i.e., the condition in Proposition 2). In the free-entry $S$-equilibrium, therefore, $\varphi \geq h(\sqrt{t/F})$.

**Definition 2.** $N^*_M$ is the number of firms in a free-entry $M$-equilibrium if (i) $\pi_c(N^*_M) = F$; and (ii) $\pi_M(N^*_M) \equiv \pi_s(N^*_M)$.

The first condition ensures that all firms on the circle make zero profits. Proposition 2 established that $\pi_c(N) < \pi_M(N)$. It follows that only the firms on the circle must satisfy the zero-profit conditions for free-entry. The second condition guarantees that with the equilibrium number of firms in the market, exactly one firm wants to switch to the MOB. Define the following function:

$$g(N^*_M) = \frac{1}{2}(\sqrt{18tF} - \frac{t}{N^*_M - 1}).$$

(9)

The function $g(.)$ is increasing and, by (9), the equality $g(N^*_M) = \varphi$ represents the zero-profit condition for the firms on the circle. Condition (ii) in Definition 2 is equivalent to $\varphi \leq h(N^*_M)$. Therefore, $N^*_M$ satisfies the requirements (i) and (ii) of Definition 2 if and only if $g(N^*_M) = \varphi \leq h(N^*_M)$.

**Proposition 3.** Let $\pi_s(N^*_S) = \pi_c(N^*_M) = F$; then $N^*_S > N^*_M$. That is, if $N^*_M$ and $N^*_S$ are determined by the zero-profit condition, the number of firms in the $S$-equilibrium is higher than it would be in the $M$-equilibrium.

**Proof.** Suppose $N^*_S \leq N^*_M$. Since (6) is decreasing in $N$, $\pi^*_M(N^*_M) \leq \pi^*_c(N^*_S)$. It follows from Proposition 2 that $\pi^*_s(N^*_S) < \pi^*_s(N^*_M)$. The free-entry $S$-equilibrium requires that $\pi^*_s(N^*_S) = F$. But then $\pi^*_S(N^*_M) < F$, and $N^*_M$ cannot be the number of firms under a free-entry equilibrium. A contradiction.

Proposition 3 states that the number of firms in the free-entry $S$-equilibrium is larger than in the free-entry $M$-equilibrium. Therefore, the market with a MOB is more competitive. This accords with the result that nonlocalized competition yields fewer firms in a free-entry equilibrium than with localized competition (Deneckere and Rothschild, 1992).7 The conditions for an $S$- and $M$-equilibrium are now analyzed.

**Lemma.** (i) $h(3) = 0$ and $h(N) > 0$ for all $N > 3$; (ii) $g(3) \geq 0$ if and only if $t/F \leq 72$; (iii) $g'(N) > h'(N)$ for all $N > 3$; (iv) $g(N) > h(N)$ for all $N$ large enough.

**Proof of Lemma.** (i) $h(3) = 0$, obvious. $h(N) > 0$ for all $N > 3$ if and only if $N/(\sqrt{N} - 1) > 3\sqrt{2}$. Since $N/(\sqrt{N} - 1)$ is strictly increasing in $N$ and equals $3\sqrt{2}$ at $N = 3$, the result follows. (ii) $g(3) \geq 0$ if and only if $t/F \leq 72$. From evaluation of Eq. (9) at $N = 3$, we find that $t/F = 72$. Since $g(N)$ is strictly increasing, the result follows. (iii) $g'(N) - h'(N) > 0$ for all $N > 3$ if and only if $3t/(2(N - 1)^2) > 3\sqrt{2}(3N - 2)/(4N(N - 1)^{3/2})$. It can easily be checked that this holds for all

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7Assume that $F < t/18$, such that $N^*_M > 1$.

