Informative advertising and entry deterrence: a Bertrand model

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Abstract

This paper shows that strategic entry deterrence via informative advertising is impossible in a game of sequential advertising followed by simultaneous price setting in a homogenous-product market. It contrasts Schmalensee’s [Journal of Political Economy, 91 (1983) 636–653] result that when the strategic variables at the post-advertising stage are quantities, not prices, optimal entry deterrence is possible and involves underinvestment in advertising. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

Schmalensee (1983) was the first to examine whether an incumbent can deter a new firm’s entry via strategic advertising in subgame perfect equilibrium. The firms are assumed to simultaneously set a quantity after sequentially advertising though he thought ‘it would perhaps be most natural to assume that post-advertising equilibria are noncooperative in price’ (Schmalensee, 1983, p. 649). This is because the subgame has no Bertrand–Nash equilibrium in pure strategies, and the existence of the mixed-strategy Bertrand–Nash equilibrium of the subgame was unknown.

This paper examines the ‘natural’ advertising entry-deterrence game whose post-advertising competition is Bertrand, relying on Theorem 5 of Dasgupta and Maskin (1986) which guarantees the existence of the mixed-strategy Bertrand–Nash equilibrium of the post-advertising subgame. I show that, when the post-advertising strategic variables are prices, the incumbent is unable to strategically deter entry via informative advertising. This result highly contrasts Schmalensee’s (1983) that, when
the post-advertising strategic variables are quantities, optimal entry deterrence is possible and involves investing less in advertising than when there was no entry threat. It suggests that an incumbent’s ability to deter entry through informative advertising in a homogenous-product market is sensitive to whether firms compete in prices or quantities. The result of this article is due to the fact that (1) in the sequential-move advertising stage, the entrant’s profit declines with the incumbent’s advertising investment, but it levels out to be a constant; and (2) where the incumbent’s post-entry profit is maximized, the entrant’s post-entry profit is minimized.

The rest of this paper is organized as follows. In Section 2, I describe the assumptions of the game. In Section 3, I examine the incumbent’s behavior as a monopolist when new entry is impossible. In Section 4, I characterize the post-advertising Bertrand–Nash equilibrium. In Section 5, I derive the advertising best-response correspondences and investigate the possibility that the incumbent deters entry by advertising in subgame perfect Nash equilibrium. Section 6 concludes this paper.

2. Assumptions

Consider the following three-stage game. At stage 1, firm X (incumbent) sinks a fixed cost of advertising, \( f \), and chooses a level of advertising, \( x \); at stage 2, firm Y (potential entrant) chooses whether it enters the market, and, if it does, it sinks \( f \) and determines an advertising level, \( y \); and at stage 3, each firm simultaneously sets a price. The firms produce a homogeneous product. The marginal cost of production, \( c \), is a positive constant and less than 1 to both firms. There is a continuum of potential consumers whose total mass is normalized to unity. Their common demand function is \( q = \frac{1}{2p} \) for \( i = X, Y \) where \( q \) is the quantity demanded for brand \( i \) and \( 0 \leq p \leq 1 \) is brand \( i \)'s market price. The technology of informative advertising follows Butters (1977). Before receiving the advertising leaflets, consumers are assumed not to know the existence of either of the brands. Receiving one or more leaflets from a firm, a consumer knows the existence and the attributes of its brand, and the telephone number and the location of the firm. As to variable cost of advertising, following Tirole (1988), I will employ a quadratic function \( e(z) = (a/2)z^2 \) where \( z = x \) or \( y \) and \( a \) is a positive parameter.  

