Do consumers benefit from tighter price cap regulation?

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Abstract

This paper examines whether consumers necessarily benefit from tightening the price cap. We find that a tighter price cap always increases consumer welfare when demands are independent. Conversely, when demands are interdependent a tighter price cap may reduce consumer welfare. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

In recent years, price-cap regulation has been extensively applied to telecommunications and other privatized sectors in the UK and the US, and is rapidly becoming the dominant form of economic regulation. The primary objective of price-cap regulation is to restrain monopoly power and create superior incentives for cost minimization (Braeutigam and Panzar, 1989; Brennan, 1989; Weisman, 1993; Sappington and Weisman, 1996).

Price cap regulation is commonly referred to as ‘RPI-X’ regulation, where RPI is the retail price index and X is a factor that reflects anticipated productivity gains, and specifies the rate at which the regulated firm’s prices must fall in real terms on an annual basis (Bernstein and Sappington, 1999). Regulators can thus tighten the price cap by setting a higher X-factor. In telecommunications, tightening the price cap is a common regulatory response to ‘excessive earnings’ by the regulated firm. For example, the X-factor in British Telecom’s price cap plan has been ratcheted upward on three separate occasions since 1984 (Armstrong et al., 1994, pp. 224–229).
In 1995 and again in 1997, the FCC implemented a more stringent set of X-factors for the Regional Bell Operating Companies (RBOCs) in response to what were perceived as excessive financial returns (FCC, 1995, 1997). In each of these cases, tightening the price cap was seen as an avenue through which to share a greater proportion of the realized efficiency gains from price cap regulation with consumers.

The welfare effects of tightening a price cap have been examined in several studies. Armstrong and Vickers (1991) show that in a single product case, tightening an average revenue constraint (ARC) always improves welfare. However, in a multi-product setting, lowering the price cap can reduce welfare because it can result in prices set below marginal cost. Law (1995) shows that with independent demands, tightening an ARC can reduce consumer welfare when marginal costs differ between products. These previous studies focus on independent products under an ARC in which weights can be changed in response to a tighter price cap. In fact, a lagged revenue (non-contemporaneous) weight constraint is used extensively in the telecommunications industries in North America (Canada/US). In addition, the regulated firm’s products are often interdependent. Hence, an outstanding question concerns whether tighter price cap regulation necessarily increases consumer welfare under fixed weights with interdependent demands.

In this paper, we investigate the conditions under which consumers are actually made worse off by tightening the price cap constraint. We examine the consumer welfare consequences of tighter price cap regulation when demands are independent, substitutable and complementary, respectively. Our main conclusion is that whether a tighter price cap increases consumer welfare depends crucially on the particular demand relationships. A tighter price cap always increases consumer welfare when demands are independent. Conversely, for interdependent demands, tighter price cap regulation can actually reduce consumer welfare.

2. Effects of a tighter price cap

We consider a highly stylized setting in which a regulated monopolist produces two services with constant marginal cost \( c_i, i = 1 \text{ and } 2 \). The two services are bundled into a subset of the firm’s services called a basket that is subject to price cap (\( \hat{P} \)) regulation. Within the basket the firm has the flexibility to increase the price of one service and simultaneously reduce the price of the other service as long as the overall price cap constraint is satisfied. Let \( P_i \) denote the price for service \( i = 1 \) and 2. To facilitate numerical analysis, we assume that demand is given by

\[
Q_i = a_i - a_i P_i + b_i P_j
\]

where \( i \) and \( j \) are services with \( i \neq j \) and \( a_i > 0 \). To rule out degenerate cases, we assume that \( a_i > |b_j| > 0 \). We also restrict the equilibrium outputs and prices to be non-negative. The services \( i \) and

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1See also Law (1997) for an insightful discussion of strategic firm behavior in anticipation of price cap regulation.
2For example, local telephone service and long distance telephone service are complements; whereas extended area service and short-haul toll services are substitutes. Moreover, in the US, it is not uncommon for state regulators to place all services subject to price cap regulation in a single basket.
3The terms consumer welfare and consumers’ surplus are used interchangeably.
are independents, substitutes or complements when \( b_i = 0, b_i > 0 \) and \( b_i < 0 \), respectively. Let \( \pi_i(P_i, P_j) \) denote the regulated firm’s profit from service \( i \).

The firm’s problem \([F-P]\) is to maximize

\[
I = \sum_{i,j=1}^{2} \pi_i(P_i, P_j)
\]

subject to:

\[
\sum_{i=1}^{2} w_i P_i \leq \bar{P}, \quad \text{for} \quad w_i \in (0, 1) \quad \text{and} \quad \sum_{i=1}^{2} w_i = 1
\]

where \( w_i \) represents the relative price weight for service \( i \).

