An explanation for the compass rose pattern

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Abstract

The compass rose pattern cannot be observed in daily returns in the spot foreign exchange (forex) market, but is observable when intraday forex returns are used. Monte Carlo simulations show that the pattern is visible only when the tick/volatility ratio is above some threshold level, a condition that is readily met with intraday returns. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

The recent discovery of the compass rose pattern has sparked considerable interest among researchers (see, e.g., Chen, 1997; Kramer and Runde, 1997; Szpiro, 1998; and Lee et al., 1999). Crack and Ledoit (1996) (CL) first documented the existence of the compass rose pattern, whose graphical representation takes the form of evenly spaced rays emanating from the origin with the most prominent rays pointing in the major directions of the compass, hence the name. CL postulate that the pattern is caused by discrete jumps in prices. As stated by Harris (1991), the discreteness of prices has not been examined in detail, despite its persistence, yet it may have serious implications. For example, Crack and Ledoit (1996) show that chaos tests for low dimensional structure in \( n \)-histories such as the BDS test (Brock et al., 1991) are affected to the extent that evidence supporting the existence of chaos may have been completely driven by stock price discreteness. Similarly, Kramer and Runde (1997) argue that the pattern distorts the null distribution for the BDS chaos test.

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Following up on the CL study, Szpiro (1998) shows that both the compass rose and the grid structure are observable for intraday forex returns. Szpiro argues that the only necessary and sufficient condition for the pattern to exist is that prices should change in discrete jumps. The progression from randomness to the pattern as the discreteness in price changes increases, demonstrated in the Monte Carlo experiment by Kramer and Runde (1997), supports the notion that discreteness indeed is a determinant of the pattern. Kramer and Runde (1997) note that the compass rose may have been masked in previous research due to the use of an excessively large frame size in plots.

However, empirical evidence from futures markets by Lee et al. (1999) (LGM) does not support the pervasiveness of the pattern in non-equity markets. Furthermore, contrary to CL’s claim, effective tick size does not explain the presence or absence of the pattern. LGM (1999) show that the pattern does not exist when daily forex data are used, while Szpiro (1998) finds the pattern for transactions forex data. One possible source for these conflicting findings might be attributed to the different holding periods, one day for LGM and a few minutes for Szpiro. The price changes in the forex market have a typical magnitude of 0.0001 (either $/unit of foreign currency or foreign currency/$) for both daily and intraday forex rates. Despite this small magnitude, the discreteness in price changes should still uniformly manifest itself. The fact that it does not suggests that some other factor may be at work in determining whether the pattern is observed.

We argue that volatility, measured as the standard deviations of returns, may be a determinant for the pattern to exist. Specifically, it is the ratio of the tick size to the volatility of returns (hereafter, the tick/volatility ratio) that has to be above a threshold level before the pattern can be observed.

The purpose of this paper is to use Monte Carlo simulation to show that the tick/volatility ratio accounts for the presence of the compass rose patterns. The methodology is discussed in the second section, the results in the third, followed by the conclusion in the fourth section.

2. Data and methodology

2.1. Data

We obtain 52 daily spot forex rates from the Bridge-CRB Database for up to 27 years for the major currencies, and for at least five years for other currencies. We also obtain 1-min, 5-min and hourly data on the Euro, the British Pound, the Swiss Franc, and the Japanese Yen (all against the US dollar) from Dow Jones Telerate Services for a maximum period from 11/4/98 to 1/22/99.

2.2. Methodology

In general, return for period \( t \), \( R_t \), is calculated as the change of closing price of period \( t \), \( P_t - P_{t-1} \), over the closing price of period \( P_{t-1} \), i.e.,

\[
R_t = (P_t - P_{t-1})/P_{t-1}
\]  

Equivalently, considering that prices move only in discrete jumps \( h \), the ratio of next to current period returns can be expressed as

\[
R_{t+1}/R_t = [(P_{t+1} - P_{t+1})/P_{t+1}]/[(P_t - P_{t-1})/P_{t-1}]
\]  

(2)
which can be simplified to $n_{t+1}h/n_t = n_{t+1}/n_t$, where $n_t$ is the number of ticks that the price change represents.

