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Labor supply under wage uncertainty

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Abstract

We consider an agent in an atemporal setting facing an increase in the uncertainty of her uncertain wage. More labor is supplied, given small initial wage uncertainty, by a person with a CES utility function with an elasticity greater than unity. We relate this result to an extended measure of decreasing relative risk aversion. A Cobb–Douglas person makes no change in her labor supply in response to increased wage uncertainty. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

Does more wage uncertainty, ex ante, lead to more labor committed, or less? This is the topic we investigate in the simple, single agent, single period model of labor–leisure choice. The case of the agent with a Cobb–Douglas utility function provides us with a benchmark because wage uncertainty has no effect on labor supply for this case. Our approach is to consider the elasticity of substitution between ‘consumption’ and leisure and to see when increased wage uncertainty leads to more labor supplied. The answer turns out to require considerable qualification. The clearest result we get is that for ‘small’ initial uncertainty, a mean preserving spread in the uncertain wage results in more labor being supplied, when the agent’s elasticity of substitution is greater than unity.

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It is obvious that a worker who works as a private contractor incurs more income variability over, say, a period of years than if she worked for a large organization, on salary. In fact, as a private contractor she probably discovers her implicit wage for the past year only when she files her income tax. She has implicitly committed herself to a labor supply, contingent on a wage that gets revealed at the end of the ‘period’ of work. This is a framework of labor supply committed in advance of the realization of an uncertain wage.

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The agent must commit, in advance, to a specific labor supply. The wage she receives is revealed at the end of the period. She knows the distribution of her uncertain wage. There are two goods, leisure, $s = (L - N)$, and ‘consumption’, $x$ with $\$1x$ equal to $wN$, where $w$ is the wage rate, $N$ is hours worked, and $L$ is time available. We observe that for an increase in wage uncertainty to result in more (less) labor put out, it is sufficient that the agent’s first-order condition is convex (concave) in the random wage and that the slope of the first-order condition, with respect to the wage, has a single sign. These conditions turn out not to be satisfied for an agent with a CES utility function, with an elasticity of substitution, $\sigma$, not equal to unity. For the special case of Cobb–Douglas preferences ($\sigma = 1$), we observe that an increase in uncertainty in the uncertain wage induces no change in labor supply. (This is a consequence of a Cobb–Douglas person selecting her labor supply independently of the wage.) For the case, $\sigma < 1$, we observe that the agent’s first-order condition, under certainty, is not uniformly concave or convex in the wage, given $N$ held constant. The case of $\sigma$ greater than unity turns out to involve a uniformly convex first-order condition but one that is not monotonic in the wage. Here, however, we get the weak result: for small amounts of initial wage uncertainty,$^2$ an increase in wage uncertainty (a mean preserving spread) results in more labor put out by the agent. But, in general, the assumption of CES preferences is not sufficient to sign the direction of the agent’s response to more wage uncertainty by ‘direct’ means. We mean ‘by direct means’, the use of our sufficiency condition in Eq. (1). Since CES preferences are representable in homothetic indifference curve maps, we can rule out homothetic indifference curves as sufficient to sign the direction of the agent’s response to more wage uncertainty by direct means. In carrying out our analysis, we meet an expression for relative risk aversion, which is an extension of the Arrow–Pratt measure. In the case where an agent takes on more risk, in an intuitive sense, we observe that her extended measure of relative risk aversion is decreasing in the uncertain wage. A CES agent’s elasticity of substitution figures directly in the question of whether she works more or less in response to a wage increase in a certainty environment. Thus, there are a number of links in the chain of ‘reasons’ for an agent’s response to increased wage uncertainty.

