Leasing versus selling and firm efficiency in oligopoly

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Abstract

We examine sales and leasing of a durable good in an asymmetric duopoly. We show that the inefficient firm leases more than the efficient firm, and that an increase in unit costs implies a higher ratio of leased units to sales. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

Leasing has been viewed as a possible solution to the “durable goods monopoly” problem, since it allows the monopolist to maintain ownership of the units, making the “promise” to not overproduce in future periods credible (see Bulow, 1982).\textsuperscript{1} In reality, durable goods markets are primarily oligopolistic rather than monopolistic, and firms sell as well as lease goods.\textsuperscript{2} This note focuses on strategic considerations that determine the choice between leasing and selling. That this choice has strategic implications is already known from Bucovetsky and Chilton (1986) and Bulow (1986). They show that a monopolist facing the threat of entry chooses to sell part of the units supplied (or,

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\textsuperscript{1}It is well known that a durable good monopolist faces a time inconsistency problem which leads to lower profit relative to the profit the monopolist can earn by committing to selling the good only once (see e.g. Tirole (1988), pp. 79–87 and references therein).

\textsuperscript{2}Examples include automobiles, house appliances, computers, copy machines, and machinery equipment.
equivalently, increase the durability of its product). Bulow (1986) also shows that in a two-period oligopoly, firms use a mixture of sales and leasing (in the first period). We further explore these ideas in a dynamic asymmetric duopoly. By allowing costs of production to vary across firms, we offer new insights with respect to the strategic implications of leasing and selling.

We study a three-period quantity-setting duopoly. Each firm chooses its volume of leasing and sales in each period. By keeping the model as simple as possible, we obtain analytic solutions. The main implication is that inefficient (high-cost) firms tend to lease more. In particular, while the low-cost firm sells more than the high-cost firm in every period, the high-cost firm leases more (in each of the first two periods). Further, an increase in the unit cost of a given firm (holding constant the other firm’s cost) implies a higher ratio of leased units to sales. Similarly, when the costs are equal, this ratio increases with cost. We abstract from other factors that may influence the lease versus sell choice, such as informational asymmetries, to show how strategic incentives may differ solely because of cost differences. Based on our analysis, an empirical investigation of whether high-cost firms lease more may help determine whether the lease versus sell choice is driven by strategic considerations, or is primarily influenced by other factors.

While leasing helps solve the durable goods problem, in the presence of competition leasing becomes less attractive. Sales allow a firm to capture part of the market whereas leasing may result in loss of future market share to a rival since both firms can compete anew for consumers that leased in the past. Also note that future profit levels are affected not only by current sales (because demand is lower) but also by current leasing volumes because units leased in the past return to the firm and can be supplied again at no additional cost. In other words, in each period the marginal cost of supplying units previously leased is zero. With respect to cost differences, in a quantity-setting dynamic game with only sales, the low-cost firm sells more. When both leasing and sales contracts are possible, we find that while the low-cost firm still sells more, it is the high-cost firm that leases more. The intuition is that the high-cost firm has a strategic disadvantage in competing directly with its low-cost rival and leasing allows it to capture part of the demand in a more indirect way. For comparison purposes, the paper also considers the case where costs decrease significantly over time (as with new products). We discuss why in this case the incentives are reversed and it is the efficient firm that leases more.

2. The model

The market lasts for three periods. There is a durable good which is perfectly homogeneous. In each of the three periods, the demand for the services of the good is

\[ p_t = a - bQ_t, \]  

Equation 1

More precisely, in a two-period model where Cournot competition takes place in the second period, a monopolist in the first period sells the good in order to capture some of the second-period demand.

Three periods is the minimum length needed to explore the dynamics. The analysis of a two-period model leads to similar conclusions but is restrictive in the sense that it does not allow us to consider a period of leasing followed by another period where firms face a lease versus sell decision.

