The impact of public information on insider trading

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Received 1 December 1999; accepted 20 July 2000

Abstract

We study the impact of public information on insider trading in the context of Kyle’s speculative market. The linear Nash equilibrium is characterized analytically. We find that public information is detrimental for the insider and beneficial for the liquidity traders. The insider puts a negative weight on the public information in formulating his optimal strategy. The equilibrium price becomes more informative when there exists public information. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Insider trading; Public information; Private information; Nash equilibrium

JEL classification: C72; D43; D82

1. Introduction

The value of information is well understood in the context of single agent decision problems. In this situation, the value of information is always positive. However, in the context of multiple agents decision problems with strategic interaction, the sign of the value of information is ambiguous. Both positive and negative examples abound. The intuition lies in that in multiple agents decision situations (games), there is a trade-off of two effects between information for one agent and that for others. Better information may improve an agent’s decision, but this may also cause other agents’ decisions to strategically shift, and which in turn has an impact on the original agent’s decisions. This interpersonal interaction can generate counterintuitive and puzzling patterns in practice.

This paper explores the value of public information in Kyle’s insider trading model. In his pioneering paper, Kyle (1985) proposed a semi-strong efficient speculative market with insider trading. Now, this model has been widely used to analyze financial market microstructure and the value of information, and has elicited a large body of literature. See, for example, Admati and

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Our main result is that public information reduces the insider’s profit and the liquidity traders’ loss, and leads to more informative equilibrium prices.

This paper is structured as follows. In Section 2, we present the one-shot insider trading model with a single insider and public information. In Section 3, we identify the unique linear Nash equilibrium, and explore the welfare and informational impact of public information. Finally, Section 4 concludes.

2. The Model

The model is a natural extension of Kyle (1985). There are two periods and a single risky asset in the economy: period 0, at which the information (public and private) is released and trading takes place; and period 1, at which the risky asset payoff is realized and consumption takes place. We assume that there is no discount across the two periods, that is, the interest rate is normalized to zero.

The *ex post* liquidate value of the risky asset at period 1 is a random variable $\tilde{v}$, normally distributed with mean 0 and variance $\sigma_v^2$. In statistical notation, $\tilde{v} \sim N(0, \sigma_v^2)$. The assumption of a zero mean is a convenient normalization.

The economy is populated with three kinds of traders: a single risk neutral insider, liquidity/noise traders, and competitive risk neutral market makers. The insider observes a public signal and also has private information, he submits orders based on the public information as well as his private information. Liquidity traders have an inelastic demand for the risky asset and their trading is exogenous. The presence of the liquidity traders not only serves as a camouflage and makes it impossible for the market makers to exactly infer the insider’s private information, but also is the source of the profits to be exploited by the insider. The market makers observe the public information and the total orders, they set prices in a semi-strong efficient way. Thus the public information here is in a restrictive sense, it is only ‘public’ among the insider and market makers (not shared by the liquidity traders).

The trading proceeds as follows. At period 0, a piece of public information which can be observed by both the insider and the market makers is announced. The public information is

$$\tilde{s}_v = \tilde{u} + \tilde{e}_v.$$  

Here $\tilde{e}_v \sim N(0, \frac{1}{t^2} \sigma_v^2)$ (the notation of variance here is for latter computational convention, the parameter $1/t$ describes the relative precision of the public information). Further, the insider receives a piece of private information concerning the future payoff of the risky asset. The private information is

