Optimal monetary policy with a nonlinear Phillips curve

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Abstract

The analysis of optimal discretionary monetary policy under a non-linear Phillips curve is shown to yield results that are in marked contrast to conventional results that are drawn from the linear paradigm. Specifically, we show that there exists a deflation bias in expected output, while the inflation bias cannot be signed. Preference uncertainty, however, adds to expected inflation. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

There exist a small, but growing literature on significant nonlinearities in economic time series and in important macro-economic relationships – see, for example, Rothman (1999). From a policy perspective, moreover, there is increasing recognition that nonconvexities in relationships such as the Phillips curve and preferences gives rise to issues such as unemployment costs of disinflation, credibility, and the design of mechanisms to counter the so-called inflation bias under discretion. Recent examples include Bean (1996), Clark and Laxton (1996), Tambakis (1998), and Amano et al. (1999). To date, however, the analysis under nonlinearity has been limited by the inherent difficulties in securing closed-form solutions, and as a result, the empirical evidence is limited to simulation results.\textsuperscript{1}

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\textsuperscript{1}The simulation results in Tambakis, for the U.S and Amano et al. for the Canadian economy indicate values of around 0.7\% for the output deflation bias, and a modest though positive inflation bias of around 0.2\% for a range of shocks considered.
In this paper we re-examine the standard linear results on optimal monetary policy under discretion with a nonconvex Phillips curve. The analysis exploits a procedure due to Varian (1974) and Zellner (1992) which allows an analytically tractable closed-form solution to the issue of optimal policy under discretion. We show that a number of literature results, drawn from the the linear paradigm, as in Svensson (1995), no longer hold. Specifically, under a nonlinear Phillips curve, there exists an output deflation bias. In related analysis, Clark and Laxton draw a distinction, for policy analysis, between the natural rate and the non-accelerating inflation rate of unemployment (NAIRU). Output targeting, following Persson and Tabellini (1990) cannot therefore be applied to eliminate the inflation bias under discretion. Moreover, the (positive) inflation bias outcome under discretion does not in general hold, given that the expected inflation equilibrium is ambiguous. Preference uncertainty, however, adds to expected inflation. One consequence of these results is that the linear inflation target or Walsh contract mechanism, which delivers expected inflation of zero in the literature, no longer has the relatively tractable structure to permit plausible practical implementation.

1.1. The nonlinear Phillips curve specification

Following Varian (1974) and Zellner (1992), we propose a specification for the Phillips curve as having the following modified Linex functional form:

\[ \pi - \pi^e = \frac{e^{a(y - y_n) - 1}}{g} \] (1)

where \( \pi \) is inflation, \( y \) is real output, \( \pi - \pi^e \) is unanticipated inflation, \( y_n \) is the natural or normal level of output, corresponding to the natural rate in a Phillips curve, and \( g, a \) are positive constants. Eq. (1) is a nonlinear Phillips curve or supply schedule which approaches the familiar linear Phillips case, \( y - y_n = 1/a(\pi - \pi^e) \), as \( g \) approaches zero. To illustrate, we normalise \( y_n \) at zero and plot (1), for \( g = 0.2 \) and \( a = 2 \), in Fig. 1 below.

![Fig. 1. The nonlinear Phillips curve. Expected output under a nonlinear Phillips curve.](image-url)

Assume that expectations are rational and that output deviation is the control variable, as in Rogoff (1985), and depends linearly on the deterministic instrument, \( z \), so that

\[ y = z + u \] (2)
where $u$ is a conditionally normal error. Then, the expectation of Eq. (1) has a closed-form solution (see, e.g. Christofferson and Diebold, 1994) and is given by

$$0 = \frac{e^{ag(z+ag)/2}\sigma_u^2}{g}$$

(3)

where $\sigma_u^2$ is the conditional variance of $u$. The solution of Eq. (3) for $z$ implies that $z < 0$, so that the expected level of output is less than the natural level $y_*$, which is obtained in the case of a linear Phillips curve. Note too that the magnitude of this effect depends on the variance of the shock to output. Consequently, the nonlinearity of the Phillips curve imparts a negative bias to policy ceteris paribus.

We see also from (3) that

$$z + \frac{ag}{2}\sigma_u^2 = 0$$

(4)

so that increases in $g$, a measure of the degree of nonlinearity of the Phillips curve, leads to lower values of $z$. For a useful policy discussion on the distinction between NAIRU and the natural rate, see Clark and Laxton (1996).

1.2. The optimal discretionary policy and inflation bias

Here, we assume that the monetary authorities minimise the familiar loss function

$$L = \frac{(\pi - \pi^*)^2}{2} + \lambda\frac{(y - y^*)^2}{2} + t\pi$$

(5)

where $\pi^*$ is an inflation target and $t$ is the constant parameter of a Walsh (1995) inflation contract.