In this sense, our analysis is linked to the literature on capacity expansion (see e.g. Dixit, 1980). Here we examine a durable good and in every period firms decide how much to produce, as well as the volumes of sales and leasing.
In every period, the two firms choose their sales and leasing volumes. The units sold by firm $i$ in period $t$ are denoted $s_i^t$ and the units leased are denoted $l_i^t$, where $i = A, B$, $t = 1, 2, 3$. In the last period, leasing and selling are equivalent and we write $l_3^t = 0$. The good is assumed perfectly durable: it does not depreciate, so that consumers view units previously sold and units just produced as identical. To calculate the total production cost of firm $i$ in period $t$ (which we denote $TC_i^t$) the unit cost $c_i$ has to be multiplied by the number of units produced in that period. The units produced in period $t$ equal the difference between the units supplied (sold or leased) in that period and the units leased in the previous period, assuming that this difference is non-negative. To keep the analysis simple, we focus on the case where this difference is non-negative in every period. Direct calculations show that this condition is guaranteed if demand, as parameterized by $a$, is high enough. From the leasing prices, $p_i$, we can derive the sale prices, which we denote $\hat{p}_i$. It is easy to see that we should have

$$\hat{p}_1 = p_1 + \delta \hat{p}_2 \quad \text{and} \quad \hat{p}_2 = p_2 + \delta \hat{p}_3.$$  \hspace{1cm} (2)

These conditions reflect no-arbitrage requirements. We assume that firms maximize the present value of their profits. In period $t$ each firm $i$ chooses $s_i^t$ and $l_i^t$ taking the other firm’s choices as given. We look for a subgame perfect equilibrium.

### 3. Analysis

The usual parameter restrictions that guarantee smoothness of solutions in Cournot games are assumed to hold. We start from the last period, $t = 3$. Firm $i$, $i = A, B$, solves

$$\max_{s_3^t} p_3 s_3^t - TC_3^t$$  \hspace{1cm} (3)

where

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* Consumers have preferences over the services of the good and the fundamental relation is the flow demand in each period, while the demand for purchasing the good can be derived. Note that the residual demand curve in every period (the portion of the demand curve to the right of the current stock of units sold) captures the demand for the services of the good by consumers who have not yet purchased the good.

* Also, from the viewpoint of firms, a unit previously leased can be brought to the market without incurring any additional cost.

* This is because, if demand is not too low, firms find it profitable to supply additional units to the market in each period. If the difference mentioned above is negative, the analysis needs to be modified in a straightforward way.

* A consumer should be indifferent, for example, between paying $p_2$ to lease the good for the second period, and paying $\hat{p}_2$ to purchase the good at the beginning of the second period and selling it for $\hat{p}_3$ at the end of the second period.
$p_3 = \hat{p}_3 = a - b \left( \sum_{j=1}^{2} \sum_{k=A,B} s^j_k + s^A_3 + s^B_3 \right)$ (4)

and

$TC^i_3 = c'(s^i_3 - l^i_2)$. (5)

As discussed above, this expression for $TC^i_3$ is valid when $s^i_3 \geq l^i_2$ and $s^i_2 + l^i_2 \geq l^i_1$, which hold in equilibrium for $a$ high enough. In the third period, we derive each firm’s sales as a function of the history. For compactness, let $h_t$ denote the history of the game up to the beginning of period $t$. So $h_3$ represents $(s^i_1, l^i_1)$, $i = A, B$, $t = 1,2$. Then the equilibrium in the third period generates $s^i_3(h_3)$, $i = A, B$ from which we can also calculate the price $\hat{p}^*_3(h_3)$ as well as $II^*_3(h_3)$, the third-period profit for firm $i$.

At $t = 2$, taking as given the history of selling and leasing in period one, firm $i$ solves

$$\max_{s^i_2, l^i_2} p^i_2 l^i_2 + \hat{p}_2 s^i_2 - TC^i_2 + \delta II^*_3(h_3)$$ (6)

where

$$p^i_2 = a - b \left( \sum_{k=A,B} s^k_1 + s^A_2 + s^B_2 + l^A_2 + l^B_2 \right), \quad \hat{p}_2 = p^i_2 + \delta \hat{p}^*_3(h_3),$$ (7)

and

$$TC^i_2 = c'(s^i_2 + l^i_2 - l^i_1).$$ (8)

Note that now each firm chooses two variables, one for leasing and one for sales. Again we derive $s^i_2(h_2), l^i_2(h_2), \hat{p}^*_2(h_2)$ and $II^*_2(h_2)$.