2. Model

For a certainty situation, our agent maximizes $U(wN, L - N)$ with respect to labor time, $N$. $U(\cdot, \cdot)$ is assumed to be smoothly increasing and concave in its arguments and to possess Inada regularity conditions for the case of the arguments tending to zero. We also require well-behaved second and third derivatives. The first-order condition is $wU_w - U_\gamma = 0$. The second-order condition is $\partial Z / \partial N (\equiv E) < 0$, for $Z \equiv wU_\gamma - U_\gamma$. The comparative statics result on more or less labor time, given a wage increase, is

$$\frac{dN}{dw} = \left\{ - \frac{\partial Z}{\partial w} \right\} / E$$

at solution value $N^*$, where

$^2$‘Small amounts’ here means that the support of the distribution of wages does not range far from the mean of the wage. More on this below.
\[ \frac{-\partial Z}{\partial w} = -U_x - xu_{ss} + xu_{ss}/w \]

\[ \text{d}N/\text{d}w \text{ is positive (negative) as } \{-U_x - xu_{ss} + xu_{ss}/w\} < (>) 0 \text{ or as } I < (>) 1, \text{ for} \]

\[ I = -\frac{xU_{ss}}{U_x} + \frac{xU_{ss}}{U_x} \]

\( I \) is our extended Arrow–Pratt measure of relative risk aversion. It is a measure of the impact on the agent of variability in \( x \). The use of

\[ \frac{\partial U_x}{\partial w} = w \frac{\partial U_x}{\partial w} + U_x \]

and straightforward differentiation yields

\[ \frac{\partial I}{\partial w} = \left( \left( -\frac{\partial U_x}{\partial w} \right) I - \frac{\partial^2 U_x}{\partial w^2} + \frac{1}{w} \frac{\partial xU_{ss}}{\partial w} - \frac{xU_{ss}}{w^2} \right) / U_x \]

We refer to \( \partial I/\partial w \geq (\leq) 0 \) as increasing (decreasing) relative risk aversion. And one also obtains

\[ \frac{-\partial^2 Z}{\partial w^2} = \left( -\frac{\partial U_x}{\partial w} \right) - \frac{\partial^2 U_x}{\partial w^2} + \frac{1}{w} \frac{\partial xU_{ss}}{\partial w} - \frac{xU_{ss}}{w^2} \]

Note that \( -\partial U_x/\partial w \) is positive. It follows directly:

**Proposition 1.** \( \partial I/\partial w < 0 \text{ and } I > 1 \implies \partial^2 Z/\partial w^2 > 0. \) \( \text{And } \partial I/\partial w > 0 \text{ and } I < 1 \implies \partial^2 Z/\partial w^2 < 0. \)

Also, we obtain directly:

**Proposition 2.** \( \partial^2 Z/\partial w^2 > 0 \text{ and } I < 1 \implies \partial I/\partial w < 0. \) \( \text{And } \partial^2 Z/\partial w^2 < 0 \text{ and } I > 1 \implies \partial I/\partial w > 0. \)

This second proposition relates to our result below for the case of the agent with CES preferences, responding to increased wage uncertainty. Her behavior in one instance implies that she is exhibiting declining relative risk aversion, for the case of relative risk aversion being measured by our index, \( I \).

3. Wage uncertainty

Under wage uncertainty, the agent faces \( n \) (set at 3 here, without loss of generality) possible ‘future’ wages, with \( w_1 \) occurring with probability \( \pi_1 \), \( w_2 \) occurring with probability \( \pi_2 \), and \( w_2 \) with probability \( 1 - \pi_1 - \pi_3 \). We assume that \( w_1 < w_2 < w_3 \) and that the probabilities are each positive. Under wage uncertainty, the agent selects \( N \) to maximize

\[ \pi_1 U(w_1N,L-N) + (1-\pi_1-\pi_3)U(w_2N,L-N) + \pi_3 U(w_3N,L-N) \]
The first-order condition is:
\[ \pi_1 Z_1 + (1 - \pi_1 - \pi_3)Z_2 + \pi_3 Z_3 = 0 \]
for \( Z_i = w_i U_i (w_i N^*, L - N^*) - U_i (w_i N^*, L - N^*) \), \( i = 1, 2, 3 \). We sketch two possible equilibria in Fig. 1(a) and (b).