$$\tilde{s}_i = \tilde{u} + \tilde{e}_i.$$  

Here $\tilde{e}_i \sim N(0, \sigma_i^2)$. The random variables $\tilde{u}$, $\tilde{e}_v$ and $\tilde{e}_i$ are mutually independent. Thus the insider’s information (public and private) is $(\tilde{s}_v, \tilde{s}_i)$. After receiving the information, the insider chooses his trading strategy by submitting an order $\tilde{x} = X(\tilde{s}_v, \tilde{s}_i)$ based on the public information as well as on his private information. In the meantime, the liquidity traders collectively submit an order $\tilde{u}$, which is a normal random variable independent of $\tilde{u}$, $\tilde{e}_v$ and $\tilde{e}_i$, with mean 0 and variance $\sigma_u^2$, thus $\tilde{u} \sim N(0, \sigma_u^2)$. The total order of the insider and the liquidity traders is $\tilde{x} + \tilde{u}$. After receiving the total order $\tilde{x} + \tilde{u}$, the market makers take the opposite side of the incoming order and set the price $P = P(\tilde{s}_v, \tilde{x} + \tilde{u})$ of
the risky asset such that they expect to earn a zero profit conditional on the public information and the total order. When doing so they observe the public information $\tilde{s}_c$ and the total order $\tilde{x} + \tilde{u}$, but he cannot disentangle the amount, that is, they cannot observe $\tilde{x}$ or $\tilde{u}$ separately (nor can they observe $\tilde{s}_i$ or $\tilde{v}$). Thus the price is not fully revealing.

At period 1, the uncertainty is resolved and the risky asset payoff is realized. The structure of the economy is common knowledge.

This market model has some game-theoretic features. Firstly, we can view it as a game played by the insider and the market makers: The insider attempts to hide his private information (in the camouflage of the liquidity traders and his own strategic behavior) and make the best use of his information to maximize his profit; the market makers attempt to learn the private information from the order and set prices as efficiently as possible in order to rule out the profit opportunities of the insider.

Secondly, the market model can be regarded as a zero-sum two-person game played by the insider on one side and the liquidity traders on the other. Of course, the insider is an active player and has a lot of strategies, while the liquidity traders are collectively a passive player and have a trivial strategy. The subtle point here is the intermediate role of the market makers.

Finally, we can also view the market model as a monopolistic market: the single insider has superior private information and thus has some monopoly power in exploiting profit from the liquidity traders. The inverse demand functions in the standard monopolistic market are now replaced by the market makers’ pricing rules.

We are interested in characterizing the insider’s optimal trading strategies and the market makers’ pricing rules, and the welfare and informational implications of the public information. The concept of an equilibrium is as that of Kyle (1985).

**Definition.** A Nash equilibrium of the above market model consists of a pair

\[
\{\tilde{x} = X(\tilde{s}_c, \tilde{s}_i), \quad P = P(\tilde{s}_c, \tilde{x} + \tilde{u})\},
\]

the insider’s trading strategy and the market makers’ pricing rule, such that the following conditions hold:

1. Profit maximization for the insider: for insider’s any alternate trading strategy $\tilde{x}'$,

\[
E[(\tilde{v} - P)\tilde{x}|\tilde{s}_c, \tilde{s}_i] \geq E[(\tilde{v} - P)\tilde{x}'|\tilde{s}_c, \tilde{s}_i].
\]

2. Market semi-strong efficiency:

\[
P(\tilde{s}_c, \tilde{x} + \tilde{u}) = E[\tilde{v}|\tilde{s}_c, \tilde{x} + \tilde{u}].
\]

Here $E[ \cdot | \cdot ]$ denotes conditional expectation.

In equilibrium, the market makers make a zero profit, since by the law of iterated expectations and market semi-strong efficiency, we have
\[ E[(\tilde{u} - P)(-\tilde{x} - \tilde{u})] = -E[E((\tilde{u} - P)(\tilde{x} + \tilde{u})|\tilde{s}, \tilde{x} + \tilde{u})] \]
\[ = -E[(E[\tilde{u}|\tilde{s}, \tilde{x}]) - P(\tilde{x} + \tilde{u})] \]
\[ = 0. \]

3. The Nash equilibrium

We only consider linear Nash equilibria. The following result is a generalization of Kyle (1985). Taking \( t \to \infty \), we recover Kyle’s result.

