Growth and differentiated oligopoly

Roberto Cellini*

University of Catania, Faculty of Economics, Corso Italia 55-95129, Catania, Italy

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Abstract

The paper presents a model of endogenous growth and strategic interdependence among firms, in the form of differentiated oligopoly. It shows that the degree of interdependence among firms affects the growth rate of available varieties along a balanced growth path. The resulting link between market interdependence and varieties’ growth may be positive or negative, because opposite forces are at work. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

When models of growth abandon the hypothesis of perfect competition, they generally focus on monopolistic competition (Grossman and Helpman, 1991; Aghion and Howitt, 1992, among many others). By contrast, the size of the available literature on growth in the presence of oligopoly is rather limited (Aghion et al., 1997; Peretto, 1998, and Vencatchellum, 1998, are remarkable contributions). This is partly due to the difficulty of finding a nice formulation of the market equilibrium under oligopoly, consistent with a balanced growth path.

This paper proposes a very simple model of oligopoly and growth, with two ingredients. First, it considers firms competing in a differentiated oligopoly: as is known, this form of competition (analyzed by Spence, 1976; Singh and Vives, 1984; and Cellini and Lambertini, 1998, among others) encompasses traditional oligopoly with homogeneous goods, and monopolistic competition. Second, the model considers the creation of new varieties, according to the widely accepted technology à la Lucas (1988) and Grossman and Helpman (1991). The major limitation of the model is represented by its partial equilibrium approach.
The main question here concerns how the growth rate of available varieties is affected by the degree of strategic interdependence among firms in the market stage. The model shows that the strategic interdependence may increase or reduce the rate at which new varieties are introduced along the balanced growth path, depending on the configuration of parameters. This result resembles the results of recent models of growth with innovation that show that the relationship between intensity of market competition and growth is ambiguous (e.g., Aghion et al., 1997).

The paper is organized as follows. Section 2 presents the model, introducing – in turn – the static market equilibrium allocation, the technology for creating new varieties, and the dynamics of the system. Concluding comments are in Section 3.

2. The model

2.1. The market equilibrium in a static framework

Consider an economy with a sector producing a set of differentiated industrial goods, \( \{q_{ij}\}_{i=1}^{N} \). In particular, following the available literature, consider a continuum of differentiated goods, defined over the compact interval \([0, N]\). Assume that each differentiated industrial good is produced by a single-product firm, competing under differentiated oligopoly.

As to the formulation of demand side, we propose the following inverse demand function for the good of variety \( i \), in each time period \( t \) (index \( t \) is suppressed in this section to ease notation):

\[
p_i = \frac{A}{q_i^\alpha \left( \int_0^N q_j^\beta dq_j \right)^{1-\beta}} \tag{1}
\]

Parameters \( A \), \( \alpha \), \( \beta \) are assumed to obey: \( A > 0 \), \( 0 < \alpha < 1 \), \( 0 \leq \beta \leq 1/2 \). Conditions regarding \( \beta \) can be easily understood if we consider the demand function under the symmetry assumption \( q_i = Q/N = q \), \( p_i = p \), for any \( i \), where \( Q \) denotes the aggregate quantity of industrial goods and \( N \) the number of available varieties. Under such an assumption the demand curve for variety \( i \) turns out to be:

\[
p_i = \frac{A}{(q_i \cdot Q^{\beta(1-\beta)N})^\alpha} \tag{1'}
\]

and now it is immediate to realize that for \( \beta = 0 \) the demand for any variety is independent on the produced quantity of different varieties (so resembling monopolistic competition), while homogeneous oligopoly emerges if \( \beta = 1/2 \). Put differently, the higher the level of \( \beta \) (within the interval \([0, 1/2]\)),