7See, however, Anderson et al. (1992) p. 194 for a critical assessment of this interpretation.
Compared to Salop’s model, the number of firms entering the market is lower. The MOB competes fixed cost, irrespective of the consumer’s location. With free-entry, at most one MOB pops up. entail distance related transportation costs when buying at a store. Buying from a MOB implies a fixed cost, irrespective of the consumer’s location. With free-entry, at most one MOB pops up. Compared to Salop’s model, the number of firms entering the market is lower. The MOB competes

Proposition 4. (i) $N^*_M$ is increasing in $\varphi$, and decreasing in $F$; (ii) If a free-entry $M$-equilibrium exists, then $N^*_M \geq 3$

Proof. (i) Inspection of (9) yields the comparative static results; (ii) From the Lemma, $g'(N) > 0$ and $g(N^*_M) = \varphi \equiv h(N^*_M)$ cannot be satisfied if $g(3) > 0$. Since $\varphi \equiv 0$, $N^*_M \geq 3$ if an $M$-equilibrium exists. □

An increase in $\varphi$ implies more friction in the market and prevents the MOB from decreasing the prices drastically. Therefore, more firms can enter the market.

Proposition 5. (i) Let $F \leq t/72$; then there exists a $\varphi > 0$, such that an $M$-equilibrium with free-entry exists if and only if $0 \leq \varphi \leq \varphi$. (ii) If $F > t/72$, free-entry does not result in an $M$-equilibrium.

Proof. (i) By the Lemma, there exists a $\bar{N}$ such that $h(\bar{N}) = g(\bar{N}) = \bar{\varphi}$. Since $g(3) \leq 0$ and $g'(N) > h'(N)$ for all $N > 3$ with $g'(N) > 0$, for $\varphi \leq \bar{\varphi}$ there is a unique $\bar{N}$ such that $g(\bar{N}) = \varphi = h(\bar{N})$; (ii) Since $h(3) = 0$ and $g'(N) > h'(N)$ for all $N \geq 3$, the condition for a free-entry $M$-equilibrium $0 \leq g(N) = \varphi \leq h(N)$ (as stated in Definition 2) can never be satisfied. □

If the fixed set-up cost is too large compared to the marginal cost of transportation, the zero-profit condition for firms on the circle cannot be satisfied.

Proposition 6. Let $N^*_S$ and $N^*_M$ satisfy the zero-profit conditions of the free-entry equilibrium. (a) Let $h(N^*_M) < h(N^*_S)$. Then, (i) the $S$-equilibrium is unique if $h(N^*_S) \leq \varphi$; (ii) if $\varphi \leq h(N^*_M)$, the $M$-equilibrium is unique; (iii) if $h(N^*_S) < \varphi < h(N^*_M)$, no pure strategy equilibrium exists. (b) If $h(N^*_S) \leq h(N^*_M)$, then for all (i) $\varphi < h(N^*_S)$, the $M$-equilibrium is unique; (ii) $\varphi > h(N^*_M)$, the $S$-equilibrium is unique; (iii) $h(N^*_S) \leq \varphi \leq h(N^*_M)$ both the $S$-equilibrium and the $M$-equilibrium coexist.

Proof. (a) (i) from Definition 1, a free-entry $S$-equilibrium exists, since $\varphi \equiv h(N^*_S)$ holds, while condition (ii) of Definition 2 is violated; (ii) Similarly, no free-entry $S$-equilibrium exists, since condition (ii) of Definition 1 is violated, while Definition 2 holds; (iii) In the same fashion, both conditions for the free-entry $S$- and $M$-equilibrium are violated if $h(N^*_M) < \varphi < h(N^*_S)$. (b) can be proven in a similar fashion. □

5. Conclusion

This paper uses Salop’s circle model where firms either operate a store or a MOB. Consumers entail distance related transportation costs when buying at a store. Buying from a MOB implies a fixed cost, irrespective of the consumer’s location. With free-entry, at most one MOB pops up. Compared to Salop’s model, the number of firms entering the market is lower. The MOB competes
with every firm on the circle, and therefore engages in non-localized competition. The stores on the circle face only one local competitor – the MOB – and are engaged in localized competition.

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**References**