**Proposition 1.** The unique linear Nash equilibrium is given by

\[ \tilde{x} = \alpha \tilde{s}_c + \beta \tilde{s}_i, \]
\[ P = \gamma \tilde{s}_c + \lambda (\tilde{x} + \tilde{u}), \]

with

\[ \alpha = -\sqrt{\frac{\sigma_u^2}{(t \sigma_v^2 + (1 + t) \sigma_i^2)(1 + t)}}, \]
\[ \beta = \sqrt{\frac{(1 + t) \sigma_u^2}{t \sigma_v^2 + (1 + t) \sigma_i^2}}, \]
\[ \gamma = \frac{1}{1 + t}, \]
\[ \lambda = \frac{t \sigma_v^2}{2 \sqrt{(t \sigma_v^2 + (1 + t) \sigma_i^2)(1 + t) \sigma_u^2}}. \]

**Proof.** See Appendix A.

From Eqs. (3) and (4), we see that the insider put different weights on the public information and the private information in formulating his trading strategy. The relative weight is \( \alpha / \beta = -(1/1 + t) \). Since \( \gamma = -(\alpha / \beta) \), the market makers put a weight which is exactly the negative of the insider’s relative weight on the public information in forming their pricing rule.

When \( t \to \infty \) (so that there is actually now public information), we have

\[ \alpha \to 0, \quad \beta \to \sqrt{\frac{\sigma_u^2}{\sigma_v^2 + \sigma_i^2}}, \quad \gamma \to 0, \quad \lambda \to \frac{\sigma_v^2}{2 \sqrt{(\sigma_v^2 + \sigma_i^2) \sigma_u^2}}. \]

This is exactly Kyle’s result (1985).

An important quantity describing the informativeness of the price is the conditional variance \( \text{var}[\tilde{v}|P] \), which measures the residual variance after information (public and private) is incorporated into the price. Thus \( h(t) \equiv \text{var}[\tilde{v}] - \text{var}[\tilde{v}|P] \) measures how much information has been incorporated into the equilibrium price.
Proposition 2. We have

\[ I(t) = \frac{(t^2 + 2t)\sigma_v^2 + 2(1 + t)\sigma_i^2}{(2t^2 + 2t)\sigma_v^2 + 2(1 + t)^2 \sigma_i^2} \cdot \sigma_v^2. \]

In particular, \( I(t) \) is a decreasing function of \( t \). Consequently, the price is more informative when the public information is more precise.

Proof. See Appendix A.

Two extreme cases are of particular interest. First, when \( t = 0 \), the public information is perfect, and \( I(0) = \sigma_v^2 \), thus the equilibrium price incorporates all the information about the future payoff, this is evident since by Proposition 1, in this situation, \( P = \bar{u} \). Second, when \( t \to \infty \), there is actually no public information, and

\[ I(\infty) = \frac{\sigma_v^2}{2(\sigma_v^2 + \sigma_i^2)} \cdot \sigma_v^2 \leq \frac{1}{2} \sigma_v^2. \]

Thus the equilibrium price can at most reveal half of the future payoff information.

Proposition 3. In the equilibrium, the ex ante expected profit of the insider is \( \Pi(t) = E[(\bar{u} - P)\bar{x}] = \lambda\sigma_w^2 \), where \( \lambda \) is given by Eq. (6). Consequently, \( \Pi(t) \) is an increasing function of \( t \), and thus a decreasing function of the precision of the public information. In particular,

\[ \Pi(t) < \Pi(\infty) = \frac{\sigma_v^2}{2} \sqrt{\frac{\sigma_u^2}{\sigma_v^2 + \sigma_i^2}}. \]

Proof. By Eqs. (1) and (2), and Proposition 1, the ex ante expected profit of the insider is

\[ \Pi(t) = E[(\bar{u} - P)\bar{x}] = E[(\bar{u} - \gamma \bar{x}_c - \lambda((\alpha \bar{x}_c + \beta \bar{x}_i) + \bar{u}))((\alpha \bar{x}_c + \beta \bar{x}_i))] = (\alpha + \beta)\sigma_v^2 - \alpha \gamma(1 + t)\sigma_v^2 - \beta \gamma \sigma_i^2 - \lambda((\alpha + \beta)^2 \sigma_v^2 + \alpha^2 \sigma_v^2 + \beta^2 \sigma_i^2) = \lambda \sigma_w^2. \]

The last equality follows from observing that

\[ \gamma = \frac{1}{1 + t}, \quad \alpha = -\frac{2\sigma_v^2\lambda}{t\sigma_v^2}, \quad \beta = \frac{2\sigma_i^2(1 + t)\lambda}{t\sigma_v^2}, \]

which are implied by Eqs. (3)–(6).
4. Conclusion

In Kyle’s model of insider trading, the value of public information demonstrates quite different characteristics than those of private information. Indeed, the insider puts a negative weight on the public information in order to evade the inference of market makers. We have shown that the private information is more valuable than the public information. The insider puts a relatively larger positive weight on his private information, and a relatively smaller negative weight on the public information in formulating his trading strategies. The insider’s profit is decreasing when the public information becomes more precise. The informativeness of the price is increasing with more precise public information.