Finally, at $t = 1$, firm $i$ solves

$$\max_{s^i_1, l^i_1} p^i_1 l^i_1 + \hat{p}_1 s^i_1 - TC^i_1 + \delta II^*_2(h_2)$$ (9)

where

$$p^i_1 = a - b(s^A_1 + s^B_1 + l^A_1 + l^B_1), \quad \hat{p}_1 = p^i_1 + \delta \hat{p}^*_2(h_2),$$ (10)

and

$$TC^i_1 = c'(s^i_1 + l^i_1).$$ (11)

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10 See the discussion below for precise parameter restrictions. If written explicitly over its entire domain, the cost function for firm $i$ would have a “kink” at $l^i_1$. The marginal cost is equal to zero for supplying each of the first $l^i_2$ units and equal to $c'$ for any additional units.

11 Again, we assume that $s^i_1 + l^i_2 \geq l^i_1$.

12 $II^*_2$ represents the equilibrium profit for the last two periods, since it is defined as the value of the problem of maximizing profit from period two onwards.
Once we derive the equilibrium levels $s_i^*$ and $l_i^*$, we proceed to the appropriate substitutions and obtain the equilibrium path of leasing and sales. Clearly, there is a unique subgame perfect equilibrium. As these calculations are standard, we only report here the results:

$$s_i^A = \frac{c^b(250 + 35\delta - 39\delta^2) - c^a(375 + 30\delta - 87\delta^2) + (125 + 120\delta + 27\delta^2)a}{b(125 + 63\delta)(5 + 3\delta)},$$

$$l_1^A = -\frac{c^b(125 + 40\delta + 384\delta^2 + 189\delta^3) - c^a(125 + 40\delta - 741\delta^2 - 378\delta^3) + (250 + 330\delta + 108\delta^2)a}{3b(125 + 63\delta)(5 + 3\delta)}$$

$$s_2^A = \frac{c^b(55 + 22\delta) - c^a(70 + 41\delta) + (15 + 9\delta)a}{b(125 + 63\delta)}$$

$$l_2^A = -\frac{c^b(5 + 48\delta + 21\delta^2) + c^a(-5 + 77\delta + 42\delta^2) + (10 + 6\delta)a}{b(125 + 63\delta)}.$$

Clearly, the expressions for the quantities supplied by firm $B$ can be found by exchanging $c^A$ and $c^B$.

4. Main results

Our first result is that while $A$, the efficient firm, sells more in every period, $B$, the inefficient firm, leases more (in each of the two first periods):\textsuperscript{14}

**Proposition 1.** If $c^A < c^B$, in equilibrium we have $s_i^A > s_i^B$, $t = 1, 2, 3$ and $l_i^A < l_i^B$, $t = 1, 2$.

**Proof.** This can be shown using the expressions for the leasing and sales levels derived above. Direct calculations show that $l_1^A - l_1^B = 3(c^A - c^B)\delta^2/b(3\delta + 5) < 0$, $l_2^A - l_2^B = (c^A - c^B)\delta/b < 0$, $s_1^A - s_1^B = (2\delta - 5)(c^A - c^B)/b(3\delta + 5) > 0$, and $s_2^A - s_2^B = s_1^A - s_1^B = (c^B - c^A)/b > 0$. \hfill $\Box$

Note that, if a firm has such a great cost advantage that it is de facto a monopolist, we know from past work that it would choose to only lease. When the cost difference, however, is not too great, so that strategic considerations are important, our analysis shows that the less efficient firm is expected to lease more.\textsuperscript{15} In addition to Proposition 1, there is another sense in which “inefficient firms lease more.” As the unit production cost of a given firm increases, its ratio of leasing to sales increases:

\textsuperscript{13}The fact that $s_i^A = s_i^A$ comes from the linearity of the problem in the last two periods.

\textsuperscript{14}The aggregate quantity supplied, including both leasing and sales, is higher for the efficient firm. This can be shown with direct calculations: $(s_1^A + l_1^A) - (s_1^B + l_1^B) = (s_1^A + l_1^A) - (s_1^B + l_1^B) = (\delta - 1)(c^A - c^B)(3\delta^2 + 2\delta - 5)/b > 0$, since the quadratic term is negative for $\delta \in (0, 1)$.

\textsuperscript{15}We can also check how sales and leasing evolve over time for the same firm. Direct calculations show that $s'_i > s_i$ and $l'_i > l_i$ as long as $a$ is not too low.
Proposition 2. In equilibrium, an increase in $c^A$ (while keeping $c^B$ fixed) or a decrease in $c^B$ (while keeping $c^A$ fixed), leads to a decrease in $s^A_t$, $t = 1, 2, 3$, to an increase in $l^A_t$, $t = 1, 2$, (as long as $\delta$ is not too low) and to an increase in the $(l^A_t/s^A_t)$ ratios, $t = 1, 2$.

