Costly signal extraction and profit differentials in oligopolistic markets

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Abstract

Empirical evidence indicates that there can be persistent profit differentials between firms in an industry. We show that demand uncertainty and costly information acquisition by firms on market demand leads to significant profit differentials for intermediate levels of demand variability. \textcopyright{} 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

Empirical evidence indicates that firms in the same industry often differ in their performances. Furthermore, the difference in firm profits persists over relatively long time periods.\textsuperscript{1} This paper offers a simple explanation for this puzzling phenomenon: When information acquisition is costly, demand uncertainty can cause profits to diverge even if all firms are ex ante identical.\textsuperscript{2} Even though firms are similar in terms of technology or product space, firm profits will differ. Poorly informed firms will employ a suboptimal number of workers or invest suboptimally and therefore, will enjoy lower profits than their better informed rivals.

We consider a symmetric, two-firm Cournot model with demand uncertainty. Each firm, before choosing its output level, decides whether to acquire information at some cost. The information that

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\textsuperscript{1}See Mueller (1986, 1990).

\textsuperscript{2}For some alternative explanations, see Barney (1986) and Röller and Sinclair-Desgagné (1996).
firms acquire takes the form of a noisy signal on the true market demand. The noise level is exogenous, but a firm can increase the likelihood of acquiring the signal by spending more resources. We show that if the signal extraction technology is linear, then for intermediate levels of demand uncertainty, all pure strategy equilibria will be asymmetric: Only one firm will become informed, and thus obtain larger profits than its rival.

2. The model

Two symmetric Cournot firms produce a homogenous product under demand uncertainty. The stochastic linear inverse demand takes the following form:

\[ P(Q) = a + \Theta - bQ, \quad Q = \sum_{i=1,2} q_i, \]  

where \( a, b > 0 \), and nonstochastic, and \( q_i \) denotes firm \( i \)'s output. The demand shock \( \Theta \) is normally distributed with mean zero and variance \( \sigma^2 \). The marginal cost of production, \( c \), is constant and symmetric across firms. Before making their output decisions, each firm can engage in costly information gathering activity to learn the true market demand. Learning takes the form of observing a noisy signal on the true demand: The precision of the signal is exogenous, but each firm can increase the probability of observing this signal by spending more resources on information gathering.

Our formulation departs from the more common approach that assumes it is the signal precision that depends on the amount of resources firms spend (see Hwang, 1995). That formulation, however, can be inappropriate if there is no standard, well-established method to acquire information. Then, it is conceivable that if the method a firm develops is unsuccessful, it produces no useful results and leaves the firm with no information despite the resources it spent. Conversely, if there is success, the firm observes the signal from which information is extracted on market demand. We assume that for both firms, the cost of signal extraction is:

\[ C(p) = \alpha p \quad \text{for} \quad p \in [0,1] \]  

where \( \alpha \geq 0 \), and \( p \) denotes the probability of observing the signal. We let \( S = \Theta + \varepsilon \) denote the noisy signal, where \( \varepsilon \) is normally distributed with mean zero and variance \( \tau^2 \), which is exogenous. Letting \( \text{Cov}(\Theta, \varepsilon) = 0 \), standard results – see DeGroot (1970) – allow us to write the expectation of \( \Theta \) conditional on \( S \) as

\[ E(\Theta|S) = \lambda S, \]

where

\[ \lambda = \frac{\text{Cov}(S,\varepsilon)}{\text{Var}(S)} = \frac{\sigma^2}{\sigma^2 + \tau^2} \]

Routine computations – see Appendix A – yield the following expected profit function for firm \( i \), where the expectation is taken before the signal is observed:

\footnote{A signal extraction technology is ‘linear’, if the cost of observing the signal with probability \( p \) is a linear function of \( p \).}
\[
E(\pi_i) = \frac{(a-c)^2}{9b} + \frac{\lambda \sigma^2}{b} p_i \left[ \frac{2 - \lambda p_i}{4 - \lambda^2 p_i p_j} \right]^2 - \alpha p_i, \quad \text{for } i = 1, 2; \ i \neq j,
\]

where \( p_i \) denotes the probability that firm \( i \) observes the signal. Firm \( i \) chooses \( p_i \), i.e., decides on the level of its information gathering activity, to maximize (3), given \( p_j \). Here is our main result that characterizes the asymmetric equilibrium:

**Proposition 1.** If

\[
\frac{1}{4} \frac{\lambda \sigma^2}{b} \geq \alpha \geq \frac{1}{2} \frac{\lambda \sigma^2}{b},
\]

there is a unique equilibrium in pure strategies, up to a renaming of firms, where one and only one firm engages in the signal extraction activity. In equilibrium, this firm observes the signal with probability 1.

