Measuring the temporary component of stock prices: robust multivariate analysis

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Received 10 May 1999; accepted 29 October 1999

Abstract

We identify the temporary and permanent components of US stock prices through appropriate restrictions on a vector autoregression of real stock returns and changes in interest rates, employing alternative robust estimation procedures designed to allow for non-Gaussian innovations. © 2000 Elsevier Science S.A. All rights reserved.

Keywords: Stock prices; Mean reversion; Permanent–temporary decomposition; Robust estimation

JEL classification: G14; C15

1. Introduction

Recent empirical evidence has lent strong support to the hypothesis of mean-reversion in stock prices. In particular, Fama and French (1988) and Poterba and Summers (1988) report impressive findings that stock prices are mean-reverting (i.e., contain a slowly decaying temporary component) and induce returns characterised by large negative autocorrelations at long horizons.

The reliability of the multi-period return tests employed in these studies has recently been questioned (see, for example, Richardson and Stock, 1989; Kim et al., 1991) and more recently, vector autoregressive analysis of stock prices and dividends has also been used to identify the permanent and temporary components of stock prices (Cochrane, 1994; Lee, 1995).

In this letter we seek to contribute to this literature in a number of ways. First, we employ a vector

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PII: S0165-1765(99)00258-X
autoregression (VAR) of real stock prices and nominal interest rates in order to identify the temporary and permanent components of stock price movements (an approach we justify in Section 2). Further, by decomposing real stock price movements into mutually orthogonal temporary and permanent components, we are able to measure the size and statistical significance of the mean-reverting component of US stock prices. Finally, we examine the sensitivity of the mean-reverting component using robust estimation procedures in order to allow for possible non-normality of the innovations to stock returns and interest rates.

2. VAR decomposition technique

In order to effect a decomposition of real stock prices into permanent and temporary components, we need to consider the multivariate time series representation of stock returns together with another appropriate variable. We prefer not to consider the multivariate representation of stock returns and dividend changes or stock price–dividend yields in this paper primarily because the primary underlying rationale for considering such a representation — namely the present value model of stock prices — has not been well supported empirically (see Campbell et al., 1997 for a survey of the evidence). Rather, we consider a general, unrestricted multivariate analysis of real stock returns and changes in nominal interest rates. While the presence of a strong empirical relationship between these variables is uncontroversial and well documented, our analysis is sufficiently general to allow for various anomalies and puzzles which have been recorded. For example, while the standard present value model suggests a close association between stock returns and interest rates (see again Campbell et al., 1997), the real-world relationship between these two families of financial returns appears to be highly complex and subject to anomalies such as the equity-premium puzzle (Mehra and Prescott, 1985). In addition, when one considers the relationship between stock returns and nominal interest rates, further complications arise because of complex interactions between real and nominal interest rates, aggregate price inflation and stock returns — see for example the literature on the stock return–inflation puzzle (e.g., Fama, 1981; Marshall, 1992) or on inflation, stock returns and the Fisher equation (e.g., Shiller and Beltratti, 1992).

Therefore, by considering an unrestricted multivariate time series representation of real stock returns and changes in nominal interest rates, we can be sure of a strong empirical relationship between the two variables while remaining agnostic concerning the precise nature of that relationship.

If changes in real log stock prices $\Delta q_t$ and changes in the nominal interest rate $\Delta r_t$ are each stationary processes (where $\Delta$ denotes the first difference operator), then by the multivariate form of Wold’s decomposition (Hannan, 1970), there will exist a bivariate moving average representation of the form:

$$\begin{bmatrix} \Delta q_t \\ \Delta r_t \end{bmatrix} = \sum_{i=0}^{\infty} L_i \begin{bmatrix} \psi_{11i} & \psi_{12i} \\ \psi_{21i} & \psi_{22i} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

(1)

where $\psi_{k_it}$ represents the parameters of the multivariate moving average representation and $\varepsilon_{1t}$ and $\varepsilon_{2t}$ represent white noise innovations. We can identify $\varepsilon_{1t}$ and $\varepsilon_{2t}$ as temporary and permanent innovations to real stock prices, respectively, and recover these from the VAR residuals in the following way.

