A currency crisis model with a misaligned central parity: a stochastic analysis

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Abstract

The currency crisis model outlined in this paper assumes state-contingent reserve dynamics which depend on the deviations of the exchange rate from a misaligned central parity. The size of the misalignment is affected by an underlined fundamental which embodies money aggregates and a stochastic shock. The main result, in the presence of an irreversible switch of regime, is that the size of the attack is amplified by the feedback effect from the composite fundamental to the exchange rate. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

The collapse of the Exchange Rate Mechanism (ERM) in 1992–1993 and the recent crisis episodes in South-East Asia have drawn widespread attention to speculative attacks on pegged exchange rate regimes. To improve our understanding of these events, researchers have undertaken new theoretical and empirical studies which are outlined in a recent paper by Flood and Marion (1998). There is an important intergenerational link between the currency crises literature but still some crises may be particularly difficult to predict using currently popular methods.

The first generation models of Currency Crisis (CC) emphasize the role played by limited reserves in defining the switch from a fixed to a floating rate system (Krugman, 1979). They also predict the timing and the probability of speculative attacks endogenously and forecast lower bounds for the post-collapse exchange rates in a stochastic framework (Flood and Garber, 1983). This research area has been extended to deal with a variety of issues, including alternative post-collapse regimes...
(Dornbusch, 1988), uncertainty about domestic credit growth (Flood and Garber, 1984), imperfect asset substitution and sticky prices (Willman, 1988). The so called second-generation models are generally based on optimizing policy makers; they have been developed mainly to explain the 1992–1993 crises of the ERM (Obstfeld, 1994; Ozkan and Sutherland, 1998). Other recent works (Bensaid and Jeanne, 1997) focus on the vicious circle which arises when a government tries to defend its currency by raising the nominal interest rate in a fixed exchange rate system. They present a stylized model where raising the nominal interest rate helps to maintain the parity, but it is costly for the government. The speculators are aware that this cost provides incentive for the government to stop defending the parity, which in turn reinforces the speculation against the currency.

The present paper offers an explanation for the ERM crises in the light of first-generation models of currency crises modified to take into account the role of a misaligned central parity. It stresses the important role of speculators and it also recognizes that the government’s commitment to a fixed exchange rate is constrained by the dynamics of the underlined fundamentals.

2. A currency crisis model with endogenous fundamentals

The model assumes the existence of a symmetric band for the nominal exchange rate with respect to the central parity. This captures the main feature of the ERM which had the aim of limiting fluctuations of the exchange rate inside a relatively small region.¹

The reserve-dynamics are state-contingent and are an increasing function of the deviation of the nominal exchange rate with respect to the central parity. Speculators are aware that the reserves used to defend the parity are limited and they eventually bet that the authorities will abandon the fixed exchange rate regime.

Agents have perfect foresight and the assets available to domestic residents are domestic money, domestic bonds and foreign money. The monetary authorities have a stock of official reserves (the foreign currency) to defend the regime; it is assumed that the Central Bank (CB) will defend the band by reducing the stock of international reserves.

The analysis can be considered as a generalization of Krugman’s approach (1991). The main difference is that the latter assumes a recursive structure for the exchange rate dynamics (the exchange rate is affected by the expected future fundamentals but does not itself affect the evolution of the fundamentals) while this paper takes into account the possibility that the order of magnitude of the collapse of the ERM can be explained by assuming an additional ‘feedback’ between the exchange rate and the fundamental.

The resulting system with saddle path dynamics is examined between realignments; the central parity is assumed to be fixed and the model analyses the dynamics of the exchange rate and the fundamental within the exchange rate band.

2.1. The model between realignments

Assuming away the foreign exchange risk premium the domestic nominal interest rate must satisfy the following condition:

¹Many countries declared a central parity for their own currency and then prevented the exchange rate from deviating by more than ±2.25% with respect to the central parity.
where the foreign interest rate $i^*$ is normalized to zero and $[(E \frac{ds(t)}{d(t)})]$ is the expected change of the nominal exchange rate with respect to time $t$. The nominal exchange rate can be expressed as follows:

$$s_t = \varphi_t + c_t$$

Eq. (2) defines the normalized exchange rate $\varphi_t$ as the (log) deviation of the nominal exchange rate $s_t$ from the central parity $c_t$. It simply states that the position of the exchange rate within the zone at time $t$ can be defined in terms of its central parity $c_k(t)$, where $k$ is the number of realignments that have occurred since time zero. The size of a realignment at $t$ can be written in terms of the change in the central parity, $c_{k+1}(t) - c_k(t)$, where $c_{k+1}(t)$ is the value of the central parity after the realignment. While a jump can occur in the log-deviation of the exchange rate with respect to the central parity, this is not the case for the nominal exchange rate; thus $s(t) = s(t')$ and $\varphi(t) = \varphi(t') = c(t) - c(t')$, where the index $k$ is omitted for convenience. This implies that when a realignment occurs there is a discrete jump in the central parity that affects the deviation of the nominal exchange rate with respect to the central parity. The interest parity condition can be defined in the following way:

$$i_t dt = E[d\varphi + dc_t] = E[d\varphi] + E[dc_t]$$

As long as there is not a constant probability of a jump at every moment of time, it is possible to examine the dynamics between realignments. In this case the second term on the right hand side of Eq. (3), $E[dc_t]$, is zero.