Advertising creates consumer segmentation in terms of knowledge of product existence. Suppose firm X has informed \( 0 \leq x \leq 1 \) fraction of all the consumers of the existence of brand X, and firm Y has informed \( 0 \leq y \leq 1 \) fraction of all the consumers of the existence of brand Y. Then, \( x(1-y) \) fraction of all the consumers know the existence of only brand X; \( y(1-x) \) fraction of consumers know the existence of only brand Y; \( xy \) fraction of all the consumers know the existence of brands X and Y; and \( (1-x)(1-y) \) fraction of all the consumers know neither of the brands. The perfectly informed consumers can compare the prices set by both the firms. They each purchase \( q_X = 1 - p_X \), \( p_X \in [0, 1] \), units of brand X and nothing from firm Y if brand X’s price is less than brand Y’s; otherwise buy \( q_Y = 1 - p_Y \), \( p_Y \in [0, 1] \), units of brand Y and nothing from firm X. On the other hand, the asymmetrically informed consumers are insensitive to the price differences because they know the existence of only one of the brands. Thus, those who know only brand X (Y) have a demand function,  

\(^1\)Under this advertising cost condition, it is possible to have closed-form solutions of the game for all \( x \) and \( y \). If I use Butters’s (1977) type advertising cost function \( (\alpha \log[1/(1 - z)]) \) as Schmalensee (1983) did, numerical investigation is required.
3. Monopoly equilibrium

Suppose that firm Y cannot enter the market and firm X has informed 0 ≤ x ≤ 1 fraction of all the consumers of brand X via advertising. Then, firm X’s profit in the post-advertising stage is \( x(1-p)(p-c) \) where I neglect the subscript X for simplicity. The equilibrium price, quantity, and profit at the post-advertising stage are \( p^m = (1 + c)/2 \), \( q^m = x(1 - c)/2 \), and \( \pi^m = x(1 - c)^2/4 \), respectively. Consequently, firm X’s profit at the advertising stage is \( \Pi^m(x) = \pi^m(x) - e(x) - f = x(1-c)^2/(4r) - (\alpha/2)x^2 - f \) where \( r \) is the relevant discount rate. For the sake of simplicity of the analysis, let me define \( A = 4\alpha r/(1-c)^2 \) and \( F = 4fr/(1-c)^2 \), and work with \( [4r/(1-c)^2]\Pi^m(x) \equiv x - (A/2)x^2 - F \equiv V^m(x) - F \) as Schmalensee (1983) did. Firm X’s optimal level of advertising can be obtained by maximizing \( V^m(x) \):

\[
x^m = \begin{cases} 
 1/A & \text{if } 1 \leq A \\
 1 & \text{if } 0 \leq A < 1.
\end{cases}
\]

Schmalensee (1983) focused on the situation where firm X as a monopolist never informs all the consumers of its product by restricting the value of \( A \) to a certain range. To compare my result with his, therefore, I limit my attention to the case where \( 1 \leq A < \infty \). With this restriction, the monopoly equilibrium profit in the whole game is given by

\[
V^m(x^m) - F = 1/(2A) - F.
\]

4. Post-advertising Bertrand–Nash equilibrium

To examine the subgame perfect equilibrium of the multi-stage game, I solve the game recursively. As I mentioned in the Introduction, the Bertrand game at the last stage has no pure-strategy Nash equilibrium, but it is not difficult to prove the existence of mixed-strategy Bertrand–Nash equilibrium of the subgame in reference to Dasgupta and Maskin’s (1986) Theorem 5. Hence, what is required for characterization of the equilibrium is to obtain the explicit formulas for the mixed-strategy Bertrand–Nash equilibrium cumulative distribution functions, and the expected profits. Nevertheless, similar games have been already analyzed; particularly, Golding and Slutsky (undated) characterized the pricing equilibrium of the games of general class containing this subgame, and Ireland (1993) analyzed a two-stage advertising-then-pricing game similar to the model here. Therefore, I simply state the mixed-strategy Bertrand–Nash equilibrium profits for the later analysis.

**Proposition 1 (post-advertising Bertrand–Nash equilibrium).** The simultaneous-move price-setting game in the post-advertising stage has a unique mixed-strategy Bertrand–Nash equilibrium. If \( x > y \), firm X tends to set high prices more frequently and earns more than firm Y since firm X has a larger captive consumer segment than firm Y (this can be explicitly confirmed by deriving the pricing equilibrium cumulative probability distribution functions). The expected equilibrium profits are
\[ \pi_x^* = \pi_y^* = (1 - y)(1 - c)^2 / 4 \]  (3)

When \( x < y \), the expected equilibrium profits can be derived from the above with arguments and subscripts transposed.