Appendix A

We first recall a well-known regression result which one will use.

Lemma. Let \( X_1 \) ad \( X_2 \) be two normal random vectors,

\[
\begin{pmatrix}
X_1 \\
X_2
\end{pmatrix} \sim N(\mu, \Sigma) \quad \text{with} \quad \mu = \begin{pmatrix}
\mu_1 \\
\mu_2
\end{pmatrix}, \quad \Sigma = \begin{pmatrix}
\Sigma_{11} & \Sigma_{12} \\
\Sigma_{21} & \Sigma_{22}
\end{pmatrix}.
\]

Then the random variable \( X_1 \) conditional on \( X_2 \) (we denote this as \( X_1 | X_2 \)) has a normal distribution. More precisely,

\[
X_1 | X_2 \sim N(\mu_1 + \Sigma_{12} \Sigma_{22}^{-1}(X_2 - \mu_2), \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}).
\]

In particular,

\[
E[X_1 | X_2] = \mu_1 + \Sigma_{12} \Sigma_{22}^{-1}(X_2 - \mu_2).
\]

Proof of Proposition 1. First, we prepare some calculations for latter use. By assumption, we have

\[
\begin{pmatrix}
\tilde{v} \\
\tilde{s}_c \\
\tilde{s}_i
\end{pmatrix} \sim N\left( \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}, \begin{pmatrix}
\sigma_v^2 & \sigma_v^2 & \sigma_v^2 \\
\sigma_v^2 & (1 + t)\sigma_v^2 & \sigma_v^2 \\
\sigma_v^2 & \sigma_v^2 & \sigma_v^2 + \sigma_i^2
\end{pmatrix}\right).
\]

Thus, by the Lemma, we have

\[
E[\tilde{v} | \tilde{s}_c, \tilde{s}_i] = \begin{pmatrix}
\sigma_v^2, \sigma_v^2
\end{pmatrix}^{-1} \begin{pmatrix}
(1 + t)\sigma_v^2 \\
\sigma_v^2
\end{pmatrix} \begin{pmatrix}
\tilde{s}_c \\
\tilde{s}_i
\end{pmatrix}
\]

\[
= \frac{1}{t\sigma_v^2 + (1 + t)\sigma_i^2} (\sigma_v^2 \tilde{s}_c + t\sigma_i^2 \tilde{s}_i).
\]

Second, we conjecture that the linear Nash equilibrium is given by Eqs. (1) and (2) with the
parameters $\alpha$, $\beta$, $\gamma$ and $\Lambda$ being constants which need to be determined. We will identify the parameters and verify the conjecture.

Since $\bar{x}$ is a measurable function of $\bar{s}_c$ and $\bar{s}_i$, and $\bar{u}$ is independent of $\bar{s}_c$ and $\bar{s}_i$, the insider's expected profit conditional on his information is

$$E[(\bar{u} - P)\bar{x} | \bar{s}_c, \bar{s}_i] = E[(\bar{u} - \gamma \bar{s}_c - \lambda (\bar{x} + \bar{u}))\bar{x} | \bar{s}_c, \bar{s}_i]$$

$$= (E[\bar{u} | \bar{s}_c, \bar{s}_i] - \gamma \bar{s}_c - \lambda \bar{x}) \bar{x}. $$