For investigating influence of the tick/volatility ratio on the presence of pattern, the following Monte Carlo simulation is performed. Six sets of simulated lognormal return series are generated. The mean and standard deviation of these six sets vary, ranging from 0 to 0.0005 for mean, and 0.0002 to 0.006 for standard deviation. These means and standard deviations are chosen to correspond to the actual observed values for daily and intraday returns in forex markets. Within each set, there are 13 separate series, each corresponding to a tick/volatility ratio of 1/100, 1/50, 1/10, 1/5, 1/4, 1/2, 1/1, 2/1, 3/1, 4/1, 5/1, 10/1, and 20/1.

3. Results

3.1. Daily and intraday returns in spot forex markets

The compass rose pattern is not observed for any of the 52 daily rates examined. However, for the intraday holding periods, a clear grid pattern is present for the 1-min and 5-min data intervals. A blurry grid pattern is observed for the 1-h data interval. A grid pattern, rather than the compass rose pattern, emerges due to the use of only 4318 observations. A larger sample size would provide the number of observations needed for the grid pattern to converge to the compass rose pattern. These results affirm LGM’s results for daily forex data, and Szpiro’s results for intraday forex data. These results are not central to this paper and thus are not reported in detail here, but are available from the authors.

4. Monte Carlo simulation

The above results show that despite the discreteness in price changes, the pattern can be observed only with high frequency intraday data and not with daily data. Price discreteness, thus, does not account for these conflicting results. Since tick sizes are of the same magnitude in daily and intraday data, the tick sizes cannot account for the observed difference. We examine volatility as a likely candidate for the phenomenon reported above. The tick/volatility ratio reported in Table 1 shows that this ratio decreases as the holding period for the currencies increases.

Fig. 1 reports the results for the first set of simulated series with mean zero and standard deviation 0.0002. This mean-standard deviation combination is chosen for reporting because it is very close to the actual intraday mean-standard deviation values reported in Table 2. Fig. 1A shows that when the tick/volatility ratio is 1/100, the pattern fails to emerge. However, when the ratio is 1/50, a grid pattern begins to emerge, as shown in Fig. 1B. The pattern becomes clearer as the ratio increases to 1/10, and 1/4, as shown in Figs. 1C and D. The results from Fig. 1 suggest that the compass rose pattern can easily emerge in return series that have very small standard deviations, such as 0.0002. Under these circumstances, the threshold value for the tick/volatility ratio for the pattern to start to emerge is as small as 1/50. Since 1-min and 5-min and hourly returns have small volatility, they easily satisfy the requirement that their tick/volatility ratios exceed the threshold value, hence the existence of the pattern in these intraday series.
Table 1
Means and standard deviations of returns and tick/volatility ratios of intraday forex rates