Proof. From (12) and (14) it is easy to see that an increase in $c^A$ (or a decrease in $c^B$) leads to a decrease in $s^A_t$ for $t = 1, 2, 3$. With respect to leasing, simple calculations involving the coefficients of the relevant parameters show the following. An increase in $c^A$ increases $l^A_1$ at least as long as $\delta > 0.4$, and a decrease in $c^B$ increases $l^A_1$ for all $\delta$. An increase in $c^A$ leads to a higher $l^B_2$ at least as long as $\delta > 0.1$ and a decrease in $c^B$ increases $l^A_2$ for all $\delta$. Similarly, it is possible to show that an increase in $c^A$ or a decrease in $c^B$ always leads to a higher ratio $l^A_t/s^A_t$, $t = 1, 2$. □

A related question of interest is what happens when the two firms are equally efficient and their unit cost increases.

Proposition 3. Suppose that $c^A = c^B = c$. Then a higher $c$ leads to higher $(l_t/s_t)$ ratios, $t = 1, 2$.

Proof. Substituting $c^A = c^B = c$ into (12)–(15) and collecting common factors gives the following leasing to sales ratio for each of the two firms: $l_t/s_t = [c(-50+14\delta + 63\delta^2) + (50 + 36\delta)a]/[3(c(-25 + 16\delta) + (25 + 9\delta)a)]$ and $l_t/s_t = [c(-10 + 29\delta + 21\delta^2) + (10 + 6\delta)a]/[c(-15 - 19\delta) + (15 + 9\delta)a]$. Differentiation with respect to $c$ delivers the results. □

Since we have derived explicit expressions, we can easily calculate numerical examples. For illustration, suppose that $a = 80$, $b = 2$, $c^A = 7$, $c^B = 8$, and $\delta = 0.8$. Then, we find $s^{A*}_1 = 7.20$, $s^{A*}_2 = 4.67$, $l^{A*}_1 = 5.94$ and $l^{A*}_2 = 3.74$. Similarly, for the inefficient firm $s^{B*}_1 = 6.97$, $s^{B*}_2 = 4.17$, $l^{B*}_1 = 6.07$ and $l^{B*}_2 = 4.14$. Using these sales and leasing volumes, we calculate that firm $A$'s total profit approximately equals 579 while that of firm $B$ equals 552. It is useful to compare these numbers with the equilibrium levels of sales when leasing is not possible. In other words, we solve the model again when the two firms compete only in sales in every period. With sales only, in equilibrium, firm $A$ sells 11.22, 4.48, and 2.02 in periods one, two, and three respectively, while the levels for firm $B$ are 11.00, 4.25, and 1.52. Total profit in this case is 366 for firm $A$ and 340 for firm $B$. Additional examples suggest that this is a typical pattern. In particular, the ability to lease increases the equilibrium profit of both firms and, as illustrated by a comparison of the ratio of profits, it is the high cost firm that benefits relatively more from the ability to lease. This is consistent with our result that the high cost firm chooses to lease relatively more.\textsuperscript{16}

5. Discussion and conclusion

The analysis is presented under the assumption that there is (positive) production in every period. As mentioned above this assumption holds for $a$ high enough. Let us now examine more carefully this

\textsuperscript{16}As these observations are only based on numerical results, they are only suggestive. Analytic expressions at this point become too complicated to allow us to derive a general result.
parameter restriction. Four inequalities need to be satisfied, \( s_2^i + l_2^i \geq l_1^i \) and \( s_1^i \geq l_2^i \) for \( i = A, B \). Using the equilibrium expressions from the above analysis, it can be shown that both the first and the second constraints are more binding for the inefficient firm than for the efficient firm and, furthermore, that the second constraint is more binding than the first.\(^{17}\) It is then easy to derive an explicit lower bound for the demand intercept: all four of the required inequalities are satisfied if and only if \( a \) is not lower than \([c^b(42\delta^2 + 118\delta + 65) - c^i(21\delta^2 + 70\delta + 60)]/(5 + 3\delta)\). We can further show that this parameter restriction is very mild when the cost asymmetry is small, and that it becomes more severe the larger the asymmetry between the firms and the larger the discount factor. Of course, as \( a \) becomes low (below the critical value given above) and when the cost asymmetry is large, the inefficient firm’s total supply (that is, sales) in period three becomes low. In this case, the inefficient firm’s period three sales volume may be lower than its leasing volume in period two. At the margin, this would create an incentive to decrease the leasing volume in period two (given that it is no longer true that all the leased units can be brought again to the market after period two) and part of this decrease would be translated as an increase in sales. However, we expect our main results to remain qualitatively valid at least for intermediate values of the parameters.

