Mean reversion in the real exchange rates

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Abstract

Robinson’s [J. Am. Stat. Assoc. 89 (1994) 1420] fractionally-based tests for unit roots and other hypotheses are applied to real exchange rate data between U.S. and five industrialized countries. The results indicate that the series are fractionally integrated with mean reversion if the disturbances are autocorrelated. © 2000 Elsevier Science S.A. All rights reserved.

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JEL classification: C22

1. Introduction

The work on fractional differencing suggests that many macroeconomic time series might be modelled as I(\(d\)) processes where \(d\) can be any real number (see, e.g. Diebold and Rudebusch, 1989; Sowell, 1992; Gil-Alana and Robinson, 1997, etc.). A series is said to be I(\(d\)) if it becomes I(0) after applying the difference operator \((1 - L)^d\), which can be defined in terms of its binomial expansion as

\[
(1 - L)^d = \sum_{j=0}^{\infty} (-1)^j \binom{d}{j} L^j = 1 - dL + \frac{d(d-1)}{2!} L^2 - \cdots
\]

This type of process belongs to the class of long-memory processes, so-named for their ability to display significant dependence between observations widely separated in time, as opposed to I(0) or short-memory processes, where autocorrelations decay fairly rapid. The distinction between I(\(d\)) processes with different values of \(d\) is also important from an economic point of view: If \(d\) belongs to the interval (0, 0.5), the process is stationary and mean-reverting; if \(d\) belongs to [0.5, 1) is

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nonstationary but still mean reverting, while \( d \geq 1 \) means nonstationary and non-mean-reverting. Thus, allowing fractional differencing, we can study a wide range of mean reverting behaviours. In this paper we present some results concerning fractional differencing in the real exchange rate market, using Robinson’s (1994) univariate tests. These tests are briefly described in Section 2. Section 3 applies the tests to historical annual data of real exchange rates between U.S. and five industrialized countries for the 1914–1983 period, using the same dataset as in Cheung and Lai (1993). Section 4 contains some concluding remarks.

2. Tests of fractional integration

Robinson (1994) proposes a very general testing procedure for testing unit roots and other hypotheses in raw time series. Unlike most of unit root tests embedded in autoregressive alternatives, Robinson’s (1994) tests are nested in a fractionally integrated model,

\[
(1 - L)^{d+\theta} x_t = u_t, \quad t = 1, 2, \ldots
\]

\[
x_t = 0, \quad t \leq 0,
\]

where \( d \) is a given real number; \( u_t \) is an \( I(0) \) process with parametric density function \( f \) of form

\[
f(\lambda; \sigma^2; \tau) = \frac{\sigma^2}{2\pi} g(\lambda; \tau), \quad -\pi < \lambda \leq \pi,
\]

where the scalar \( \sigma^2 \) and the \((q \times 1)\) vector \( \tau \) are unknown but \( g \) is of known form; and \( x_t \) is the time series we observe from \( t = 1, 2, \ldots n \). Thus, under the null hypothesis,

\[
H_0: \quad \theta = 0
\]

\( x_t \) in (1) is \( I(d) \) and the residuals are \( \tilde{u}_t = (1 - L)^d x_t \).

Unless \( g \) is a completely known function (e.g., \( g \equiv 1 \), as when \( u_t \) is white noise) we have to estimate the nuisance parameter \( \tau \), for example by \( \hat{\tau} = \arg \min_{\tau \in T} \sigma^2(\tau) \), where \( T \) is a suitable subset of \( \mathbb{R}^q \) and

\[
\sigma^2(\tau) = \frac{2\pi}{n} \sum_{j=1}^{n-1} g(\lambda_j; \tau)^{-1} I(\lambda_j), \quad \text{with} \quad I(\lambda_j) = \frac{1}{2\pi n} \left| \sum_{t=1}^{n} \tilde{u}_t e^{i\lambda_j t} \right|^2 \quad \text{and} \quad \lambda_j = \frac{2\pi j}{n}.
\]

The test statistic, which is derived from the Lagrange multiplier principle is:

\[
\hat{\tau} = \frac{n^{1/2}}{\sigma^2} \tilde{A}^{-1/2} \hat{a},
\]

where

\[
\hat{a} = -\frac{2\pi}{n} \sum_{j=1}^{n-1} \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j),
\]

\( \psi(\lambda) \) is the characteristic function of the radial variable \( \tilde{u}_t \).