**Proof.** Differentiate (3) with respect to 

\[
p_i^* = \frac{\lambda \sigma^2}{b} \left( \frac{2 - \lambda p_i}{4 - \lambda^2 p_i p_j} \right) - \alpha.
\]

Set \( p_2 = 0 \) and simplify: \( E(\pi_1)^* = 1/4 \cdot \lambda \sigma^2/b - \alpha \). Our assumption \( 1/4 \cdot \lambda \sigma^2/b \geq \alpha \) implies \( E(\pi_1)^* \geq 0 \) for all \( p_1 \). This, in turn, implies that \( E(\pi_1) \) is maximized at the corner solution where \( p_1 = 1 \). Next, we show that if \( p_1 = 1 \), then \( E(\pi_2) \) will be maximized at \( p_2 = 0 \). Set \( p_1 = 1 \). By symmetry, we have \( E(\pi_2)^* = \lambda \sigma^2/b \left( 2 - \lambda \right)^2 \left( 4 + \lambda^2 p_2/4 - \lambda^2 p_3 \right) - \alpha \). Since \( E(\pi_2)^* \) is increasing in \( p_2 \), only corner solutions are possible: \( p_2 = 0 \), or \( p_2 = 1 \). Our assumption \( \alpha \geq \lambda \sigma^2/b \cdot (2 + \lambda)^2 \) implies that \( E(\pi_2) \) at \( p_2 = 0 \) is larger than \( E(\pi_2) \) at \( p_2 = 1 \). This proves that \( (p_1 = 1 \text{ and } p_2 = 0) \) is an equilibrium. To show uniqueness, suppose there is another equilibrium. It is immediate that neither \( (p_1 = p_2 = 1) \) nor \( (p_1 = p_2 = 0) \) can be an equilibrium under the assumption of the proposition. Therefore, in equilibrium, the first order condition must hold with equality. Consider firm 1: A close inspection of \( E(\pi_1)^* \) reveals that \( E(\pi_1)^* \) is increasing in \( p_1 \) for all \( p_1, p_2 \in (0, 1) \), therefore, there can be no equilibrium with \( p_1 \in (0, 1) \). An identical argument shows that there can be no equilibrium with \( p_2 \in (0, 1) \) either. This proves the claim.

According to Proposition 1, for the asymmetric equilibrium to exist and thus profit differentials to emerge, the necessary and sufficient condition is that the marginal cost of signal extraction, \( \alpha \), is in an intermediate range relative to the level of demand uncertainty, \( \sigma^2 \).

**Corollary.** If \( \alpha < 1/(2 + \lambda)^2 \lambda \sigma^2/b \), the unique equilibrium is \( p_1 = p_2 = 1 \). If \( \alpha > \lambda \sigma^2/4b \), the unique equilibrium is \( p_1 = p_2 = 0 \).

**Proof.** Similar to the proof of Proposition 1, therefore omitted.
Let $\Delta \pi = E(\pi_i) - E(\pi_j) = \lambda \sigma^2 / 4b - \alpha$ denote the profit differential. $\Delta \pi$ will be large, if $\alpha$ is small. Since $\alpha \geq 1/(2 + \lambda)^2 \lambda \sigma^2 / b \equiv \alpha^\text{min}$ must hold for an asymmetric equilibrium, the largest $\Delta \pi$, which is $(\lambda \sigma^2) / 4b + 4 + \lambda(2 + \lambda)^2$, is obtained for $\alpha = \alpha^\text{min}$. To see whether such a difference is economically significant, we compute the ratio $\Delta \pi / \pi_j$. This simplifies to

$$\frac{9}{4} \frac{\sigma^2}{(a-c)^2} \cdot (4 + \lambda) \left( \frac{\lambda}{2 + \lambda} \right)^2.$$ 