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1We can not assume the widely used linear function à la Spence (1976): \( p_i = A - Bq_i - D \sum_{j} q_j \), \( (A > 0, B > 0, 0 \leq D \leq B) \), consistent with a quadratic utility function. In Spence’s formulation, parameter \( D \) represents the symmetric degree of substitutability between any pair of varieties; if \( D = 0 \) products are independent and each firm behaves as a monopolist. Unfortunately, the linear formulation of market demand turns out to be inconsistent with any reasonable form of balanced growth, and so we have to propose here a different formulation.
the higher the degree of interdependence among firms. More formally, notice that \(-dp_i/dq_j\) is zero when \(\beta=0\), and then it is increasing in \(\beta\) over the sensible interval. An interpretation of \(\beta\), with respect to the underlying preference structure of consumers, is immediate: it may be seen as an indicator for the (symmetric) degree of substitutability between any pair of different goods. Needless to say, the higher the substitutability among goods, the higher the firms’ interdependence in market. Note also that, given the assumption of continuity, the quantity of any single \(q\) has no influence on \(Q\).

Assume that each industrial good is produced through labor \((l)\) and normalize the input in such a way that its average and marginal productivity are equal to one:

\[ q_i = l_i, \quad (2) \]

Hence, the corresponding cost function is \(c(q_i) = wq_i\), (where \(w\) denotes the wage rate), and the labor force devoted to the production of industrial goods, \(L_Q\), is equal to \(Q\).

Each firm chooses its level of production to maximize its own profit and perceives the level of production of different goods (and hence the global level of industrial goods’ production) as given. The problem can be summarized as follows:

\[
\text{Max}_{q_i} \quad \pi_i = p_i q_i - wq_i \\
\text{s.t.:} \quad \text{Eq. (1)}
\]

The first order condition of the above problem gives the reaction curve for firm \(i\), i.e., the optimal decisions of firm \(i\) for any given levels of \(q_j, j \neq i\):

\[ q_i = \left(\frac{A(1-\alpha)^{1/\alpha}}{w}\right) \cdot \frac{1}{\left(\int_0^N q_j^\beta dj\right)^{1/\beta}} \quad (4) \]

Under the symmetry assumption \(q_i = q\forall i\), Eq. (4) becomes:

\[ q = \left(\frac{A(1-\alpha)^{1/\alpha}}{w}\right) \cdot \frac{1}{Q^{\beta/(1-\beta)N}} \quad (4') \]

and the Nash–Cournot equilibrium level of production of any single good may be easily obtained, by substituting \(Q/N = q\):

\[ q = \frac{1}{N} \left(\frac{A(1-\alpha)}{w}\right)^{1-\beta/\alpha} \quad (5) \]

The corresponding maximum profit is

\[ \pi^* = \frac{\alpha}{N} A^{1-\beta/\alpha} \left(\frac{1-\alpha}{w}\right)^{1-\beta/\alpha} \quad (6) \]

Note that it is sensible to impose \(\alpha + \beta < 1\), in order to have maximum profit decreasing in \(w\).

The particular formulation of the demand curve leads to an optimal quantity produced by each firm which is inversely proportional to the number of existing varieties, \(N\). The same holds for the profit of any single firms. On the contrary, the equilibrium price of any good, (namely, \(p = w/[1-\alpha]\)), and the
sum of profits of all industrial firms do not depend on $N$. The same holds for the consumer surplus evaluated at the equilibrium point of the markets for the differentiated goods. In this respect, the demand side considered in the present model does not reflect a preference for the variety, differently from what happens in formulations à la Dixit and Stiglitz (1977). However, this is not tantamount to saying that variety is not positive for the society. On the opposite, we may well assume a positive, external effect of $N$ on the social welfare. Moreover, notice that the higher is $\beta$, the lower is $q$: this result – namely, the inverse relationship between the degree of substitutability among goods and the optimal production under Cournot behaviour – is common in the models of differentiated oligopoly (see, e.g., Singh and Vives, 1984).

2.2. The production of new varieties

Assume that a sector producing innovation in varieties is operative, according to the technology

$$\frac{dN_t}{dt} = \frac{1}{a} \cdot (L^r)_t \cdot N_t$$

(7)

where $a$ is a positive parameter, $L^r$ is the labor force employed by this sector, $N$ is the number of existing varieties, and $t$ stays for time. Eq. (7) is clearly borrowed from Grossman and Helpman (1991), who based it on a similar formulation proposed by Lucas (1988) for the accumulation of human capital; its features and shortcomings are very well known (see, e.g., Solow, 1992).