Minimising (5) subject to (1), with respect to $y$ we obtain the first-order condition

$$(\pi - \pi^* + t)a\pi + \lambda(y - y^*) = 0$$

(6)

Eq. (6) defines the first order condition for the optimal policy rule under discretion. To obtain the solution for expected inflation, $\pi^e$, it is simplest to substitute for $\pi$ in (6) from (1) and take the expected value of the expression

$$ae^\pi(t - \pi^* + \pi^e + \frac{e^\pi - 1}{g}) + \lambda(y - y^*)$$

(7)

This gives, using (3), that

$$\pi^e = \pi^* - t + \frac{y^*}{a} - \frac{z}{a} + \frac{1}{g}(1 - e^{2ag(z+ag\sigma_u^2)})$$

(8)

or substituting for $z$ that

$^2$For a similar result, derived from a model with adaptive expectations, and employing a Taylor expansion of a nonlinear Phillips curve, see Bean (1996).
As will be recognised, the first term on the right-hand side of (9) is the standard inflationary bias obtained in the case of a linear Phillips curve. In general, however, the difference between the second and third terms is ambiguous—the sign will depend on both the variance of the real shock and the relative weight given to the stabilization of output, $\lambda$. Consequently, under a nonlinear Phillips curve there is no unambiguous prediction as to expected inflation under discretion. The conditions under which the conventional (linear) outcome of positive inflation bias pertains can be derived from the partial derivative of expected inflation with respect to the degree of nonlinearity of the Phillips curve, indexed by the parameter $\gamma$. This is given below as

$$
\frac{\partial \pi^e}{\partial \gamma} = 0.5 \left[ \left( g^2 \sigma_u^2 (\lambda - 4a^2 e^{a^2 \gamma^2} \sigma_u^2) - 2(1 - e^{a^2 \gamma^2} \sigma_u^2) \right) \right]
$$

We observe from (10) that for the positive bias outcome to exist under a nonlinear Phillips curve, the following sufficient condition, $\lambda \geq 4a^2 e^{a^2 \gamma^2} \sigma_u^2$ has to be satisfied.

### 1.3. Output targeting and preference uncertainty

In the linear paradigm it can be shown, as in Persson and Tabellini (1990), p. 27, n. 18) that setting the output target to the natural rate so that $y^* = 0$ removes the inflationary bias. An inspection of Eq. (9) however, will show that $y^*$ will need to be set at a level which makes the RHS of Eq. (9) to equal zero. This result underscores the issue of whether policymakers’ insistence that their output objectives is maintaining output at the natural rate, as in King (1996) and Blinder (1997) is appropriate if the Phillips curve exhibits nonlinearity.

A related issue, as detailed in Briault et al. (1996), and Nolan and Schaling (1996) is the discretionary policy outcome and mechanisms under preference uncertainty. It is fairly straightforward, within our framework to consider the implications of preference uncertainty. If we let the relative weight on output exhibit multiplicative uncertainty so that $\lambda = \lambda e^x$ where $x$ is a random variable with mean zero and constant variance $\sigma_x^2$ then the solution for $\pi^e$ is given by

$$
\pi^e = \lambda \left( \frac{y^*}{a} + \frac{g \sigma_u^2}{2} \right) e^{x^2/2} + \frac{1}{g} (1 - e^{a^2 x^2} \sigma_u^2)
$$

when it is assumed that the covariance between $u$ and $e$ is zero. We observe from (11) that expected inflation unambiguously increases with preference uncertainty. In principle, therefore, the linear inflation bias result may be secured by sufficient preference uncertainty. Consider next an alternative

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To examine whether the ambiguity arises from the use of the Linex formulation of the supply curve, we have re-evaluated under a cubic form which approximates to any underlying function. We find that the ambiguity still persists. The advantage of the Linex form, of course, is the we can secure a closed-form solution.
situation in which the central bank is solely concerned with inflation so that $\lambda = 0$. We observe from Eq. (9) that the outcome is now a negative inflation bias, since the last term is always negative under a nonlinear Phillips curve.

Finally, consider the implications of our analysis for the prominent inflation targeting and linear inflation contract mechanisms, following Walsh (1995) and Svensson (1995). It will be evident from Eq. (9) that implementing such mechanisms to eliminate the inflation bias will require a framework which reflects all the parameters determining $\pi^e$. As a matter of practical implementation, such mechanisms are likely to be too complex to design and monitor.

2. Concluding remarks

In this paper we have examined the implications of a simple but analytically tractable form of a nonlinear Phillips curve for the discretionary inflation outcome. We were able to derive a number of interesting implications. We demonstrated that expected output would be below the natural rate — the authorities cannot, therefore, remove the inflation bias by having an output target equal to the natural rate as is the case with a linear Phillips or supply curve. In general we showed that the effect on expected inflation in the discretionary inflation equilibrium is ambiguous. Finally we find that the inflation target or Walsh contract which generate expected inflation of zero no longer have a relatively simple structure as is the case with a linear Phillips curve.

References