The profit-maximizing prices for \([F-P]\) are given by

\[
P_i^* = \frac{\bar{P}[2a w_i + (b_i + b_j)w_j] + [b_i c_i - a_i c_j - \alpha_i w_i w_j - [b_j c_j - a_i c_i - \alpha_i]w_j^2]}{2[(b_i + b_j)w_j + a_i w_i^2 + a_j w_j^2]}.
\]

The effect of a change in the price cap on the firm’s profit maximizing prices under the price cap constraint is

\[
\frac{dP_i^*}{dP} = \frac{2a w_i + (b_i + b_j)w_j}{2[(b_i + b_j)w_j + a_i w_i^2 + a_j w_j^2]}.
\]

where \( 2[(b_i + b_j)w_j + a_i w_i^2 + a_j w_j^2] > 0 \) by sufficient second-order conditions for a maximum.

Proposition 1 follows directly from (5) upon substituting \( k_i = 2a w_i + (b_i + b_j)w_j \).

**Proposition 1.** At the solution to \([F-P]\):

(i) \( \frac{dP_i}{dP} > 0 \) and \( \frac{dP_j}{dP} > 0 \), for \( b_i \geq 0, \quad i \neq j; \)

(ii) \( \frac{dP_i}{dP} < 0 \) when \( b_i < 0 \) for \( b_i < 0 \).

**Proof.** When \( b_i = 0, \) (5) reduces to \( a_i w_i/(a_i w_i^2 + a_j w_j^2) > 0 \). When \( b_i > 0, \) it is straightforward to show that \( dP_i/dP \geq 0 \) for \( i = 1 \) and 2 from (5) since \( k_i > 0 \) for \( i = 1 \) and 2. However, when \( b_i < 0, \) either \( k_i > 0 \) or \( k_i < 0 \) is possible, resulting in \( dP_i/dP > 0 \) or \( < 0. \)

Note that at least one of \( k_i \) and \( k_j \) must be positive to satisfy the second-order condition. Hence, there exist three possible combinations: (1) \( dP_1/dP > 0 \) and \( dP_2/dP > 0; \) (2) \( dP_1/dP > 0 \) and \( dP_2/dP < 0; \) or (3) \( dP_1/dP < 0 \) and \( dP_2/dP > 0. \)

Abstracting from income effects, total consumers’ surplus is given by

\[\text{\footnote{The result for } k_i = 0 \text{ or } k_j = 0 \text{ can be derived in similar fashion.}}\]
\[ \text{CS} = \sum_{i,j=1}^{2} \left( \int_{\hat{P}_i}^{P_j} Q_i(z_{ij}, P_j) \, dz_i \right), \]  
\[ \text{where } \hat{P}_i \text{ is the minimum price at which demand for service } i \text{ is identically zero. For linear demand, the expression for total consumers' surplus becomes} \]

\[ \text{CS} = \sum_{i,j=1}^{2} \left[ \frac{(\alpha_i + b_i P_i^*)^2}{2a_i} - \alpha_i P_i^* + \frac{a_i(P_i^*)^2}{2} - b_i P_j^* P_j^* \right], \]

\[ \text{where } P_i^* \text{ is the profit maximizing price for service } i \text{ under the price cap constraint.} \]

The effect of a change in the price cap on total consumers' surplus is given by

\[ \frac{d\text{CS}}{dP} = \sum_{i,j=1}^{2} \left( \frac{dP_j^*}{dP} \frac{b_i}{a_i} - \frac{dP_j^*}{dP} \right). \]

Substituting (5) into (8) yields

\[ \frac{d\text{CS}}{dP} = \sum_{i,j=1}^{2} \frac{b_i [2a_i w_j + (b_i + b_j) w_j] - a_i [2a_i w_j + (b_i + b_j) w_j]}{2a_i [(b_i + b_j) w_j + a_i w_i^2 + a_i w_j^2]} \]

Proposition 2 follows directly from (9) upon appealing to the definition of \( k_i \).

**Proposition 2.** At the equilibrium in [F-P]:

(i) \( \frac{d\text{CS}}{dP} < 0 \) \( \forall P > 0 \) for \( b_i = 0 \);

(ii) \( \text{CS}(P) \) is non-monotonic \( \Rightarrow \text{sgn}(b_i k_j - a_i k_j) > 0 \) for \( b_i \neq 0, i = 1 \text{ or } 2, i \neq j \);

(iii) \( \frac{d\text{CS}}{dP} > 0 \) \( \forall P > 0 \) when \( \text{sgn}(b_i k_j - a_i k_j) > 0 \) for \( b_i < 0, i = 1 \text{ and } 2, i \neq j \).

