Don’t fool yourself to believe you won’t fool yourself again

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Abstract

A possible solution to the absent minded driver problem is that the decision maker optimizes taking into account the utility he will actually have in the future, and not the utility level he \textit{should} have. © 2000 Elsevier Science S.A. All rights reserved.

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In their analysis of the paradox of the absent minded driver, Piccione and Rubinstein (1997) offered the following problem of decision making with imperfect recall. While sitting in a bar, a decision maker knows that on his way home there are two similar intersections. Leaving the road at the first leads to a payoff of 0, while leaving the road at the second lead to a payoff of 4. If he does not leave the road at all, his payoff will be 1. The decision maker is also aware of the fact that when he reaches an intersection, he will not be able to tell whether it is the first or the second, and moreover, he will not remember through how many intersections he already went. While in the bar, he has to decide with what probability to leave the main road when an intersection is reached. Denote this probability $1-p$ and obtain the lottery

\begin{eqnarray*}
(0, 1-p; (4, 1-p; 1, p), p) = (0, 1-p; 1, p^2; 4, p(1-p))
\end{eqnarray*}

The expected utility from this lottery is $4p - 3p^2$, which is maximized at $p = 2/3$. Observe that the probability of leaving the road must be the same at both intersections, as the decision maker will not be able to tell where he is.

Suppose now that the decision maker is on his way home, and that he finds himself at an

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intersection. Again he decides to leave the road with probability $1 - p$. What is the optimal value of $p$ now? Given that he is at an intersection, he participates in the following lottery $T$. He believes that with probability $1 - \alpha$ he is at the second intersection, facing the lottery $S = (1, p; 4, 1 - p)$. With probability $\alpha$ he is at the first intersection, where with probability $1 - p$ he will leave the road and receive zero and with probability $p$ he will continue to the second intersection and face the lottery $S$. In other words, the decision maker is facing the lottery

$$(S, 1 - \alpha; (0, 1 - p; S, p)) = ((1, p; 4, 1 - p), 1 - \alpha; (0, 1 - p; (1, p; 4, 1 - p), p), \alpha) = (0, \alpha(1 - p); 1, p(1 - \alpha + \alpha p); 4, 1 - p - \alpha + 2\alpha p - \alpha^2)$$

The expected utility of this lottery is

$$4 - 3\alpha p^2 + 7\alpha p - 3p - 4\alpha$$

and the optimal value of $p$ is $(7\alpha - 3)/6\alpha$. This value equals $2/3$ only when $\alpha = 1$, that is, only when the decision maker knows for sure that he is at the first intersection. But to reach this certainty, he has to leave the road at the first intersection with probability one, hence the paradox.

I disagree with the above analysis. Of course, if the driver is at the first intersection, then with probability $p$ he will reach the second intersection. However, at this intersection his utility will not be the one obtained from the lottery $S$, but the one obtained from the lottery $T$. The reason is that upon reaching this intersection, he will be exactly at the same situation he is now, therefore his utility will be the same as it is now. The decision maker may decide to stick to his original plan. But given that he is willing to reconsider now, he should not fool himself to believe that he will not reconsider again.

I do not claim that the decision maker thinks that he will face the lottery $T$ again, because he knows that this is impossible. However, the utility level he will reach at the second intersection, if he will reach it, will be the same utility level he is receiving from the lottery $T$ now. Since the decision maker is an expected utility maximizer, we can create an equivalent (recursive) lottery $T^*$, given by $T^* = (S, 1 - \alpha; (0, 1 - p; T^*; p), \alpha)$. Hence, by computing the probabilities in the (infinite stage) lottery $T^*$ we obtain that

$$T^* = \left(0, \frac{(1 - p)\alpha}{1 - p\alpha}; 1, \frac{p(1 - \alpha)}{1 - p\alpha}; 4, \frac{(1 - p)(1 - \alpha)}{1 - p\alpha}\right)$$

The expected value of this lottery is

$$\frac{(4 - 3p)(1 - \alpha)}{1 - \alpha p}$$

What is missing is an analysis of the value $\alpha$. Piccione and Rubinstein claim that it does not matter what the value of $\alpha$ is. In their analysis, unless $\alpha = 1$, the optimal $p$ is different from $2/3$, the value of $p$ the decision maker initially planned to use. In their analysis, they optimize with respect to $p$, holding $\alpha$ fixed. I disagree with this point as well (see also Aumann et al., 1997). The decision maker’s beliefs should depend on the value of $p$, where

$$\frac{1 - \alpha}{\alpha} = p \Rightarrow \alpha = \frac{1}{1 + p}$$
Optimizing Eq. (1) subject to the constraint of Eq. (2) yields optimizing $4p - 3p^2$, hence $p = 2/3$ and $\alpha = 3/5$.

But should $\alpha$ really be a function of $p$? After all, standard optimization in decision theory requires the decision maker to form his beliefs before he is choosing an optimal strategy. In other words, his beliefs should be part of the input and not of the output of the optimization process. However, this is not the case here, and rightly so. The decision maker has to form his beliefs about where he is, but he is aware of the fact that the probability $\alpha$ depends on the value of $p_1$, the probability he is using the first time he will reach an intersection. Of course, he does not know whether he already used $p_1$ (that is, he is now at the second intersection) or not. But he knows that the optimal $p$ he is going to compute now must equal $p_1$. Otherwise, it must follow that he can tell the difference between the two intersections. So in forming his beliefs about the value of $\alpha$, he must realize and take into consideration that $\alpha$ depends on $p_1$. Hence his beliefs on $\alpha$ must depend on $p_1$, and since $p = p_1$, his beliefs on $\alpha$ must depend on $p$. At least in expected utility theory, beliefs about probabilities are just probabilities, hence the value of $\alpha$ depends on $p$.

To summarize, the above analysis differs from that of Piccione and Rubinstein in two aspects.

1. I believe that the decision maker can commit himself to his plan, or he can decide to re-optimize. What he cannot do is to re-optimize and assume that he will not do it again if he reaches the second intersection later on.
2. When optimizing, the decision maker must realize that $\alpha$ and $p$ are not independent variables.

When these two points are satisfied, the re-optimization will lead to the original plan, that is, to set $p = 2/3$.

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