Write $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t})'$, and denote the bivariate vector of innovations recovered from the vector
autoregressive representation for \((\Delta q_r \Delta r_t)\)' as \(\eta\). Since the VAR representation is simply an inversion of the Wold representation (1), \(\eta\) will in general be a linear function of \(\epsilon\), \(\eta = A \epsilon\) say, where \(A\) is a \(2 \times 2\) matrix of constants. To recover the underlying temporary and permanent stock price innovations from the VAR residuals then requires that the four elements in \(A\) be identified, which requires four identifying restrictions. Three restrictions can be obtained by normalizing the variances of \(\epsilon_t\), and \(\epsilon_{2t}\), to unity and setting their covariances to zero (see Blanchard and Quah, 1989, for a defence of these restrictions). The fourth, crucial identifying restriction, which effectively identifies \(\epsilon_t\) as the temporary stock price innovation, is the requirement that \(\epsilon_t\) has no long-run effect on the level of real stock prices, although it may affect the long-run nominal interest rate. The latter restriction on the Wold representation (1) may be written:

\[
\sum_{i=0}^{\infty} \psi_{11i} = 0.
\]

These four restrictions are then sufficient to recover the underlying temporary and permanent innovations to real stock prices (see Blanchard and Quah, 1989, for further details). That part of the Wold decomposition corresponding to, respectively, permanent and temporary past innovations in \(\Delta q_t\) can then be cumulated and taken as the permanent and temporary components of \(q_t\).

3. Robust estimation of the VAR decomposition

Least-squares (LS) estimation is inefficient when the disturbances are non-Gaussian, a characteristic of most financial data series where innovations are generally known to have fat-tailed and, perhaps, skewed distributions (Loretan and Phillips, 1994; Phillips et al., 1996; Taylor and Peel, 1998). We investigate the sensitivity of our VAR decomposition as estimated by LS to alternative robust estimation procedures — the least absolute deviation (LAD) and the more recent residuals augmented least square (RALS) estimators (see Judge et al., 1988, Chapter 22; and Im, 1996).

The LAD estimator is sometimes used as an alternative to LS particularly when the disturbances may be distributed as Cauchy or Student’s \(t\) (i.e., fat-tailed). Consider the following simple regression,

\[
y_t = \phi' \xi_t + u_t
\]

where \(t = 1, \ldots, T\), \(\xi_t = (1, x_t)'\), \(x_t\) is a \((k - 1) \times 1\) vector of time series observed at time \(t\), \(\phi = (\alpha \beta')'\) is the parameter vector where \(\alpha\) is the intercept and \(\beta\) is the \((k - 1) \times 1\) vector of parameters of interest, and the residuals \(u_t\) are iid with distribution function symmetric around zero. The LAD estimator \(\beta_L\) is equivalent to finding that \(\beta\) which minimizes \(\Sigma |y_t - \beta' \xi_t|\). The properties of the LAD estimator are well known (Judge et al., 1988).

The RALS estimator can be interpreted as a generalized method of moments (GMM) estimator based on the moment conditions \(E[z_t u_t] = 0\), \(E[z_t (u_t^2 - \mu_z)] = 0\), \(E[z_t (u_t^3 - \sigma_z^2)] = 0\), where \(\sigma_z^2\) is the variance and \(\mu_z\) denotes the \(i\)th central moment of \(u_t\). Im (1996) shows that this GMM estimator of \(\beta\),

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1Because the LS procedure minimizes squared deviations, it places a relatively heavy weight on outliers, and in the presence of errors that are not normally distributed (for example, a more leptokurtic distribution) can lead to estimates that are extremely fragile.
\( \beta^* \) say, can in fact be simply computed from LS applied to (3) augmented by \( \hat{w}_i = ((\hat{u}_i - 3\hat{\sigma}^2 \hat{u}_i)(\hat{u}_i - \hat{\sigma}^2))' \):

\[
y_i = \alpha + \beta' z_i + y' \hat{w}_i + e_i
\]

where \( \hat{u}_i \) denotes the LS residual and \( \hat{\sigma}^2 \) the standard residual variance estimate obtained from LS applied to (3). The RALS estimator is given by

\[
\beta^* = (\tilde{X}' M_\tilde{w} \tilde{X})^{-1} \tilde{X}' M_\tilde{w} Y
\]

where the idempotent matrix \( M_\tilde{w} \) is \( M_\tilde{w} = I_T - \tilde{W}' (\tilde{W}' \tilde{W})^{-1} \tilde{W} \) where \( I_T \) is the \( T \times T \) identity matrix and \( \tilde{N} = (\tilde{n}_1 \tilde{n}_2 \ldots \tilde{n}_T)' \), \( \tilde{n}_t = n_t - T^{-1} \sum_{t=1}^{T} n_t \) for \( (N,n) = (X,x),(Y,y),(W,w) \) and \( t = 1, \ldots, T \). The asymptotic distribution of the RALS estimator is given by \( \sqrt{T}(\beta^* - \beta) \to N(0, \sigma_A^2 \text{Var} \chi_i)^{-1} \) where