The second-order condition is \( D < 0 \), for
\[ D = \pi_1 \frac{\partial Z_1}{\partial N} + (1 - \pi_1 - \pi_3)\frac{\partial Z_2}{\partial N} + \pi_3 \frac{\partial Z_3}{\partial N} \]

A mean preserving spread in the uncertain wage is a pair \((d\pi_1, d\pi_3)\), both positive, satisfying
\[ d\bar{w} = [w_1 - w_2]d\pi_1 + [w_3 - w_2]d\pi_3 = 0 \]
where \( \bar{w} \) is the mean wage. This gives us
\[ d\pi_3 = \left[ \frac{w_2 - w_1}{w_3 - w_2} \right] d\pi_1 \]

The mean preserving spread induces a change, \( dN \), satisfying
\[ DdN + [Z_1 - Z_2]d\pi_1 + [Z_3 - Z_2]d\pi_3 = 0 \]
We substitute for \( d\pi_3 \) to obtain
\[ dN = \left\{ \left[ \frac{Z_2 - Z_1}{w_2 - w_1} \right] - \left[ \frac{Z_3 - Z_2}{w_3 - w_2} \right] \right\} [w_2 - w_1]d\pi_1 / D \]  

(1)

Now

Fig. 1. First-order condition \((Z=0)\) and random wave, \( w \).
\[
\left[ \frac{Z_2 - Z_1}{w_2 - w_1} \right]
\]
is the discrete approximation of \( \partial Z / \partial w \) at an appropriate point. Similarly for
\[
\left[ \frac{Z_3 - Z_2}{w_3 - w_2} \right]
\]
Hence, if \( Z \) is both monotonic in \( w \), and uniformly concave or convex in \( w \) for \( w_1 \leq w \leq w_3 \) we can sign \( dN \), using Eq. (1).\(^3\)

4. CES utility function

4.1. Cobb–Douglas case

For \( U = (wN)^\alpha (L - N)^{1-\alpha} \) the agent’s choice of \( N \), given wage \( w \), satisfies
\[
\left\{ \frac{\alpha}{N} - \frac{(1 - \alpha)}{L - N} \right\} = 0
\]
or is independent of the wage. Given the solution \( N^* \), it follows that \( \partial Z(w; N^*) / \partial w = 0 \). Hence the agent makes no change in her labor supply, in response to an increase in wage uncertainty.

4.2. CES case

For \( U = [a(wN)^{-\beta} + b(L - N)^{-\beta}]^{1/(1 + \beta)} \), \( a \) and \( b \) positive and \( \beta > -1 \), \( \beta \neq 0 \), we have \( \sigma = 1 / (1 + \beta) \) as the elasticity of substitution. One obtains straightaway, for an interior solution \( N^* \),\(^4\) the comparative statics result \( dN^*/dw > (\langle \rangle 0 \) as \( \sigma > (\langle \rangle 1 \). One can see these results in an indifference curve diagram, with the Cobb–Douglas case as a benchmark. We proceeded to calculate \( Z(w; N^*) \), namely \( wU_x - U_y \), for various choices of parameters. Over the range of \( 0 < w < \infty \) we could not rule out \( \partial Z(w; N)/\partial w \) changing sign as \( w \) was increased for either \( \sigma > 1 \) or \( \sigma < 1 \). See Fig. 2(a) for the case of \( \sigma > 1 \) (\( a = 0.5, b = 0.5, L = 24, N = 23 \), and \( \beta = -0.9 \)).

However, \( Z \) is uniformly convex in \( w \) for this case, \( \sigma > 1 \).\(^5\) In Fig. 2(b), we have results for the case of the elasticity of substitution less than unity, i.e. \( \beta \) set at 1.05 and \( N = 10 \), and then \( N = 15 \). We observe the sign reversal for \( \partial Z/\partial w \) as \( w \) is moved down toward zero. In addition \( Z \) is neither

\(^3\)This is a variant of the Rothschild and Stiglitz (1971) algorithm: a mean preserving spread in a random variable causes the agent to increase (decrease) the value of her control, as the first-order condition is convex (concave) in the random variable.