Now we express the labor force employed in the research sector as the difference between the industrial labor force $L$ and the labor force employed in the production of differentiated goods: $L^r = L - L^Q$. From Eq. (2), together with the symmetry condition, we derive $L^r = L - Q$. Denoting percentage growth rate by a hat, Eq. (7) may be rewritten as:

$$\hat{N}_t = \frac{1}{a} (L - Q_t)$$

(8)

($L$ is assumed to be constant over time). Consistently, the cost of the creation of a new variety at time $t$ is: $C_C = a/N_t$. The fact that $N_t$ enters the innovation cost function inversely is due to the positive external effect of the number of existing varieties on the process of discovering new varieties.

Still following Grossman and Helpman (1991) we assume free entry in the research sector, so that the actual value of future profits deriving from any variety discovered at time $t$, denoted by $v_t$, is equal to its creation cost; formally:

$$v_t = \int_0^\infty \pi(\tau) e^{-\tau t} d\tau = \frac{a}{N_t}$$

(9)

Provided that $a$ is a constant parameter, we immediately derive:

$$\hat{v}_t = -\hat{N}_t$$

(10)

2Trivial algebra leads to find the consumer surplus in the sector of differentiated goods, $CS = A^{1-\beta}w_0/\alpha/(1-\alpha)^{1+\beta}$.\footnote{Trivial algebra leads to find the consumer surplus in the sector of differentiated goods, $CS = A^{1-\beta}w_0/\alpha/(1-\alpha)^{1+\beta}$.}
Verbally, the percentage variation of the actual value of profit of new varieties has to be the opposite of the percentage variation of the number of existing varieties.

The Fisher equation, where \( r \) denotes the interest rate, is in the present case:

\[
\frac{\pi_t}{v_t} + \frac{\dot{\pi}_t}{v_t} = r_t \implies \frac{\alpha}{A^\beta} \left( \frac{1 - \alpha}{\omega} \right) \frac{1 - \alpha - \beta}{\alpha} - \dot{N}_t = r_t
\]  

(11)

As usual, it states the equality between the current interest rate, and the sum of current return of any new variety and the associate capital gain.

2.3. Dynamics

The dynamics of the industrial sector is represented by the system of Eqs. (8) and (11). Notice that, unlike in the Grossman–Helpman model, \( Q \) does not enter the Fisher equation, so that it is pointless to look for a steady state in \((Q, \dot{N})\). In fact \( Q \), is necessarily constant over time in the present model; this is due to the fact that the demand structure leads firms to modify individual production in such a way that \((dq/dt)/q = -(dN/dt)/N\).

We may interpret the dynamic system of Eqs. (8) and (11) in different ways. First, we could take \( r \) as endogenous and note that its value derives from the solution of the system; put differently, there is only one level of \( r \) consistent with the balanced growth rate — specifically the level obtained by substituting the constant level of \( \dot{N} \) from Eq. (8) into Eq. (11). More interestingly, we could imagine that \( L \) is a (positive) function of \( r \). In this case Fig. 1 provides a graphical representation of the system.

The behavior of the economy may be summarized as follows. The optimal market choices of firms competing in a differentiated oligopoly are such that the aggregate labor employed in production, and the aggregate production itself, are constant over time. There is a technology creating new varieties, through labor not used in the production process. The labor productivity in this process benefits from the number of already existing varieties. The higher the interest rate, the larger the labor force, and the higher, \textit{ceteris paribus}, the rate of growth of varieties. However, the arbitrage condition from the Fisher equation entails a unique level of interest rate consistent with a steady state. In this steady state, \( \dot{N} \) and \( r \) (along with \( Q \)) are constant over time. Varieties increase at the percentage growth rate \( \dot{N} \) and the production of any single variety decreases at the same rate.

