Options to quit

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Abstract

This paper develops a theoretical model of a worker’s decision problem under uncertainty about the optimal separation time, when holding a representative outside offer but facing fixed costs of quitting. Implications of the model’s closed form solution are consistent with the quit behavior of workers from a large Dutch company. © 2001 Published by Elsevier Science B.V.

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1. Introduction

Workers examine options to quit extensively and pro-actively. Evidence thereof is widely available. When more and better jobs are available, the ratio of quits to layoffs increases with value of outside opportunities (McLaughlin, 1991; Hamermesh and Pfann, 1996). Traditionally, theories of voluntary turnover are built on the net present value difference of current or expected future surpluses (Hall and Lazear, 1984; Jovanovic, 1979a). Characteristics of worker turnover, such as lumpiness — implied by stepping stone irreversibility (Rosen, 1972) or fixed and firm-specific costs (Burdett, 1978; Jovanovic, 1979b; uncertainty dependence (Gray, 1978; Addison and Portugal, 1987) and the value of waiting to quit determine the dynamic patterns of worker mobility.

Fig. 1 shows the evolution of the workforce of a Dutch aircraft builder, named Fokker, over the period 1987–1996. Until 1991 the company’s tenured workforce expands from 10,500 in 1987 to

People spending most their lives living in a future paradise.

Stevie Wonder, ‘Songs in the Key of Love’, 1976
12,500 in 1991, with one reorganization announced on January 15th, 1988. After 1991 the company downsizes its workforce dramatically. This process started with a new early retirement scheme for workers aged 55 years and older that was initiated on March 1st, 1991, followed by four mass staff reductions notified in advance on October 1st, 1992, April 23rd, 1993, April 13th, 1994, and June 19th, 1995, respectively. On March 15th, 1996 Fokker’s construction plants were declared bankrupt. Fig. 2 displays the rates of quits and lay-offs relative to the company’s total workforce and the exact dates of the announced downsizing operations. The subsequent voluntary quit rate is large and lumpy and increases after each consecutive downsizing operation. Fig. 2 shows that from time to time, shocks hitting workers have a more drastic impact on worker mobility than at other times.

In order to better understand the dynamics of corporate turnover, this paper proposes an option pricing model of idiosyncratic job turnover risk. The model derives the optimal timing to quit for a worker facing uncertainty and fixed costs of quitting. The difference between this model and the traditional NPV models of worker turnover is that a worker quits when the net present value of the difference in expected future cash flows associated with the old and the new job exceeds the costs of quitting plus the value of postponing the decision to quit the current job. Modeling the present value of lifetime earnings on a representative new job directly — and not explained in terms of fundamentals like wage — allows us to derive a closed form solution of the optimal quitting decision problem under uncertainty. An extended version of the model allows for discrete jumps to happen instantaneously, that can be interpreted as corporate events, such as announcements of a major reorganization or a plant closing, rendering the worker’s option value to quit worthless.

\[\text{1Many of the ideas developed here have their capital counterpart in irreversible investment theory (Pindyck, 1991; Dixit and Pindyck, 1994).}\]
The paper is organized as follows. Section 2 models and solves the worker’s decision problem of timely turnover under uncertainty. Section 3 discusses the model’s extension to firm-specific discrete shocks. Section 4 concludes.

2. The option value of job quitting

At present time $t = 0$ a representative risk-neutral worker values the lifetime earnings of her current job as $W_0$. In the meantime, she scans outside employment opportunities. $V_0$ is the present value of expected lifetime earnings on a representative new job.\(^5\) In the presence of fixed quit costs and uncertainty about the future development of the expected stream of earnings differences between the current and the new job, the worker is not indifferent if $W_0 = V_0$. Or, put differently, the net present value rule does not apply. Facing uncertainty about the future development of the difference in wage growth, she needs $V_0$ to be strictly greater than $W_0$.

