Testing for risk aversion: a stochastic dominance approach

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Received 27 September 2000; accepted 14 November 2000

Abstract

We conduct an experiment based on the Stochastic Dominance framework in order to investigate risk-aversion in isolation of other effects such as subjective probability distortion, the ‘certainty effect’, and ‘framing’ effects. The result is striking: most individuals are not risk-averse. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Risk-aversion; Stochastic Dominance; Decision-making under uncertainty

JEL classification: D81; C91

1. Introduction

Most models in economics and finance assume a non-decreasing utility function ($U'(w) \geq 0$) with diminishing marginal utility ($U''(w) < 0$). Diminishing marginal utility implies risk aversion, a concept which plays a key role in economic thought. Risk aversion is an important assumption in many economic and finance equilibrium models, and these models typically break down once the risk aversion assumption is relaxed. Most economists believe that peoples’ behavior can be generally characterized by risk aversion. However, there are a few exceptions. The most notable exceptions are Friedman and Savage (1948); Markowitz (1952a); Kahneman and Tversky (1979), and Tversky and Kahneman (1992).

Observing everyday behavior, and in particular, the fact that people buy lottery tickets (which are unfair games), Friedman and Savage argue that the utility function must also include a convex
segment; i.e. a segment with increasing marginal utility. By discussing hypothetical uncertain choices, Markowitz (1952a) refines Friedman and Savage’s arguments but reaches a similar conclusion regarding risk aversion: he claims that investors have a utility function with two concave segments and two convex segments. While Friedman and Savage and Markowitz make theoretical arguments regarding the shape of the utility function, Kahneman and Tversky (1979) conduct experiments to study this issue. Based on their experimental results, they conclude that investors maximize the expected value of a function with a convex segment for negative outcomes and a concave segment for positive outcomes.

Most economists ignore non-risk-averse utility functions and explicitly or implicitly assume risk aversion, i.e. \( U''(w) < 0 \). There are three main objections to the non-risk-averse utility functions. First, the behavior which seems to support the non-risk-aversion utility is typically observed in rather special circumstances which involve extreme probabilities. The behavior in these circumstances may be a result of the mental treatment of extreme probabilities rather than a result of risk-loving. Specifically, the existence of lotteries can be explained by the subjective distortion of small probabilities, and therefore does not necessarily contradict risk aversion. Secondly, many preference experiments supporting risk-loving typically involve comparing an investment alternative with a certain outcome with another alternative with an uncertain outcome. The results of these experiments may be therefore due to the ‘certainty effect’, which is the special mental treatment of probability 1 (see Kahneman and Tversky (1979)). Finally, most of the experimental evidence is based on experiments in which bets have outcomes which are either only positive or only negative. These are very unrealistic investments, because in realistic investments (e.g. stocks and bonds) the outcomes (rates of return) can be both positive or negative (von Neuman and Morgenstern, 1953).

If one accepts any of the non-risk-averse utility functions suggested by Friedman and Savage, Markowitz, or Kahneman and Tversky, most economic models are not intact anymore, which means that a substantial modification in thinking is needed regarding what we know about economic models and their implications, e.g. the risk-return relationship and equilibrium prices. Thus, it is very crucial to characterize peoples’ actual behavior. Does the assumption of risk aversion generally characterize the typical investor’s behavior in typical situations, or do we need to re-examine our economic models? In this paper we report on an experimental study which is based on a methodology fundamentally different than the one used in previous studies. Our experiment is conducted with realistic bets in which both positive and negative outcomes are possible, probabilities are relatively large, and all investment alternatives are uncertain, so that subjective probability distortion is not likely to be significant and there is no certainty effect. We show that more than 50% of the subjects are not risk averse in the whole range of wealth. This implies that some of our most basic economic intuitions and many of our economic and finance models should probably be re-assessed in light of the experimental findings.

In Markowitz’s and Kahneman and Tversky’s utility functions the current wealth level plays a special role (it is an inflection point). The utility functions with this special property are also referred to as ‘value functions’.