Since cost comparisons are central in our analysis, it is also important to check that the results do not change qualitatively with alternative specifications of the production technology. In particular, we can show that the results remain valid under an alternative formulation (Bulow, 1986) where the present value of unit costs is proportional to the number of periods the product is being used in: \( c_i^1 = 3c^i, c_i^2 = 2c^i/\delta, \) and \( c_i^3 = c^i/\delta^2 \), \( i = A, B \).\(^{18}\)

Finally, we consider markets where the unit production cost decreases significantly over time. The above analysis is relevant for “mature” industries with costs that are roughly constant. In other markets, there are substantive cost decreases over time, especially when introducing new products. To investigate the implications for our problem, we examine a simple example where for every firm and in every period the unit cost is half of its previous period level: \( c_i^1 = 4c^i, c_i^2 = 2c^i, \) and \( c_i^3 = c^i \), \( i = A, B \). Then it can be shown that the low-cost firm sells more as well as leases more in every period. In addition, the low-cost firm leases relatively more in the first period \( (s_1^A - l_1^A < s_1^B - l_1^B) \) but relatively less in the second period \( (s_2^A - l_2^A > s_2^B - l_2^B) \).\(^{19}\) The intuition is as follows. When costs decrease over time, producing in early periods is less attractive relative to later periods. While both firms prefer to sell a relatively higher volume in later periods (compared to the constant cost case), the efficient firm has a greater incentive to “preserve” the market, because it will have stronger strategic position and

\(^{17}\)This is not surprising since residual demand and thus firms’ supply levels are lower from period to period and because firm \( B \) sells less but leases more than firm \( A \). We require demand to be large enough that firm \( B \)’ total supply in period three is not less than that firm’s leasing volume in period two.

\(^{18}\)In this case we find \( l_1^A - l_1^B = (3\delta + 8)(1 - \delta)(c^A - c^B)/\delta b(3\delta + 5) < 0, l_2^A - l_2^B = (1 - \delta)(c^A - c^B)/\delta^2 b < 0, s_2^A - s_2^B = 8(c^A - c^B)/\delta b(3\delta + 5) > 0, \) and \( s_1^A - s_1^B = (c^A - c^B)/\delta^2 b > 0 \). Note that the cost formulation in the main analysis follows Bucovetsky and Chilton (1986). In reality, costs may be in between the two bounds considered here. The continuity of the solutions implies that our qualitative results hold for such other in-between cases and are not crucially dependent on how this aspect of the game is formulated.

\(^{19}\)We have \( l_1^A - l_1^B = 2(\delta + 1)(3\delta - 5)(c^A - c^B)/b(5 + 3\delta) > 0, l_2^A - l_2^B = (\delta - 1)(c^A - c^B)/b > 0, s_2^A - s_2^B = (2\delta - 10)(c^A - c^B)/b(5 + 3\delta) > 0, \) and \( s_1^A - s_1^B = (c^A - c^B)/b > 0 \). Also \( (s_2^A - l_2^A) - (s_2^B - l_2^B) = 6\delta(1 - \delta)(c^A - c^B)/b(5 + 3\delta) < 0 \) and \( (s_2^A - l_2^A) - (s_2^B - l_2^B) = -\delta(c^A - c^B)/b > 0. \)
thus enjoy greater profit. Due to its greater concern to preserve the market, the efficient firm sells relatively less early on.20

The main result in this paper is that, when the choice between sales and leasing is driven primarily by strategic considerations, we should expect inefficient firms to lease more (while efficient firms sell more). Of course, additional factors may also affect the choice between leasing and sales. Also, our results have been obtained within a parametric (although standard) formulation. However, the simplicity of the model allows us to identify some key implications of strategic behavior in this context.

References


20Continuity implies that, as we vary the rate at which costs decrease over time, there is a critical rate below which the inefficient firm leases more whereas above which the pattern is reversed.