How large this will be depends on first how big $\sigma$ is relative to $a-c$; it is immediate that as $\sigma$ increases, $\Delta \pi / \pi_j$ increases. Second, it depends on the precision of the signal. Showing this is slightly more involved. First, note that $\Delta \pi / \pi_j$ is increasing in $\lambda$, which follows from the fact that $\lambda(2 + \lambda)$ is increasing in $\lambda$. Since $\lambda \in [0,1]$, $\Delta \pi / \pi_j$ is maximized at $\lambda = 1$, $\lambda \to 1$ as $\tau^2 \to 0$, thus the maximum value of $\Delta \pi / \pi_j$ is $5/4 \sigma^2 / \sigma^2(a-c)^2$, and is obtained at $\tau^2 = 0$.

**Numerical example.** Let $\alpha = \alpha^\text{min}$, $a-c = 3$, and $\tau^2 = 0$, i.e., the signal is fully informative. For $\sigma = 0.5$, $\sigma = 0.7$, and $\sigma = 1$, we have $\Delta \pi / \pi_j = 0.035$, $\Delta \pi / \pi_j = 0.068$, and $\Delta \pi / \pi_j = 0.139$, respectively.

### 3. Conclusion

To explain persistent differences in firms’ profits, one doesn’t need to appeal to asymmetric information, or to ad hoc differences in costs, capabilities, etc. Using a Cournot model with demand uncertainty where firms spend resources to increase the likelihood of acquiring information of a given reliability, we showed that for intermediate levels of demand uncertainty, all pure strategy equilibria are asymmetric: Only one firm acquires information, and obtains larger profits than its rival. Our analysis shows that when the issue is whether a signal of a particular reliability can be acquired at all, rather than the reliability level itself, asymmetric equilibria, and hence profits differentials, emerge. This points out to the importance of the information acquisition structure – increasing the reliability of the information vs. increasing the probability of obtaining information of given reliability – in determining the nature of the equilibrium.

In order to capture dynamics well, it is certainly desirable to consider a multi-period setting where demand shifts randomly so firms need new observations periodically. Also, more general cost and demand functions need to be considered. This is delegated to future work.

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Appendix A

We conjecture that if firm $i$ observes the signal $S_i$, it will produce $q_i^o = q_i^{no} + K_i S_i$, $K_i$ is the unknown constant, and $q_i^{no}$ is the output if firm $i$ does not observe the signal. Firm $i$'s expected profit conditional on $S_i$, gross of the cost of signal extraction, is $E(\pi_i|S_i) = E(\pi_i|S_i, q_i^{no} + K_i p_j S_j - c)q_i | S_i)$, for $i = 1, 2; i \neq j$. Maximizing $E(\pi_i|S_i)$ with respect to $q_i$ yields: $q_i(S_i = 1/2b(a - c + \lambda S_i - b q_i^{no} - b K_i p_j S_j))$. The method of undetermined coefficients yields $q_i^{no} = a - c - b q_i^{no}/2b$ and $q_j^{no} = a - c - b q_j^{no}/2b$. Solving these two equations yields $K_i = 1/b - \lambda p_j S_j$. Let $m = a - c/3b$. Prior to engaging in the signal extraction activity, firm $i$'s expected profit conditional on $S_i$ is $E(\pi_i) = \{a + \Theta - c - b(m + K_i S_i) - b(m + p_j K_j S_j)\}(M + K S_j)$. Substituting for $K_i$ and simplifying, yields

$$E(\pi_i) = \frac{(a - c)^2}{9b} + \frac{\lambda \sigma^2}{b} \left[ \frac{2 - p_j \lambda}{4 - p_j \lambda^2} \right]^2.$$ 

The signal is observed with probability $p_j$. Thus expected profits prior to engaging in the signal extraction activity is

$$E(\pi_i) = \frac{(a - c)^2}{9b} + p_j \frac{\lambda \sigma^2}{b} \left[ \frac{2 - p_j \lambda}{4 - p_j \lambda^2} \right]^2.$$ 

References