This was experimentally found by Edwards (1953, 1954). Viscusi (1989) suggests Prospective Reference Theory which provides a theoretical foundation for subjective probability distortion. He also argues that probabilities of 0 and 1 are treated as ‘special’. For other studies which reject the possibility of a risk-seeking segment of the utility function and attempt to reconcile gambling with risk-aversion see Hirshleifer (1965); Yaari (1965), and Hakansson (1970). In addition, the existence of lotteries may also be explained by the shear ‘pleasure of gambling’.

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2. Methodology

To study the shape of the utility function, it is common to present the subjects a choice between a certain payoff and an uncertain payoff with only two possible outcomes. To figure out whether the utility function is concave or convex, generally the subjects are asked questions regarding their choices between alternatives in which only positive outcomes are possible, and then they are asked separate questions regarding their choices between alternatives in which only negative outcomes are possible. For example, Kahneman and Tversky ask subjects to choose between a sure gain of $3000 or an 80% chance to gain $4000, then they separately ask subjects to choose between a sure loss of $3000 and an 80% chance to lose $4000 (see Kahneman and Tversky (1979) p. 268, and Tversky and Kahneman (1992) p. 313).

One problem with such questions is that they are very unrealistic. In the financial market, investors never, or virtually never, face investments with only negative outcomes, or alternatively with only positive outcomes. Kahneman and Tversky themselves show that ‘framing’ the alternatives in various ways affects peoples’ choices. Would their experimental results have changed if the experimental questions were ‘framed’ more realistically? Another problem with the standard methodology is that it compares uncertain prospects with certain prospects, so the results may be driven (at least to some extent) by the ‘certainty effect’. In this paper we employ Stochastic Dominance criteria with realistic uncertain investments in which both positive as well as negative outcomes are possible, which is a setting typical of investments in the stock market. Using the Stochastic Dominance framework, we show experimentally that individuals are not generally characterized by risk aversion.

3. Stochastic dominance rules

The following Stochastic Dominance rules are well-known in the economic literature. Consider two investment alternatives, $F$ and $G$, with a stochastic outcome $x$, which is bounded in the range $[a, b]$. Denote the cumulative probability distributions of these two alternatives by $F(x)$ and $G(x)$.

3.1. First degree stochastic dominance (FSD)

$F$ is said to dominate $G$ by FSD if and only if $F(x) \leq G(x)$ for all $x$, with a strict inequality for at least one value $x_0$. If $F$ dominates $G$ by FSD then all investors with non-decreasing utility functions (concave, convex, or with both concave and convex segments) prefer $F$ over $G$. Thus, the FSD criterion corresponds to all types of utility functions as long as they are non-decreasing in wealth. FSD only relies on the fact that investors are rational in the sense that they prefer more rather than less wealth (also known as the monotonicity axiom).

*Experiments which are restricted to outcomes which are either only negative or only positive were necessary to draw a clear-cut conclusion regarding the shape of the utility function. Otherwise, the researchers would need a large number of questions to figure out the shape of the utility function, and even then the results are not always clear (see Swalm (1966)). Employing the Stochastic Dominance methodology, we can test for risk aversion with a single bet, which includes both negative as well as positive possible outcomes.*
3.2. Second-degree stochastic dominance (SSD)

\( F \) dominates \( G \) for all non-decreasing risk averse utility functions (i.e. all functions with decreasing marginal utility), if and only if:

\[
\int_a^x [G(t) - F(t)] \, dt \geq 0 \text{ for all } x, \text{ with a strict inequality for at least one value } x_0
\]

For a proof of these Stochastic Dominance rules and for further discussion, see Fishburn (1964); Hadar and Russell (1969); Hanoch and Levy (1969); Rothschild and Stiglitz (1970), and Levy (1998). We employ these two Stochastic Dominance rules in our experiment.