\[
\hat{A} = \frac{2}{n} \left( \sum_{j=1}^{n-1} \psi(\lambda_j)^2 - \sum_{j=1}^{n-1} \psi(\lambda_j) \hat{\xi}(\lambda_j) \left( \sum_{j=1}^{n-1} \hat{\xi}(\lambda_j) \hat{\xi}(\lambda_j) \right)^{-1} \sum_{j=1}^{n-1} \hat{\xi}(\lambda_j) \psi(\lambda_j) \right),
\]

\[
\psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|, \quad \hat{\xi}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau})
\]

Robinson (1994) established under regularity conditions that

\[
\hat{r} \xrightarrow{d} N(0, 1) \quad \text{as} \ n \rightarrow \infty
\]

and thus, an approximate one-sided $100\alpha\%$ test of (3) against the alternative $\theta > 0$ rejects $H_0$ if $\hat{r} > z_\alpha$, where the probability that a standard normal variate exceeds $z_\alpha$ is $\alpha$. Conversely, a test of (3) against $\theta < 0$ rejects $H_0$ if $\hat{r} < -z_\alpha$. He also showed that the tests are efficient in the Pitman sense, that when directed against local alternatives: $H_1$: $\theta = \delta n^{-1/2}$ for $\delta \neq 0$, the limit distribution is Normal with variance 1 and mean which cannot be exceeded in absolute value by that of any rival regular statistic. Thus, we are under standard situations, unlike most of tests for unit roots where a nonstandard limit distribution and lack of efficiency theory is obtained. A diskette containing the FORTRAN codes for the tests is available from the author on request.

3. The real exchange rates

The data examined are real exchange rates for the period 1914–1989 taken from Cheung and Lai (1993). We look at five bilateral intercountry relations between the U.S. as the home country and Canada, UK, Japan, France and Italy as the foreign countries. We will employ throughout the model (1) and (2), testing (3) for different hypothesized values of $d$, from 0 through 2.25 with 0.25 increments, and modelling the I(0) disturbances as white noise and autoregressions of orders from 1 to 5.

Table 1 resumes the results of $\hat{r}$ in (4). Assuming white noise disturbances, we observe that the unit root null hypothesis is never rejected, and the only extra non-rejection value occurs for the UK when $d = 1.25$. However, assuming AR(k) disturbances, we see several non-rejections with $d < 1$. Presentation of all the results in this context of weakly autocorrelated disturbances would take up a lot of space. Thus, we present only a subset of them, with a single $k$ for each country across all $d$, and choosing the $k$ for each series which produces the smallest value of $|\hat{r}|$ across $d$. This enables better comparisons with the case of white noise disturbances and indicates the strongest support for any one hypothesis, while also having a tendency to be accompanied by relatively small $|\hat{r}|$ throughout, thereby providing an impression of relatively lower power. Starting with Canada, we observe that the null hypothesis (3) is not rejected if $d = 0.25$ and 0.50 with AR(1) $u_t$. The real exchange rate in France appears clearly nonstationary, with non-rejection values at $d = 0.75$ and 1 with AR(3) disturbances. The results for the UK and Japan show non-rejection values when $d = 0, 0.25$ and 0.50. Finally, the real exchange rate in Italy seems to be I(0) stationary, rejecting the null hypothesis (3) for all positive values of $d$. 
Table 1
Testing the order of integration in the real exchange rate market

<table>
<thead>
<tr>
<th>Country</th>
<th>Values of $d$</th>
<th>AR disturbances, $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>White noise disturbances</td>
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<td></td>
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<tr>
<td>Canada</td>
<td>8.66</td>
<td>6.46</td>
</tr>
<tr>
<td>France</td>
<td>11.38</td>
<td>10.82</td>
</tr>
<tr>
<td>Italy</td>
<td>12.12</td>
<td>10.02</td>
</tr>
<tr>
<td>UK</td>
<td>10.72</td>
<td>10.59</td>
</tr>
<tr>
<td>Japan</td>
<td>13.75</td>
<td>9.61</td>
</tr>
<tr>
<td>AR disturbances, $k$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>2</td>
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</tr>
<tr>
<td>Japan</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>1.81</td>
<td>$^*$-0.33</td>
</tr>
<tr>
<td></td>
<td>6.45</td>
<td>4.70</td>
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<td></td>
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<td>$^*$-0.77</td>
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<tr>
<td></td>
<td>1</td>
<td>$^*$0.42</td>
</tr>
</tbody>
</table>

$^*$ Non-rejection values of the null hypothesis (3) at the 95% significance level.

4. Conclusions

We can conclude by saying that mean reverting behaviour is observed in the real exchange rate series in all the countries when the disturbances are weakly autocorrelated. The real exchange rate in France seems to be the most nonstationary series, followed by Canada, Japan and the UK, whereas the exchange rate in Italy seems to be the most stationary one, with short memory behaviour.

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References