\[
\sigma_A^2 = \sigma^2 - \frac{\mu_1^2(\mu_6 - 6\mu_4 \sigma^2 + 9\sigma^6 - \mu_5^2) - 2\mu_3(\mu_4 - 3\sigma^4)(\mu_6 - 4\mu_4 \sigma^2) + (\mu_3 - 3\sigma^4)^2(\mu_5 - \sigma^4)}{\mu_4 - \sigma^4)(\mu_6 - 6\mu_4 \sigma^2 + 9\sigma^6 - \mu_5^2) - (\mu_5 - 4\mu_4 \sigma^2)^2}
\]

In practice, \( \sigma_A^2 \) can be consistently estimated by replacing each of the \( \mu_i \) with the corresponding sample moments, yielding \( \hat{\sigma}_A^2 \), and the covariance matrix for \( \beta^* \) can be consistently estimated by \( \hat{\text{Var}}(\beta^*) = \hat{\sigma}_A^2 (\tilde{X}' M_\tilde{w} \tilde{X})^{-1} \). The efficiency gain from employing RALS as opposed to LS can be gauged from the statistic \( \hat{\eta} = \hat{\sigma}_A^2 / \hat{\sigma}^2 \) (which is small for large efficiency gains).

4. Empirical results

Monthly data for the US were obtained from the International Monetary Fund’s International Financial Statistics data base for the period 1957:1 through 1997:12. The data series of interest are the natural logarithm of the real New York Stock Exchange stock price index, \( q_t \), and the monthly rate of return on 3-month Treasury bills, \( r_t \).

4.1. Least-squares results

A VAR of \( (\Delta q_t, \Delta r_t)' \) was estimated\(^1\) by least-squares preliminary to effecting the decomposition.\(^4\) The chosen lag depth was 14. Given the estimates of the VAR parameters and the covariance matrix of VAR residuals, we then carried out the Blanchard–Quah decomposition. The impulse response

\(^1\)The real stock price index is constructed using the consumer price index. The monthly rate of return on 3-month Treasury bills is calculated geometrically by \( r_t = (r_t^* + 1)^{1/12} - 1 \), where \( r_t^* \) is the annualized rate of return on 3-month t-bills in period \( t \).

\(^4\)Seasonal dummies were included in the VAR.

\(^5\)Standard unit root and cointegration tests support the null hypothesis that the interest rates and real stock prices series are first-difference stationary and are not cointegrated.
functions for stock prices are presented in Fig. 1. A one-unit (standard deviation) temporary shock to real stock prices has a half-life of 7 months.

In the VAR decomposition, by construction, changes in real stock prices $\Delta q_t$, are decomposed into three components: the permanent component, $\Delta q_{Pr}$, the temporary component, $\Delta q_{Tr}$, and the deterministic (drift and seasonal) component, $\Delta q_{Dr}$:

$$\Delta q_t = \Delta q_{Dr} + \Delta q_{Tr} + \Delta q_{Pr}$$ (6)

The $t$-statistics obtained from regressing each of these components in turn on $\Delta q_t$ provide a test of the statistical significance of each component. Regressing $\Delta q_t$ onto $\Delta q_{Tr}$, for example, provides a test of the statistical significance of the temporary component of real stock prices as the $t$-ratio of the slope coefficient. Given that the components are by construction mutually orthogonal, the $R^2$ values associated with each regression estimates the proportion of total variation in real stock returns explained by each component and the $R^2$ values must add up to unity.

Table 1 reports that the mean-reverting (temporary) component is in fact statistically significant at

![Permanent Shock to Real US Stock Prices](image1.png)

![Temporary Shock to Real US Stock Prices](image2.png)

Fig. 1. Impulse response functions. US, 1957:1–1997:12.
standard significance levels for real stock prices, explaining 25% of the variation. Seventy-two percent of the variation in real stock prices is explained by permanent innovations while a significant 3% is explained by trend and seasonal components.