To maximize the above expression over $\bar{x}$, the first order condition is

$$\bar{x} = \frac{1}{2\lambda}(E[\bar{u} | \bar{s}_c, \bar{s}_i] - \gamma \bar{s}_c)$$

$$= \frac{1}{2\lambda} \left( \frac{\sigma^2 \bar{s}_c + t\sigma^2 \bar{s}_i}{\tau^2 + (1 + t)\sigma^2} - \gamma \bar{s}_c \right)$$

$$= \frac{1}{2\lambda} \left( \frac{\sigma^2 \bar{s}_c + t\sigma^2 \bar{s}_i}{\tau^2 + (1 + t)\sigma^2} - \gamma \bar{s}_c + \frac{t\sigma^2 \bar{s}_i}{\tau^2 + (1 + t)\sigma^2} \bar{s}_i \right).$$

(7)

The second order condition $-\lambda < 0$, which is satisfied as will be seen from Eq. (16).

Comparing Eq. (7) with Eq. (1), we have,

$$\alpha = \frac{1}{2\lambda} \left( \frac{\sigma^2}{\tau^2 + (1 + t)\sigma^2} - \gamma \right)$$

$$\beta = \frac{1}{2\lambda} \left( \frac{t\sigma^2 \bar{s}_i}{\tau^2 + (1 + t)\sigma^2} \bar{s}_i \right).$$

(8)

(9)

Now we compute $P = E[\bar{u} | \bar{s}_c, \bar{x} + \bar{u}]$. Since by Eq. (1),

$$\bar{x} + \bar{u} = \alpha \bar{s}_c + \beta \bar{s}_i + \bar{u} = (\alpha + \beta)\bar{u} + \alpha \bar{e}_c + \beta \bar{e}_i + \bar{u},$$

we have

$$\text{var}[\bar{x} + \bar{u}] = (\alpha + \beta)^2 \sigma^2 + \alpha^2 t\sigma^2 + \beta^2 \sigma^2 + \sigma^2_v.$$ 

Furthermore

$$\begin{pmatrix} \bar{u} \\ \bar{s}_c \\ \bar{x} + \bar{u} \end{pmatrix} \sim N \begin{pmatrix} 0 \\ \sigma^2_v \\ (\alpha + \beta)\sigma^2_v \end{pmatrix}, \begin{pmatrix} \sigma^2_v & \sigma^2_v & (\alpha + \beta)\sigma^2_v \\ \sigma^2_v & (1 + t)\sigma^2_v & (\alpha + \beta)\sigma^2_v + \alpha t\sigma^2_v \\ (\alpha + \beta)\sigma^2_v & (\alpha + \beta)\sigma^2_v + \alpha t\sigma^2_v & \text{var}[\bar{x} + \bar{u}] \end{pmatrix}. $$
We have the determinant
\[
D = \begin{vmatrix}
(1 + t)\sigma_v^2 & (\alpha + \beta)\sigma_v^2 + at\sigma_v^2 \\
(\alpha + \beta)\sigma_v^2 + at\sigma_v^2 & \text{var}[\tilde{x} + \tilde{u}]
\end{vmatrix}
\]
\[= \beta^2(\sigma_v^2 + (1 + t)\sigma_i^2)\sigma_v^2 + (1 + t)\sigma_u^2\sigma_v^2. \tag{10}\]

By market semi-strong efficiency and the lemma, we have
\[
P(\tilde{s}_c, \tilde{x} + \tilde{u})
= E[\tilde{u}|\tilde{s}_c, \tilde{x} + \tilde{u}]
\]
\[= (\sigma_v^2/(\alpha + \beta)\sigma_v^2)\begin{pmatrix}
(1 + t)\sigma_v^2 & (\alpha + \beta)\sigma_v^2 + at\sigma_v^2 \\
(\alpha + \beta)\sigma_v^2 + at\sigma_v^2 & \text{var}[\tilde{x} + \tilde{u}]
\end{pmatrix}^{-1}\begin{pmatrix}
\tilde{s}_c \\
\tilde{s}_i
\end{pmatrix}
\]
\[= \frac{\sigma_v^2}{D}((-\alpha\beta t\sigma_v^2 + \beta^2\sigma_i^2 + \sigma_u^2)\tilde{s}_c + \beta t\sigma_v^2(\tilde{x} + \tilde{u})).
\]