**Proof.** When \( b_i = 0 \), (9) reduces to 

\[ - \sum_{i,j=1}^{2} a_i w_i P_j^* (a_i w_i^2 + a_i w_j^2) < 0. \]

When \( b_i > 0 \), the \( \text{sgn}(b_i k_j - a_i k_j) \) varies. When \( \text{sgn}(b_i k_j - a_i k_j) > 0 \) then \( \text{sgn}(b_i k_j - a_i k_j) < 0 \) in order to satisfy the given restrictions on \( a_i \) and \( b_i \). When \( \text{sgn}(b_i k_j - a_i k_j) < 0 \), then \( \text{sgn}(b_i k_j - a_i k_j) > 0 \) or \( < 0^5 \). Hence, there are three possibilities for the effect of a tighter price cap on service-specific consumers' surplus when \( b_i > 0 \): (1) \( \text{dCS}_i / d\hat{P} > 0 \) and \( \text{dCS}_j / d\hat{P} < 0 \); (2) \( \text{dCS}_i / d\hat{P} < 0 \) and \( \text{dCS}_j / d\hat{P} > 0 \); or (3) \( \text{dCS}_i / d\hat{P} < 0 \) and \( \text{dCS}_j / d\hat{P} < 0 \). Cases 1 and 2 can result in total consumers' surplus decreasing with a tighter price cap. When \( b_i < 0 \), \( k_i \) and \( k_j \) may be positive or negative. Suppose that \( k_i > 0 \) and \( k_j > 0 \). Then, \( \text{sgn}(b_i k_j - a_i k_j) < 0 \) for both \( i = 1 \) and 2, resulting in \( \text{dCS} / d\hat{P} < 0 \). Suppose that \( k_i > 0 \) and \( k_j < 0 \).

\(^5\)The result for \( b_i k_j - a_i k_j = 0 \) or \( b_i k_j - a_i k_j = 0 \) can be derived in similar fashion.
Then, there exist three possible combinations for the \( \text{sgn}\{b_i k_j - a_i k_j\}, \) \( i = 1 \) and \( 2: \) (1) \( \text{sgn}\{b_i k_2 - a_i k_1\} > 0 \) and \( \text{sgn}\{b_2 k_1 - a_2 k_2\} > 0; \) (2) \( \text{sgn}\{b_1 k_2 - a_1 k_1\} < 0 \) and \( \text{sgn}\{b_2 k_1 - a_2 k_2\} < 0; \) or (3) \( \text{sgn}\{b_2 k_2 - a_2 k_1\} < 0 \) and \( \text{sgn}\{b_2 k_1 - a_2 k_2\} > 0. \) Case 1 results in \( \text{dCS}/\text{d}P > 0 \) and \( \text{dCS}/\text{d}P > 0. \) Case 2 results in \( \text{dCS}/\text{d}P < 0 \) and \( \text{dCS}/\text{d}P < 0. \) Case 3 results in \( \text{dCS}/\text{d}P < 0 \) and \( \text{dCS}/\text{d}P > 0. \) Case 1 does result and Case 3 can result in total consumers’ surplus decreasing with a tighter price cap. Analogously, when \( k_i < 0 \) and \( k_j > 0, \) similar results can be derived in like fashion. \( \square \)

Proposition 2 reveals that we can examine the partial effects of changes in the price cap on service-specific consumers’ surplus by evaluating \( \text{sgn}\{b_i k_j - a_i k_j\} \) for \( i = 1 \) and \( 2. \) This analysis also reveals that total consumers’ surplus is increasing monotonically with a tighter price cap when demands are independent (\( b_i = 0), \) but it is not necessarily increasing monotonically when demands are interdependent. Finally, the sufficient condition for total consumers’ surplus to decrease with a tighter price cap is that \( \text{sgn}\{b_i k_j - a_i k_j\} > 0 \) for \( i = 1 \) and \( 2. \) This sufficient condition can be satisfied only when demands are complementary (Proposition 2(iii)).

It is instructive to develop the economic intuition for our central finding. The results of the analysis depend on two fundamental constructs: \( k_i \) and \( b_i k_j - a_i k_j. \) Proposition 1 reveals that the sign of \( k_i \) determines the direction of the price change for service \( i \) upon a change in the price cap constraint. The term \( b_i k_j - a_i k_j \) specifies the demand changes corresponding to these price changes. Specifically, the term \( a_i k_j \) represents the change in the demand for service \( i \) with respect to a change in the price for service \( j. \) We may refer to this as the own effect. The term \( b_i k_j \) represents the change in the demand for service \( i \) with respect to a change in the price for service \( j, \) \( i \neq j. \) We may refer to this as the cross-effect. The sum of the own and cross-effects is given by \( b_i k_j - a_i k_j. \) The sign of this term indicates whether output for service \( i \) rises or falls as a result of the associated price changes. As consumers’ surplus changes are perfectly correlated with output changes, the sign of this term indicates whether consumers’ surplus rises or falls as a result of the associated price changes. Hence, when \( \text{sgn}\{b_i k_j - a_i k_j\} > 0 \) for \( i = 1 \) and \( 2, \) output declines for both services as a result of a tighter price cap. This implies that consumers’ surplus for both services, and hence total consumers’ surplus, declines as a result of a tighter price cap constraint. \(^8\)