<table>
<thead>
<tr>
<th>Holding period</th>
<th>Currency</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EURO</td>
<td>CHF</td>
<td>GBP</td>
<td>JPY</td>
</tr>
<tr>
<td>1-MIN Standard deviation</td>
<td>0.00043</td>
<td>0.00055</td>
<td>0.00024</td>
<td>0.00038</td>
</tr>
<tr>
<td>Mean</td>
<td>-8E−07</td>
<td>8.5E−06</td>
<td>7.7E−07</td>
<td>4.2E−07</td>
</tr>
<tr>
<td>Tick/volatility</td>
<td>1/4.3</td>
<td>1/5.5</td>
<td>1/2.4</td>
<td>1/3.8</td>
</tr>
<tr>
<td>5-MIN Standard deviation</td>
<td>0.00022</td>
<td>0.00069</td>
<td>0.00039</td>
<td>0.0008</td>
</tr>
<tr>
<td>Mean</td>
<td>-5E−06</td>
<td>2.4E−06</td>
<td>-3E−06</td>
<td>-1E−06</td>
</tr>
<tr>
<td>Tick/volatility</td>
<td>1/2.2</td>
<td>1/6.9</td>
<td>1/3.9</td>
<td>1/3.8</td>
</tr>
<tr>
<td>HOUR Standard deviation</td>
<td>0.00136</td>
<td>0.00172</td>
<td>0.00104</td>
<td>0.00218</td>
</tr>
<tr>
<td>Mean</td>
<td>-1E−05</td>
<td>-2E−05</td>
<td>3.3E−05</td>
<td>-7E−05</td>
</tr>
<tr>
<td>Tick/volatility</td>
<td>1/13.6</td>
<td>1/17.2</td>
<td>1/10.4</td>
<td>1/21.8</td>
</tr>
<tr>
<td>DAY Standard deviation</td>
<td>0.00555</td>
<td>0.00688</td>
<td>0.006</td>
<td>0.0073</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0001</td>
<td>-0.0001</td>
<td>4.1E−05</td>
<td>0.00012</td>
</tr>
<tr>
<td>Tick/volatility</td>
<td>1/55.5</td>
<td>1/68.8</td>
<td>1/60</td>
<td>1/73</td>
</tr>
</tbody>
</table>

*Tick/volatility: Tick/Standard Deviation, where tick has a value of 0.0001.

Simulation results with mean 0.0005 and standard deviation 0.006 are shown in Fig. 2. Again, this combination of mean and standard deviation is chosen for reporting because it is closer to those of the daily returns in the forex markets. Figs. 2A–B report the graphs for the returns when the tick/volatility ratio is 1/100 and 1/10. No patterns are noticed for these cases.

However, as the frequency of the data increases (i.e., the tick/volatility ratio increases) the pattern is brought into focus. In Fig. 2C, when the ratio is 1/2, the pattern is clearly present, notably for the main directions of the compass. The pattern comes into much sharper focus in Fig. 2D, when the ratio is 1/1. Fig. 2 shows that for returns with larger standard deviations, as in the case of daily series, the threshold value for the emergence of the pattern is much higher. In view of the standard deviations and the tick/volatility ratio of daily returns reported in Table 1, the absence of the pattern is due to the fact that these returns have a tick/volatility ratio smaller than the threshold values that separate the regimes of pattern and no-pattern. Similar results are obtained for the other series mentioned in the methodology section.

5. Conclusion

The results show that the compass rose pattern is not observable for daily data, but is observable for intraday data. These results suggest that a factor other than the discreteness of price changes for varying holding periods may account for the differences in results. Monte Carlo simulations of the tick/volatility ratio show that the pattern is observed only if the tick/volatility ratio exceeds some threshold level. For the case that the standard deviation is 0.0002, the threshold level is very small. Hence, we see that the pattern emerges when the ratio is 1/50. With a greater value for the standard deviation, such as 0.006, the threshold is 1/4. Since intraday returns fall into the former category of a smaller standard deviation while daily returns fall into the latter, the simulation results explain why
Fig. 1. Monte Carlo simulation results. (a) Mean = 0; Standard Deviation = 0.0002; Tick/Volatility ratio = 1/100; (b) Mean = 0; Standard Deviation = 0.0002; Tick/Volatility ratio = 1/50; (c) Mean = 0; Standard Deviation = 0.0002; Tick/Volatility ratio = 1/10; (d) Mean = 0; Standard Deviation = 0.0002; Tick/Volatility ratio = 1/4.
Fig. 2. Monte Carlo simulation results. (a) Mean = 0.0005; Standard Deviation = 0.006; Tick/Volatility ratio = 1/100; (b) Mean = 0.0005; Standard Deviation = 0.006; Tick/Volatility ratio = 1/10; (c) Mean = 0.0005; Standard Deviation = 0.006; Tick/Volatility ratio = 1/2; (d) Mean = 00005; Standard Deviation = 0.006; Tick/Volatility ratio = 1/1.
the pattern exists in intraday series but not in daily series. The evidence presented demonstrates that the tick/volatility ratio is a determinant of the compass rose pattern.

References