Table 1
The alternatives presented to the subjects

Suppose that you decided to invest $10,000 either in stock \( F \) or in stock \( G \). Which stock would you choose, \( F \), or \( G \), when it is given that the dollar gain or loss one month from now will be as follows:

<table>
<thead>
<tr>
<th align="left">Task I</th>
<th align="left">Which would you prefer, ( F ) or ( G ), if the dollar gain or loss one month from now will be as follows:</th>
</tr>
</thead>
<tbody>
<tr>
<td align="left">( F )</td>
<td align="left">Gain or loss Probability ( -500 ) ( +2500 )</td>
</tr>
<tr>
<td align="left">( G )</td>
<td align="left">Gain or loss Probability ( -500 ) ( 2500 )</td>
</tr>
</tbody>
</table>

Please write \( F \) or \( G \)

\[ \Box \]

Task II
Which would you prefer, \( F \) or \( G \), if the dollar gain or loss one month from now will be as follows:

<table>
<thead>
<tr>
<th align="left">Task II</th>
<th align="left">Which would you prefer, ( F ) or ( G ), if the dollar gain or loss one month from now will be as follows:</th>
</tr>
</thead>
<tbody>
<tr>
<td align="left">( F )</td>
<td align="left">Gain or loss Probability ( -500 ) ( +500 ) ( +1000 ) ( +2000 )</td>
</tr>
<tr>
<td align="left">( G )</td>
<td align="left">Gain or loss Probability ( 0 ) ( 2500 ) ( 1500 )</td>
</tr>
</tbody>
</table>

Please write \( F \) or \( G \)

\[ \Box \]

* The alternatives given in Table 1 were presented to the subjects. They had to choose one alternative in each Task. The following paragraph accompanied the questionnaire: Investment Choice Questionnaire. We are conducting an experimental study about the preferences of individuals regarding various investment alternatives. This study investigates the investment preferences of fund managers, and finance students. The study is anonymous and will be used for academic purposes only. We would appreciate it if you would take a few minutes to answer the questions in the next page. Thank you for your cooperation.
4. The experiment and results

We focus in this study on testing risk-aversion by the SSD criterion. However, to test the reliability of our experimental results we also test FSD to examine the degree of the subjects’ rationality (in the sense that they prefer more to less). The subjects in our experiment are 62 mutual fund managers and financial analysts, and 132 business school students from UCLA, the Hebrew University, and the University of Washington. Table 1 provides two tasks presented to our subjects.

Notice that in each task investments with both positive and negative outcomes are involved. All investments are uncertain, so that the ‘certainty effect’ does not play any role in our experiment. In addition, all probabilities are fairly large ($p \geq 0.25$), so subjective probability distortion, which takes place mainly for small probabilities, is not likely to play an important role here.\(^5\)

Fig. 1 provides the cumulative distributions corresponding to the two alternatives in each of the two tasks. From Fig. 1 and the Stochastic Dominance rules presented above it is easy to verify that in Task I alternative $F$ dominates $G$ by FSD, and we expect all subjects who prefer more over less and act rationally to choose $F$. In Task II, $G$ dominates $F$ by SSD, and if risk-aversion prevails, we expect $G$ to be selected.

Table 2 summarizes the results of the choices made in the two tasks. In Task I, there is an FSD dominance of $F$ over $G$. Indeed, only 4% of the students and only 6% of the fund managers selected the ‘wrong’ alternative, alternative $G$. Thus, from Task I we conclude that the subjects conform with the monotonicity axiom, because the vast majority of them selected the FSD dominating alternative, which is the rational choice for any person who prefers more rather than less. The fact that the alternative which is dominant by FSD is selected is consistent with any non-decreasing utility function, and no further conclusions can be drawn from Task I regarding the shape of the utility function or regarding risk-aversion.