Finally, it is useful to delineate the conditions under which a tighter price cap constraint is most likely to result in a decline in consumer welfare. Specifically, this is most likely to occur when (i) demands are complementary; (ii) the cross-effects are pronounced and (iii) the interaction of the price cap weights and the demand coefficients are such that \( k_i < 0 \) for \( i = 1 \) or \( 2. \) \(^9\) The following example provides an illustration.

\(^8\)When \( k_i > 0 \) and \( k_j < 0, \) the case of \( \text{sgn}\{b_i k_2 - a_i k_1\} > 0 \) and \( \text{sgn}\{b_2 k_1 - a_2 k_2\} < 0 \) does not arise because of given restrictions on \( a_i \) and \( b_i. \)

\(^9\)When \( k_i < 0 \) and \( k_j > 0, \) the case of \( \text{sgn}\{b_i k_2 - a_i k_1\} < 0 \) and \( \text{sgn}\{b_2 k_1 - a_2 k_2\} > 0 \) does not arise because of given restrictions on \( a_i \) and \( b_i. \)

\(^9\)The driving force behind our main result is fundamentally different from that of Law (1995). In our model, the price changes that cause consumers’ surplus to decline with a tightening of the price cap operate principally through the demand interdependencies. Conversely, the driving force behind this result in Law’s model is the endogeneity of the price weights to the price (and demand) changes induced by a tightening of the ARC. See Footnote 9 below.

\(^9\)This is precisely the price pattern for which Law (1995) discovered that consumers’ surplus could actually decrease in response to a tighter ARC when demands are independent. In our modeling framework a tighter price cap always increases consumers’ surplus when demands are independent (Proposition 2(i)).
Table 1
Simulation results for numerical example

<table>
<thead>
<tr>
<th>Price cap</th>
<th>$P_1^*$</th>
<th>$P_2^*$</th>
<th>$Q_1^*$</th>
<th>$Q_2^*$</th>
<th>CS</th>
<th>Profit</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>27.7</td>
<td>15.3</td>
<td>58.5</td>
<td>88.0</td>
<td>1986.9</td>
<td>943.1</td>
<td>2930.0</td>
</tr>
<tr>
<td>20</td>
<td>25.2</td>
<td>17.8</td>
<td>59.0</td>
<td>87.5</td>
<td>1986.5</td>
<td>1019.6</td>
<td>3006.1</td>
</tr>
<tr>
<td>21</td>
<td>22.7</td>
<td>20.3</td>
<td>59.5</td>
<td>87.0</td>
<td>1986.3</td>
<td>1091.1</td>
<td>3077.5</td>
</tr>
<tr>
<td>22</td>
<td>20.2</td>
<td>22.8</td>
<td>60.0</td>
<td>86.5</td>
<td>1986.4</td>
<td>1157.6</td>
<td>3144.0</td>
</tr>
</tbody>
</table>

3. Example

There are two services in the price cap basket with demand functions given by $Q_1 = 150 - 2.2P_1 - 2P_2$ and $Q_2 = 220 - 3.2P_2 - 3P_1$, respectively. The constant marginal cost for these services are $15$ and $13$, respectively. The exogenous weights for $P_1$ and $P_2$ are 0.3 and 0.7, respectively. For this example, $k_1 = -1.58$, $k_2 = 1.58$, $b_1k_2 - a_1k_1 = 0.316$ and $b_2k_1 - a_2k_2 = -0.316$. Hence, the necessary conditions for non-monotonicity (Proposition 2(ii)) are satisfied, but the sufficient conditions (Proposition 2(iii)) are not. Table 1 reveals that tightening the price cap from $\hat{P}_1 = 22$ to $\hat{P}_1 = 21$ causes both consumers' surplus and total economic welfare to decline. This is a local result only, however, as a 'sufficiently large' reduction in the price cap causes consumers' surplus to increase.

Conclusion

Tightening the price cap (respectively, raising the X-factor) is a common policy tool used by regulators to reduce the regulated firm’s profit and presumably to increase consumer welfare. These results reveal that whether tighter price cap regulation increases consumer welfare depends critically on the underlying demand relationships. When demands are independent, a tighter price cap increases consumer welfare. When demands are interdependent, however, a tighter price cap can [actually] reduce consumer welfare. Consequently, a tighter price cap can produce welfare effects diametrically opposite to those that regulators likely intend.

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