To obtain $V_0$, the worker must incur a fixed cost $Q$ that becomes sunk upon quitting. $Q$ consists of the capitalized loss of firm-specific knowledge, search costs, and the costs of changing social and geographical environments. $P_t$ is defined as the additional expected future cash flow of a worker who gives up $W$ in exchange for $V$ at time $t$. $P_t$ is the expected present value of the difference between the current and the offered job when quitting at $t$. $P_t$ is known with most certainty at time $t = 0$, but

\(^5\) $V$ can also be interpreted as a non-participation earnings stream from social security, unemployment benefits, or a pension scheme.
becomes increasingly uncertain the further $t$ lies into the distant future. Hence $P$ is assumed to follow a geometric Brownian motion:

$$dP/P = \alpha dt + \sigma dz$$

(1)

where $\alpha$ is the expected growth of the stream of value differences between the current job and the representative outside offers through time, $\sigma$ is the per unit of time variance of $\alpha$, $dt$ is the evolution of the offer probability through time, and $dz$ is the random change in the offer probability assumed to come from a standard Wiener diffusion process $z$. Eq. (1) implies that the present value of quitting may be different if the decision to quit is postponed. $P$ follows a log-normal distribution through time:

$$E[P] = P \exp(\alpha t),$$

(2)

where $E$ denotes the expectation at time $0$. The log-normal distribution of the expected earnings difference through time is a standard assumption in the empirical labor research. For example, if $V$ and $W$ share a common trend and the transition from $W$ to $V$ is a structural shift, then $V$ and $W$ are co-integrated and $\alpha = 0$. Then, the decision to quit will depend on the expected difference between $W$ and $V$ at time zero, the costs $Q$, and the evaluation of uncertain development of $P$. Another relevant example, in which $V$ grows faster than $W$ and $\alpha > 0$, is provided by Jacobson et al. (1993).

At $t = 0$, a worker will evaluate at what point it is optimal to pay $Q$ in return for an additional income stream $P$. The worker’s dynamic optimization problem is to decide whether to quit today or postpone the decision until later, and is comparable to a financial call option (McDonald and Siegel, 1986). Waiting will give additional information about $P$ at the cost of losing the discounted income difference as of today. The worker will maximize this value when considering the decision to quit.

The novelty of this application of option pricing to the theory of labor turnover is that the turnover decision problem under uncertainty can be solved analytically. We assume that the offer is infinitely lived (this assumption will be tightened in the next section), and can be interpreted as the existence of a permanent representative offer available at the neighborhood’s employment agency. If $T = \infty$, the expected net present value of the difference in salary between the current and the offered job is

$$X(P) = \max_{t \geq 0} E[(P_t - Q)e^{-\tau t} | \Omega_0],$$

(3)

subject to (1), where $t$ is the unknown moment of quitting, $\tau$ is the discount rate, and $\Omega_0$ is the information available to the worker at $t = 0$. In general, when at the beginning of period $t$ the worker decides to quit the total return she receives during that period is $\tau X$. The return of postponing that decision is equal to the expected rise in the value of future offers $X$ during the period. The first order condition for this problem is $\tau X dt = E[dX | \Omega_t]$, which can be rewritten as

$$\frac{1}{2} \sigma^2 P^2 X_p^2 + \alpha P X_p - \tau X = 0.$$  

(4)

The analytical solution of Eq. (4) yields

$^{1}$Jovanovic (1979a) also assumes $V(0)$ as the present value of lifetime earnings on a representative new job. He fails to get a closed form solution because his learning process has a declining incremental variance due to the diminishing impact of additional signals on beliefs.

$^{4}$The solution is widely available in the option pricing literature. For real options we refer to McDonald and Siegel (1986).
\[ P^* = \left( \beta_1 / (\beta_1 - 1) \right) Q, \] 

with \( \beta_1 = 1/2 - \alpha / \sigma^2 + \{(\alpha / \sigma^2 - 1/2)^2 + 2\tau / \sigma^2 \}^{1/2} \). The fact that \( \beta_1 > 0 \) implies \( P^* - Q > 0 \).

The model predicts that a worker, facing fixed cost of quitting, will exchange jobs if the net present value of the difference in expected future cash flows associated with the old and the new job exceeds the costs of quitting plus the value of postponing the decision to quit the current job.

The model also predicts that a worker with lower fixed costs is more likely to quit, since

\[ \partial P^* / \partial Q > 0. \]

Moreover, the model encompasses the uncertainty dependence of turnover. Since

\[ \partial P^* / \partial \sigma > 0, \]

higher uncertainty of the development of future earnings differences coincides with a lower probability of job quitting, ceteris paribus.