5. Advertising equilibrium and entry decisions

In order to find the subgame perfect equilibrium of the advertising stage, one need derive the advertising best response correspondence. Using the simplification in Section 3, one has firm X’s profit function at the advertising stage from Proposition 1,

\[ V_x(x, y) = \begin{cases} 
  x(1 - y) - (A/2)x^2 & \text{if } 1 \geq x \geq y \geq 0, \\
  x(1 - x) - (A/2)x^2 & \text{if } 0 \leq x < y \leq 1. 
\end{cases} \]  (4)

The first sub-profit function is maximized at \( x = (1 - y)/A \) and its maximum is \((1 - y)^2/(2A)\); the second sub-profit function is maximized at \( x = 1/(A + 2) \) and its maximum is \( 1/[2(A + 2)] \). Comparing these local maximal profits and finding the critical value of \( y \) at which these two local maximums are equal, one can have firm X’s advertising best-response correspondence,

\[ x^*(y) = \begin{cases} 
  (1 - y)/A & \text{if } 0 \leq y \leq 1 - \sqrt{A/(A + 2)}, \\
  1/(A + 2) & \text{if } 1 - \sqrt{A/(A + 2)} \leq y \leq 1. 
\end{cases} \]  (5)

By using the fact that firm Y’s advertising best-response correspondence is symmetrical to (5), one can have firms X’s and Y’s (re-scaled) profit functions in the sequential advertising stage,

\[ V_x(x, y^*(x)) = \begin{cases} 
  [(A + 1)/(A + 2)]x - (A/2)x^2 & \text{if } 1 - \sqrt{A/(A + 2)} \leq x \leq 1, \\
  x - [(A + 2)/2]x^2 & \text{if } 0 \leq x < 1 - \sqrt{A/(A + 2)}. 
\end{cases} \]  (6)

\[ V_y(x, y^*(x)) = \begin{cases} 
  1/[2(A + 2)] & \text{if } 1 - \sqrt{A/(A + 2)} \leq x \leq 1, \\
  (1 - x)^2/(2A) & \text{if } 0 \leq x < 1 - \sqrt{A/(A + 2)}. 
\end{cases} \]  (7)

Fig. 1 has these two advertising profit functions. In the sequential-move game, the equilibrium is determined by the location of the maximizer of the first mover’s profit. From (6), it is easy to find the maximizer of firm X’s profit, which is graphically apparent in Fig. 1.

Proposition 2 (sequential-move informative advertising subgame perfect equilibrium). In the game where firms expect the post-advertising Bertrand–Nash equilibrium and sequentially run informative advertisements, there is a unique asymmetric pure-strategy advertising subgame perfect
Nash equilibrium: \((x^*, y^*(x^*)) = ((A + 1)/[A(A + 2)], 1/(A + 2))\). The firms' equilibrium profits are \(V_X(x^*, y^*(x^*)) = (A + 1)^2/[2A(A + 2)^2]\) and \(V_Y(x^*, y^*(x^*)) = 1/[2(A + 2)]\).^1

This proposition shows that (a) the incumbent advertises less than when new entry is impossible but it advertises more than the entrant: \(x^* = [(1/2A) - (1/(A + 2))] = y^*(x^*)\) for \(A \geq 1\). (b) There is a first to mover advantage but the incumbent earns less than when it is a monopolist: \(V_Y(x^*) = [(A + 1)^2/[2A(A + 2)^2]]1/(2A + 2)] = V_Y(x^*, y^*(x^*))\) for \(A \geq 1\).