4.2. Robust estimation results

In estimating the VAR by LS, and in common with previous empirical work on stock price mean reversion, we have so far ignored the fact that the associated residuals may exhibit significant non-normality. In fact, the Jarque–Bera (1980) test statistic (distributed as $\chi^2(2)$ under the null hypothesis of normality) applied to the VAR residuals were 155.24 and 2296.04 for the stock return and interest rate equations, respectively, strongly rejecting normality. We, therefore, re-estimated the VAR using the RALS and LAD estimators described above and then carried out the decomposition using the resulting estimates.

From the RALS and LAD estimation of the VAR decomposition, the temporary, permanent and deterministic components of the real stock price series, were calculated as before. We again estimated the significance of the mean-reverting component by regressing each component in turn on the change in real stock prices using the appropriate robust estimator. The results are reported in Table 2.

The RALS and LAD procedures suggest that about 30 and 40%, respectively, of the variation in real stock price movements can be explained by the mean-reverting component: the non-normality in the VAR residuals causes the size of the mean-reverting component to be underestimated when estimated by LS. However, the earlier qualitative findings of a significant mean-reverting component in US stock prices appear to be robust to the outliers in the VAR residual distributions.

We estimate the efficiency statistic $\eta$ to be 0.88 and 0.78 in the VAR regressions of real stock returns and changes in interest rates, respectively, indicating efficiency gains of around 10 and 20%, respectively, in using the RALS over the LS estimation procedure.

The orthogonality condition empirically holds for the shocks estimated by either the LS, RALS or LAD procedure. In addition, the estimated half-lives of the temporary and permanent components generated by the robust procedures appeared broadly similar in magnitude to those generated using least-squares.
Table 2  
Robust estimation: \( t \)-statistic of permanent and temporary components in real stock prices*  

<table>
<thead>
<tr>
<th>Decomposition by</th>
<th>( \Delta q_i = \alpha \Delta q_{tr} + \epsilon_i )</th>
<th>( i = P, T, D )</th>
<th>( R^2 )</th>
<th>SSE</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>RALS</td>
<td>( \Delta q_i = 0.99 \Delta q_{tr} )</td>
<td>( R^2 = 0.30 )</td>
<td>( (0.07) )</td>
<td>1.76</td>
<td>13.67</td>
</tr>
<tr>
<td></td>
<td>( \Delta q_i = 0.99 \Delta q_{tr} )</td>
<td>( R^2 = 0.66 )</td>
<td>( [29.45] )</td>
<td>2.04</td>
<td>29.45</td>
</tr>
<tr>
<td></td>
<td>( \Delta q_i = 0.92 \Delta q_{tr} )</td>
<td>( R^2 = 0.66 )</td>
<td>( (0.03) )</td>
<td>0.27</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td>( \Delta q_i = 0.87 \Delta q_{tr} )</td>
<td>( R^2 = 0.04 )</td>
<td>( [3.88] )</td>
<td>1.88</td>
<td>3.88</td>
</tr>
<tr>
<td>LAD</td>
<td>( \Delta q_i = 0.92 \Delta q_{tr} )</td>
<td>( R^2 = 0.40 )</td>
<td>( (0.06) )</td>
<td>1.81</td>
<td>16.28</td>
</tr>
<tr>
<td></td>
<td>( \Delta q_i = 0.93 \Delta q_{tr} )</td>
<td>( R^2 = 0.55 )</td>
<td>( [22.21] )</td>
<td>2.08</td>
<td>22.21</td>
</tr>
<tr>
<td></td>
<td>( \Delta q_i = 0.87 \Delta q_{tr} )</td>
<td>( R^2 = 0.05 )</td>
<td>( (0.22) )</td>
<td>0.77</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>( \Delta q_i = 0.87 \Delta q_{tr} )</td>
<td>( R^2 = 0.93 )</td>
<td>( [3.95] )</td>
<td>1.91</td>
<td>3.95</td>
</tr>
</tbody>
</table>

* See notes to Table 2. Estimation is by residuals augmented least squares (RALS) or least absolute deviation (LAD), as indicated.

5. Conclusion

The evidence presented in this letter supports the mean-reversion hypothesis that US stock prices contain a statistically significant mean-reverting component, explaining around 30–40% of the variation in real returns and thus returns are to some extent predictable. Temporary innovations to real stock prices tend, however, to be quite persistent, with a half-life of about 7 months.

These findings are robust to non-normality of the innovations in the vector autoregressive representations — a feature to be expected in financial markets. Failure to allow for this non-normality feature causes the size of the mean-reverting component to be slightly underestimated.

References