We are interested in performing comparative statics exercises. In particular we want to evaluate how parameter \( \beta \) affects the steady state growth. To this end, consider the position of the two loci in

![Fig. 1.](image-url)
Fig. 1. The higher is $\beta$, the lower the position of the negative curve corresponding to Eq. (11) in the space $(r, \hat{N})$, and the higher the position of the positive curve corresponding to Eq. (8). Accordingly, the interest rate is the lower, the higher is $\beta$, while the effect of a larger value of $\beta$ on the growth rate of varieties is ambiguous. This is the core result of the paper: the degree of strategic interdependence among firms in the market-competition phase affects the percentage growth rate of new varieties along the balanced growth path. The sign of this effect may be positive or negative. From one side, the higher is the degree of interdependence, the lower the level of production, and thus the larger the amount of (labour) resources available for R&D; this effect corresponds to the upward shift of the positive curve in Fig. 1. From the other side, the higher is the degree of interdependence, the lower the current profit and the smaller the incentive to introduce new varieties; this effect corresponds to the downward shift of the negative curve in Fig. 1.

A result with similar flavor – the ambiguous relation between the market competition and the growth rate of new goods’ creation – emerges also in recent models of growth; in particular, Aghion et al. (1997) show that, unlike in several available Schumpeterian models, more intense market competition (and/or imitation) may be growth-enhancing. In a static framework, Boone (1998) shows that a rise in competition may increase firms’ incentive to innovate.

An alternative way to ‘close’ our system is to consider $r$ as an exogenous parameter and $L$ as a function depending (positively) on $w$. In such a case, a diagram similar to that of Fig. 1 is appropriate, presenting $w$ instead of $r$. (The locus corresponding to Eq. (11) in the space $(w, \hat{N})$, is negatively sloped as long as $\beta + \alpha$ is smaller than 1). The intersection between the two loci determines the level of $w$ and $\hat{N}$ consistent with balanced growth. Also in this case, comparative statics leads to the conclusion that a larger degree of strategic interdependence may be associated with a smaller, as well as a larger, growth rate of new varieties’ creation.

The consideration of either $r$ or $w$ as an exogenous parameter may be justified by the partial equilibrium approach of the present model. Nevertheless, we are ready to admit that this shortcoming should be overcome by a more complex model.

Lastly, it is worth noting that this paper does not study the relationship between the fierceness of competition, in the sense of Cournot vs. Bertrand behaviour, for a given degree of interdependence, and the performance of growth; to this respect, we refer to Aghion et al. (1997), and (for the case of differentiated oligopoly) to Cellini et al. (1999). Moreover, this model does not deal with the issue of the efficiency of market allocation. In this respect, Benassy (1998) shows that the too low level of research (which is deemed to be a typical result in models of endogenous growth with expanding varieties) derives from the specific values of parameters capturing ‘returns to specialization’ and ‘monopolistic markup’; in the general case, no a-priori ranking exists between the market growth rate and the socially optimal growth rate. However, it is worth stressing that, in the mentioned strands of literature, there is no room for the degree of market interdependence among firms in shaping the growth of output and varieties.

\footnote{Proof: Recall from Eq. (5) that $Q = \left(\frac{A(1 - \alpha)}{w}\right)^{1-\beta/\alpha}$, and note that the higher is the degree of market interdependence, $\beta$, the smaller the equilibrium output of Cournot oligopolists, $Q$, and consequently the larger the intercept of the curve corresponding to Eq. (8) in Fig. 1.}
3. Concluding comments

We have analyzed a simple model that joins imperfect competition among firms in the form of differentiated oligopoly (that encompasses traditional oligopoly à la Cournot and monopolistic competition à la Chamberlin), and growth in the available number of goods’ varieties. A balanced steady state growth may exist under reasonable formulation for the market demand side.

The main result concerns the ambiguous effect of the degree of strategic interdependence among firms (in the market stage) upon the growth rate of varieties along a balanced growth rate. Opposite forces are at work, so that the association between the growth rate of new varieties and the interdependence among firms in quantity-setting may be positive as well as negative. From one side, a higher interdependence among firms entails lower current instantaneous returns rate and – via the Euler equation – a higher capital gain in the form of variation of actual value of profits from new variety, to preserve equilibrium. This translates into a lower innovation rate. On the other side, a higher interdependence leads to a lower level of production, as it is usual in models of differentiated oligopoly. This effect leaves more (labor) resources available to research, and so possibly translates into a higher innovation rate.

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