To derive the stochastic system it is necessary to define a composite fundamental. Note that differently from the traditional target zone literature the fundamental incorporates not only the domestic credit but also the endogenous reserves and an index of uncertainty coming form the status of the whole economy.

The money demand is defined as follows:

$$m^d_t = -\delta i_t + p_t + \sigma B_t$$

where $\delta > 0$. Eq. (4) expresses the demand for real money balances as a function of the nominal interest rate $i_t$, of the price level $p_t$ and of a stochastic component which follows a Wiener process, $\sigma B_t$.

From Eq. (4) it is evident that the interest rate is no longer deterministic since it is affected by the stochastic shock in the money demand equation:

$$i_t = \frac{p_t + \sigma B_t - m^d_t}{\delta}$$

The money supply equation is given by:

$$m^*_t = \lambda(R_t + s_t) + (1 - \lambda)D_t$$

where $0 < \lambda < 1$. The money supply depends on the level of the foreign assets of the Central Bank.
and on the level of domestic credit; \( \lambda \) represents the fraction of CB assets held in form of foreign reserves.

The price level is determined according to the following modified Purchasing Power Parity (PPP) condition:

\[
p_r = p_t^* + \theta (e_t + c_t) + (1 - \theta)c_t = p_t^* + c_t + \theta (s_t - c_t)
\]

(7)

where the foreign price level \( p_t^* \) is normalized to 0 and \( 0 < \theta < 1 \). The domestic price level depends on the foreign level \( p_t^* \), on the central parity (where the PPP is placed) and on a fraction \( \theta \) of the deviation of the nominal exchange rate from the central parity. Thus, a positive gap between \( s_t \) and \( c_t \) will increase the price level.

The following policy instruments are given:

\[
dR_t = - \gamma (s_t - c_t) \, dt
\]

(8)

\[
dD_t = \mu \, dt
\]

(9)

Eq. (8) gives the dynamics of the reserves (where \( \gamma > 0 \)). The left-hand side of Eq. (8) defines the reaction function of the monetary authorities (Flood et al., 1998); they sell reserves to defend the band when the exchange rate depreciates. As the nominal exchange rate gets weaker with respect to the central parity the monetary authorities are forced to intervene more frequently but such interventions in presence of limited reserves speed up the crisis. Note that the right hand side of this equation defines the private agents’ exit from the domestic assets to the foreign currency in the presence of an increase in the gap between the nominal exchange rate and the misaligned central parity. Finally Eq. (9) specifies the dynamics of the domestic credit in the form of a smooth time trend \( \mu \).

In equilibrium \( m_t^d = m_t^* \). By substituting Eqs. (2), (6) and (7) in (5):

\[
i_t = \frac{(\theta - \lambda)e_t + (1 - \lambda)c_t}{\delta} - \frac{\lambda R_t + (1 - \lambda)D_t - \sigma B_t}{\delta}
\]

(10)

Let \( G_t \) be the composite fundamental where:

\[
G_t = \frac{\lambda R_t + (1 - \lambda)D_t - \sigma B_t}{\delta}
\]

(11)

Thus:

\[
dG_t = \frac{1}{\delta} \{ \lambda \, dR_t + (1 - \lambda) \, dD_t - \sigma \, dB_t \}
\]

(12)

The substitution in (12) of the policy instruments (8) and (9) gives the dynamics of the composite fundamental.

The analysis is related to the saddle point systems (Miller and Weller, 1995), whose basic feature is the presence of a linear stochastic differential equation expressing the evolution of the fundamental as the weighted sum of four components — a constant, a Wiener process, the current value of the composite fundamental and an asset price that in this framework is represented by the exchange rate. Formally:
\[ i_t = \frac{E \, d \varphi(t)}{dt} = \frac{(\theta - \lambda) \varphi_t + (1 - \lambda) c_t - G_t}{\delta} \]  
\[ dG_t = \frac{1}{\delta} \left\{ -\lambda \gamma (s_t - c_t) dt + (1 - \lambda) \mu dt - \sigma dB_t \right\} \]  
\[ G_t = \frac{\lambda R_t + (1 - \lambda) D_t - \sigma B_t}{\delta} \]  

where Eq. (13) is the arbitrage equation based on the assumption that the exchange rate is the rational expectation forecast of the future fundamentals and Eq. (14) defines the evolution of the fundamental itself. Eq. (15) defines the composite fundamental as a function of the money demand net of the stochastic shock. The differentiation of (15) with respect to \( t \) gives Eq. (14). The variables and operators are defined as follows: \( G_t \), the composite fundamental; \( s_t \), the nominal exchange rate (domestic currency per unit of foreign currency); \( dB_t \), increment of a standard Wiener process with mean zero and unit variance; \( E_t \), an expectation operator conditional on information at time \( t \).