I am now ready to examine the incumbent’s strategic advertising and the entry decision by the potential entrant. If the sunk fixed cost outweighs the post-entry profit, a firm will not enter the market. Thus, when \(F \geq V^m(x^m)\), even the incumbent does not enter the market. When \(V^m(x^m) \geq F \geq V_Y(x^m, y^*(x^m))\), firm Y’s entry is blocked in the terminology of Bain (1956). Notice that if firm Y enters the market and optimally sets its advertising level, firm Y’s overall profit, \(V_Y(x^m, y^*(x^m)) - F\), is non-positive. Thus, firm X sets \(x^m\) to earn \(V^m(x^m)\), and firm Y does not enter the market. If \(0 \leq F \leq V_Y(x^m, y^*(x^m))\), entry is easy in Bain’s (1956) terminology. In this case, firm Y knows that it can earn a positive overall profit regardless of firm X’s advertising strategy since \(V_Y(x^m, y^*(x^m)) = \min V_Y(x, y^*(x)) = V^m(x, y^*(x))\). Thus, firm X expects that firm Y enters the market, and simply chooses \(x^m\).

The above examination suggests that Schmalensee’s (1983) ‘natural’ strategic entry deterrence game has no possibility of strategic entry deterrence by the incumbent’s precommitment to investment.

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\(^1\)It can be shown that the simultaneous-move advertising game has two asymmetric pure-strategy equilibria and one symmetric mixed-strategy equilibrium: \((x^*, y^*) = (y^*, x^*) = (1/(A + 2), (A + 1)/(A(A + 2)))\); and \(\sigma^x = \sigma^y = [\text{Prob}(1/(A + 2)) = (A + 2)/(2A + 1)]\). Thus, the sequential-move advertising subgame perfect equilibrium coincides with one of simultaneous-move pure-strategy advertising subgame perfect Nash equilibria. This is due to the special property that a firm’s advertising best response becomes a constant when its rival’s advertising level is beyond the critical value.
in informative advertising. This differs from the prediction of Schmalensee’s (1983) ‘Cournot’ entry-deterrence model that the incumbent can optimally deter entry through precommitting to underinvestment in advertising. The peculiar result hinges upon a property that, where \( V^*_x(x, y^*(x)) \) is maximized, \( V^*_x(x, y^*(x)) \) is minimized; given \( V^*_x(x^*, y^*(x^*)) > F \geq 0 \), no \( x \) exists such that \( V^*_x(x, y^*(x)) - F \leq 0 \).

Summarizing considerations so far, I have the following proposition.

**Proposition 3 (Entry subgame perfect equilibrium).** In Schmalensee’s (1983) ‘natural’ strategic entry deterrence game in which two firms sequentially advertise before simultaneously announcing a price, strategic entry deterrence via informative advertising by the incumbent is impossible. The model predicts only three possible entry equilibria, depending on the parameter values of \( A \) and \( F \): (1) no firms enter the market; (2) entry is blockaded; and (3) entry is easy in the terminology of Bain (1956).

### 6. Conclusions

Schmalensee (1983) examined whether an incumbent can prevent entry through strategic precommitment to investment in informative advertising in a homogenous-product market. He wished to examine the ‘natural’ entry-deterrence game where at the post-advertising stage, firms compete in prices. But he assumed that the post-advertising strategic variables are quantities because there is no pure-strategy Nash equilibrium of the post-advertising Bertrand game, and it was unknown that such Bertrand games have a mixed-strategy Nash equilibrium.

This paper dealt with the entry deterrence game of sequentially advertising followed by simultaneous price-setting, which Schmalensee (1983) originally wanted to investigate. The analysis of this game was made possible due to Dasgupta and Maskin’s (1986) Theorem 5 to guarantee the existence of the mixed-strategy Bertrand–Nash equilibrium of the post-advertising game. I have shown that when firms compete in prices at the post-advertising stage, the incumbent cannot strategically deter entry by precommitting to informative advertising investment. This finding highly contrasts Schmalensee’s (1983) prediction that, in the ‘Cournot’ entry-deterrence game, strategic entry deterrence is possible, and optimal entry deterrence involves strategic precommitment to underinvestment in informative advertising by the incumbent. This suggests that an incumbent’s ability to deter entry through informative advertising in a homogenous-product market is sensitive to whether firms compete in prices or quantities after advertising.

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