Task II is the core of our experiment, because it directly tests the hypothesis of risk aversion. In Task II, $G$ dominates $F$ by SSD (see Fig. 1B). Thus, any risk-averse individual should prefer $G$ over $F$. However, 59% of the students and 42% of the practitioners selected alternative $F$.\(^6\) Taking the two groups together (see part C of Table 2), we draw the following conclusions:

\(^{5}\)In Viscusi’s (1989) Prospective Reference Theory probability distortion occurs because subjects treat stated experimental probabilities as imperfect information, and reach their posterior beliefs about probabilities by integrating the stated experimental probabilities with their priors. Viscusi suggests that a natural case to consider is the symmetric-reference-point case, in which an equal prior probability is assigned to each possible outcome. Notice that in Task II in each alternative the experimentally stated probabilities are equal for all outcomes, so that according to Prospective Reference Theory with a symmetric-reference-point the subjective probabilities in Task II are exactly identical to the experimentally stated probabilities, and no probability distortion takes place. In Task I Prospective Reference Theory with a symmetric-reference-point does not distort the probabilities in $G$, but makes $F$ subjectively less attractive. In spite of this subjects choose $F$ in Task I, which is the FSD dominant alternative. This suggests that probability distortion did not play an important role in neither Task I nor II.

\(^{6}\)Note that alternative $G$ dominates alternative $F$, also by the Markowitz (1952b) mean-variance rule (with $E(x) = E(x)$ and $\sigma(x) > \sigma(x)$). Some of the fund managers who are well acquainted with the mean-variance rule probably employed it here in their decision-making, hence a higher proportion of them selected $G$, relative to the proportion of the students who selected $G$. 

Fig. 1. Cumulative distributions of Tasks I and II.

(a) 95% of the subjects selected an alternative which conforms with FSD in Task I, i.e. most people are rational in the sense that they prefer more rather than less money, $U'(w) > 0$.

(b) The subjects were split in their choices in Task II, with only 44% selecting the alternative with the SSD dominance ($G$). This indicates that 56% of the subjects are not risk-averse.

5. Concluding remarks

Risk-aversion is a fundamental component of economic theory, and, in particular, of models dealing with decision-making and equilibrium pricing under conditions of uncertainty. Risk-aversion is
Table 2

The experimental results

<table>
<thead>
<tr>
<th>Task no.</th>
<th>F</th>
<th>G</th>
<th>Indifferent</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>A: Students. Number of subjects: 132</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>96</td>
<td>4</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>II</td>
<td>59</td>
<td>39</td>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>B: Practitioners. Number of subjects: 62</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>94</td>
<td>6</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>II</td>
<td>42</td>
<td>56</td>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>C: Aggregate. Number of subjects: 194</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>95</td>
<td>5</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>II</td>
<td>54</td>
<td>44</td>
<td>2</td>
<td>100</td>
</tr>
</tbody>
</table>

Numbers in the tables are in percent, rounded to the nearest integer.

inconsistent with buying lottery tickets, hence Friedman and Savage and Markowitz suggest a utility function which includes a risk-seeking segment as well as a risk-averse segment. Friedman and Savage and Markowitz do not support their hypothesis experimentally. Kahneman and Tversky find a risk-seeking segment in their experimental studies. These three papers are the exceptions rather than the rule. Most economists do not accept these types of utility functions and continue to develop models which assume risk aversion in the whole range of wealth. The existence of lotteries is explained in the face of risk aversion by assuming that people distort small probabilities, i.e. the subjective belief that they have a chance to win is larger than the correct objective probability. Kahneman and Tversky’s results may be due to the fact that the investment alternatives which they present to their subjects are unrealistic as the outcomes are confined either only to the positive range or only to the negative range, while the outcomes of actual investments can be both positive or negative. In addition, as they typically compare certain prospects with uncertain prospects, their results may be influenced by the ‘certainty effect’.

In this paper we employ the Stochastic Dominance methodology which allows us to study preferences with realistic investment alternatives which have negative as well as positive possible outcomes, as typical of actual investments in the stock market. Also, in our study there are no small probabilities or outcomes which are certain, thus it is unlikely that subjective probability distortion plays an important role. With these realistic investment alternatives we find that most subjects’ choices conform with the FSD criterion, and the subjects are therefore rational in the sense that they prefer more over less. However, we strongly reject the hypothesis that risk-aversion prevails in the whole range of wealth. Thus, it would seem that many of our economic and finance models should be re-examined in light of this striking result.

Acknowledgements

The authors acknowledge the financial support of the Krueger Fund and the Zagagi Fund.
References