Comparing the above expression with Eq. (1), we have
\[
\gamma = \frac{\sigma_v^2}{D}(-\alpha\beta t\sigma_v^2 + \beta^2\sigma_i^2 + \sigma_u^2), \tag{11}
\]
\[
\lambda = \frac{\sigma_v^4\beta t}{D}. \tag{12}
\]

Substituting Eq. (12) into Eq. (9), and noting Eq. (10), we obtain
\[
\beta^2 = \frac{(1 + t)\sigma_u^2}{t\sigma_v^2 + (1 + t)\sigma_i^2}, \tag{13}
\]
thus
\[
\beta = \sqrt{\frac{(1 + t)\sigma_u^2}{t\sigma_v^2 + (1 + t)\sigma_i^2}}. \tag{14}
\]
Substituting Eq. (13) into Eq. (10), we obtain
\[
D = 2(1 + t)\sigma_u^2\sigma_v^2. \tag{15}
\]
Substituting Eqs. (14) and (15) into Eq. (12), we have
\[
\lambda = \frac{t \sigma_v^4}{2(1 + t) \sigma_u^2 \sigma_v^2} \sqrt{(1 + t) \sigma_u^2 \sigma_v^2 + \frac{(1 + t) \sigma_v^2}{t \sigma_v^2 + (1 + t) \sigma_u^2}}.
\]

(16)

From Eqs. (8) and (9), we have

\[-\alpha \beta \sigma_v^2 + \beta^2 \sigma_i^2 = \beta(\beta \sigma_i^2 - \alpha \sigma_v^2) = \frac{\beta t \sigma_v^2 \gamma}{2 \lambda}.
\]

Substituting the above expression and Eq. (15) into Eq. (11), we obtain

\[\gamma = \frac{1}{2(1 + t) \sigma_u^2} \left( \frac{\beta t \sigma_v^2 \gamma}{2 \lambda} + \sigma_u^2 \right).
\]

Substituting Eqs. (14) and (16) into the above expression, we obtain Eq. (5).

Finally, substituting Eqs. (5) and (16) into Eq. (8), we have

\[\alpha = \frac{1}{2 \lambda} \left( \frac{\sigma_i^2}{t \sigma_v^2 + (1 + t) \sigma_i^2} - \frac{1}{1 + t} \right)
= \frac{1}{2 \lambda} \cdot \frac{-t \sigma_v^2}{(t \sigma_v^2 + (1 + t) \sigma_i^2)(1 + t)}
= \frac{-\sigma_u^2}{t \sigma_v^2 + (1 + t) \sigma_i^2}(1 + t).
\]

Proof of Proposition 2. Since

\[P = \gamma \bar{x} + \lambda (\bar{x} + \bar{u})
= (\gamma + \lambda (\alpha + \beta)) \bar{w} + (\gamma + \lambda \alpha) \bar{e} + \lambda \beta \bar{e} + \lambda \bar{u},
\]

by the Lemma, we have

\[\text{var}[P] = (\gamma + \lambda (\alpha + \beta))^2 \sigma_v^2 + (\gamma + \lambda \alpha)^2 t \sigma_v^2 + \lambda^2 \beta^2 \sigma_i^2 + \lambda^2 \sigma_u^2.
\]

(17)

From Eqs. (3)--(6), we have

\[\gamma + \alpha \lambda = \frac{t \sigma_v^2 + 2(1 + t) \sigma_i^2}{2(t \sigma_v^2 + (1 + t) \sigma_i^2)(1 + t)}, \quad \lambda \beta = \frac{t \sigma_v^2}{2(t \sigma_v^2 + (1 + t) \sigma_i^2)},
\]

and
\[ \gamma + \lambda (\alpha + \beta) = \frac{t(t + 2)\sigma_v^2 + 2(1 + t)\sigma_i^2}{2(t\sigma_v^2 + (1 + t)\sigma_i^2)(1 + t)}. \]

Substituting the above expressions into Eq. (17) and into

\[ \text{var}[\bar{\nu} | P] = \text{var}[\bar{\nu}] - \frac{\left(\gamma + \lambda (\alpha + \beta)\right)^2 \sigma_v^4}{\text{var}[P]}, \]

we obtain the desired result.

References