An important point, stressed by Pindyck (1991) for irreversible capital investment decisions, is that for any \( \sigma^2 > 0 \), \( P^* \) exceeds \( Q \). The net present value rule states that a worker will quit once the expected additional income flow is larger than or equal to the costs. This rule should be extended with the opportunity costs \( X(P) \) of accepting the job offer today rather than postponing that decision until later. The difference between \( P^* \) and \( Q \) is caused by the increasing future uncertainty about \( \alpha \) and the irreversibility of \( Q \).

### 3. The possibility of drastic events

In this section, we tighten the assumption that the quit option is infinitely lived. The model is extended with a discrete jump process, that forms a second source of randomness. Next to the continuous process \( z \), the possibility of the occurrence of a drastic corporate event that renders the option value of job quitting worthless. Though finite, the worker unaware of the event’s exact timing is doubtful about the spell over which the option value should be computed. The difference between the model in Section 2 and the one presented here is a different structure for the disturbance in the \( dP/P \) equation. Deriving the solution is instructive and is motivated by Fig. 2, that shows from time to time shocks hitting workers are more intense than at other times.

Assume that the lifetime of the spell is memory-less and exponentially distributed with mean \( 1/\pi \). The stochastic process for \( P \) can be rewritten as a mixed Poisson–Wiener process. Let

\[ dP/P = \alpha dt + \sigma dz + dn, \] 

with

\[ dn = \begin{cases} 
-1 & \text{with probability } \pi dt \\
0 & \text{with probability } 1 - \pi dt,
\end{cases} \]
and the Poisson event is not correlated with $P$. An increase in the jump probability $\pi$ is logically consistent with an increase in the discount rate. The boundary value $P^*(\pi)$ is identical to $P^*$, except for a higher discount rate $\tau + \pi$ rather than $\tau$. Consequently, we have

$$P^*(\pi) = (\beta_1(\pi)/(\beta_1(\pi) - 1))Q,$$

(7)

with $\beta_1(\pi) = 1/2 - (\alpha + \pi)/\sigma^2 + [((\alpha + \pi)/\sigma^2 - 1/2]^2 + 2(\tau + \pi)/\sigma^2]^{1/2}$. Given (6), we find that

$$\partial \beta_1/\partial \tau = [2\sigma^2[(\alpha/\sigma^2 - 1/2]^2 + 2\tau/\sigma^2]^{1/2} - 1,$$

(8)

so that $\partial \beta_1/\partial \tau > 0$. Together with $\pi > 0$, this yields $\beta_1(\pi) > \beta_1$, and $\beta_1(\pi)/(\beta_1(\pi) - 1) > \beta_1/(\beta_1 - 1)$, and

$$P^*(\pi) < P^*,$$

(9)

so that an increase in $\pi$ reduces the option value to quit, lowers $P^*(\pi)$, and increases the worker’s turnover probability.

Eq. (9) shows that if the length of the discounting period is uncertain but finite, then the shorter the expected spell the smaller the difference between the current and the offered wage needs to be for a worker to quit. Thus if $\tau + \pi$ goes up, the boundary difference value necessary to quit decreases. A sudden increase of the discount rate coincides with the effect of an announcement of mass lay-off on the value of waiting to quit (see the dotted line in Fig. 2).

4. Conclusions

A theoretical model is presented that describes the optimal quitting problem for a worker who faces fixed costs of quitting and uncertainty. Other theories of voluntary turnover rely on the contemporaneous difference between the outside offer and the current wage, but do not consider the worker’s expected difference value stream between earnings at the current job and of a representative outside offer, or fail to obtain a closed form solution. In this application of option pricing theory to labor turnover, a worker will accept the outside offer only if the net present value of the difference in expected future cash flows associated with the old and the new job exceeds the discounted sunk costs of quitting plus the expected net present value of lifetime earnings on a representative new job.

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1. Here, we assume there is a nonzero probability that the net present value of $P$ discretely jumps to zero being the stopping barrier for a Brownian motion. Alternatively, one could write a model with $\theta dn$ so that at every time $dn$ is nonzero $P$, falls by $\theta$ percent.

2. See Merton (1976) for the possibility of a complete ruin, or McDonald and Siegel (1986, pp. 718–719).
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