\(^4\)With a CES utility function, one can get corner solutions when \( \sigma > 1 \). \( Z(w; N^*) \) would not in general equal zero for this case. There seems to be no light to be shed on the question of more wage uncertainty for these special cases.

\(^5\)For values of \( N \) around 10, we observed no change in the sign of \( \partial Z/\partial w \) for values of \( w \) down to 0.00001. This is supportive of our ‘result’ below: for small initial wage uncertainty, the \( \sigma > 1 \) agent puts out more labor in response to increased wage uncertainty.
uniformly concave or convex in \( w \) for this case (\( \sigma < 1 \)). (And the point of inflection is not at \( Z = 0 \).)\(^6\) It is (a) the monotonicity of \( Z \) in \( w \), and (b) the uniformity of either the convexity or concavity of \( Z \) in \( w \), for \( w \) positive, that allows us to use our condition in Eq. (1) to sign the agent’s response to more wage uncertainty. We have seen that both conditions are not satisfied in our numerical examples with the CES utility function.

For the case of \( \sigma > 1 \) and \( N^* \) around 8, we observe increasingness in \( Z \) and convexity in the

\(^6\)The case of \( \beta \to \infty \) yields Leontief indifference curves and would appear to exhibit no sign reversal in \( \text{d}Z/\text{d}w \) as \( w \) is varied systematically. However, this is a polar case with \( U_s \) and \( U_i \) not single valued and there seems no reason to make much of it, though it is a member of the CES family of cases.
neighborhood of $N^*$. Hence for the restricted case of small initial wage uncertainty and $\sigma > 1$, we have the result, making use of Eq. (1): more wage uncertainty induces the agent to work harder, i.e. $dN$ is positive under more wage uncertainty. And this case satisfies $I < 1$ (i.e. the agent is not highly risk averse or can cope more easily with increased variability in $x$). And from Proposition 2, we know that the agent exhibits decreasing relative risk aversion in this case. She certainly takes on more variability in her value of $x$, ex ante, by working harder under more wage uncertainty. This should reduce the variability ex ante in her values of $U_s$ and increase the variability in her values of $U_s$.

5. Concluding remark

The central message is that, even with tastes as regular as those associated with a CES utility function (i.e. homothetic indifference curves) we have been unable to sign, in general, the agent’s response to more wage uncertainty, by direct means. Our positive result is that for small initial wage uncertainty and an agent with an elasticity of substitution greater than unity, we can infer that more labor will be put out under increased wage uncertainty. And in this case, the agent exhibits a relatively low level of extended relative risk aversion as well as decliningness in this value of extended relative risk aversion. For the case of Cobb–Douglas preferences, the agent makes no change in her labor output, in response to increased wage uncertainty. A rough summary of this research would be: it is not obvious that increased wage uncertainty per se leads to more labor supplied. And our analysis here ignores ‘outside’ options which the agent can avail of herself to mitigate the adverse effects of an increase in wage uncertainty. It would, of course, be unreasonable to leap to conclusions about labor supply in actual economies on the basis of ‘predictions’ from our circumscribed model of individual labor supply here.

References


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7 $\partial Z/\partial w$ changes sign for extremely small values of $w$.
8 We mean that her ‘equilibrium’ values of $Z_1$, $Z_2$, and $Z_3$ are clustered close to the corresponding value of $Z$, equal to zero. See Fig. 1.
9 We do not have the result here: decreasingness in her risk aversion index implies that she increases her labor supply.
10 This result seems to capture aspects of ‘the rat race’. The added burden of increased wage uncertainty, ex ante, leads the agent to work harder.
11 Bodie et al. (1992) incorporate portfolio choice at each date for their agent, in addition to a labor–leisure choice. They also allow for a variable interval of working, before retirement.