The evolution of the composite fundamental and the expected evolution of the asset price between realignments can be summarized as follows (the time \( t \) indices are omitted for convenience):

\[
\begin{bmatrix}
  dG \\
  E(d\varphi)
\end{bmatrix} = \begin{bmatrix}
  0 & -\frac{\lambda \gamma}{\delta} \\
  -1 & \frac{(\theta - \lambda)}{\delta}
\end{bmatrix} \begin{bmatrix}
  G dt \\
  \varphi dt
\end{bmatrix} + \begin{bmatrix}
  \frac{(1 - \lambda)}{\delta} \mu \\
  \frac{(1 - \lambda)}{\delta} c
\end{bmatrix} dt + \begin{bmatrix}
  \frac{\sigma}{\delta} dB \\
  0
\end{bmatrix}
\]

(16)

The target zone solution method assumes a mapping between the exchange rate and the composite fundamental. Thus, to derive the general solution it is necessary first to postulate a deterministic functional relationship between \( \varphi \) and \( G \):

\[ \varphi = f(G) \]  

(17)

If the previous function is twice continuous differentiable by applying Ito’s lemma it follows that:

\[ d\varphi = f'(G) \, dG + \frac{1}{2} f''(G) (dG)^2 \]  

(18)

Since \( (dG)^2 = (\sigma^2/\delta^2) \, dt \) the previous expression can be written as:

\[ d\varphi = f'(G) \, dG + \frac{1}{2} \frac{\sigma^2}{\delta^2} f''(G) \, dt \]  

(19)

By taking expectations and substituting for \( E[dG] \):

\[ E \, d\varphi = \frac{1}{\delta} \left\{ -\lambda \gamma (s - c) + (1 - \lambda) \mu \right\} f'(G) \, dt + \frac{1}{2} \frac{\sigma^2}{\delta^2} f''(G) \, dt \]  

(20)

It is also valid the following relationship:

\[ E \, d\varphi = \frac{(\theta - \lambda) \varphi + (1 - \lambda) c}{\delta} - G \]  

(21)
The fundamental differential equation follows by equating Eqs. (20) and (21) and considering that \( \varphi = f(G) \):
\[
\frac{1}{2} \frac{\sigma^2}{\delta^2} \frac{f''(G)}{dt} + \frac{1}{\delta} [ -\lambda \gamma f(G) + (1 - \lambda) \mu ] f'(G) dt - \frac{(\theta - \lambda) f(G) + (1 - \lambda) c}{\delta} + G = 0
\]
(22)

Eq. (22) is a second-order non-linear equation which does not in general admit closed form solutions.

3. Stochastic solution and irreversible float

The parameters and starting values for the numerical solution of the stochastic differential equation as calculated in Eq. (22) are reported in Table 1.

By fixing an interval for the composite fundamental \( G \) and imposing suitable boundary conditions, it is possible to define the target zone solution for the exchange rate as an \( S \)-shaped curve which is tangent to the top and bottom of the band.

In Fig. 1 the log-deviation of the nominal exchange rate with respect to the central parity is plotted against the composite fundamental as defined in Eq. (15). The line \( SS' \) shows the behavior of the exchange rate in the absence of stochastic shocks. If there is a target zone band of \( \pm 2.25\% \) the solution will take the usual \( S \)-shaped curve labelled \( AA' \).

Even if the money stock is assumed to be held fixed as long as the nominal exchange rate stays strictly within the interval \( [\varphi(G), \varphi(G)] \), the money demand changes driving the exchange rate towards the edges of the band. The change in money demand is due to the increase in the domestic credit, to the reserve dynamics and to the level of uncertainty on the status of the economy embodied in the stochastic shock. The arbitrage activity increases the interest differential and weakens the exchange rate, driving it towards the edges of the band. At this point the authorities decrease the reserve level by just enough to prevent the exchange rate from leaving the band; such interventions are state-contingent since they are driven by the deviation of the exchange rate with respect to the central parity. As a consequence of these random shocks over time, the money supply shifts to the right. When reserves run out the stabilizing effect, \( KA \), that the target zone has on the fundamental ends, as it does the feedback effect from the exchange rate to the fundamental. The exchange rate jumps to the free float line \( FF' \) after a speculative attack which is assumed to be irreversible. The size of the speculative attack is given by \( AJ \).

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Parameters for the numerical solution of the Stochastic System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameters</td>
<td>Values</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>0.018</td>
</tr>
<tr>
<td>( \theta )</td>
<td>0.4</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>0.3</td>
</tr>
<tr>
<td>( c )</td>
<td>0.05</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>0.2</td>
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</tbody>
</table>
4. Conclusions

The present paper offers an explanation of the ERM crises in the light of the first-generation models of currency crises which are modified to take into account the role of a misaligned central parity. It stresses the important role of speculators and it also recognizes that the government’s commitment to a fixed exchange rate is constrained by the dynamics of the underlined fundamentals. The main result in the model with saddle path dynamics is that the size of the attack is amplified by the feedback effect from the composite fundamental to the exchange rate.

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