What education production functions really show: a positive theory of education expenditures

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Abstract

An optimizing model of expenditure allocations predicts that input use should be chosen so that the marginal product per dollar of each input is equalized. In decided contrast, the existing literature shows that the marginal product per dollar of inputs not directly valued by teachers are commonly 10 to 100 times higher than that of inputs valued by teachers. This implies that inputs which provide direct benefits to educators (like teacher wages) are over-used relative to inputs that contribute directly (but only) to educational output (like books or instructional materials). One class of positive models of expenditures consistent with this empirical finding invokes a very high ratio of teacher to parent (or student) influence in the determination of expenditures. This implies that educational reforms that shift the relative strengths of parents versus teachers in the allocation of expenditures can sometimes lead to enormous gains in the cost effectiveness of schools. [JEL I21] © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

There is broad consensus that expansion in the skills, knowledge, and capacities of individuals—increasing human capital—is a key element in developing countries’ economic progress and in raising their living standards. Primary and secondary schooling plays an important role in the expansion of human capital, and governments and international fora have often made explicit targets for the expansion of such schooling. However, there is an enormous gap between children sitting in a classroom and an increase in human capital. Quality of education is a deep concern as on internationally comparable tests some developing countries, such as Korea and China, do quite well, but others do abysmally, and in some countries students’ achievement test scores after years of instruction are little better than random guessing. If these quality deficiencies were due merely to the lack of inputs the policy prescription would be obvious: increase resources. However, a cumulation of empirical literature using comparisons of student performance across countries, across regions within countries, and across schools within regions consistently produces the puzzling and worrisome finding that resources

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1 This paper focusses on primary and secondary schooling in developing countries. The issues both for post-secondary schooling and for schooling in richer countries are likely to be qualitatively different.

2 On the comparison of reading performance of 10 years olds, Venezuela and Indonesia were an amazing 6.5 standard deviations below the average of the developed countries. On a similar assessment of science performance of 10 year olds, students from Nigeria and the Philippines were 2.6 and 3.7 standard deviations respectively below the developed country mean (Postlethwaite & Wiley, 1991; Elley, 1994).

3 This has been found for instance definitely at the fourth grade level (and to a lesser extent at even higher grades) in Ghana (Glewwe, 1998) and Kenya (Glewwe, Kremer & Moulin, 1997).
are only tenuously related to measured academic achievement.

We build on Hanushek’s argument that since budget differences do not account for performance differences, the incentives that determine how well the budget is spent likely play an important role (Hanushek, 1995). We argue the evidence is grossly inconsistent with the assumption that resources are allocated to maximize educational output (however defined). The key indicator of this misallocation is that the cost effectiveness, or achievement gain per dollar, of teacher related inputs is commonly orders of magnitude lower than alternative inputs. This relative overspending on inputs that are of direct concern to teachers is so pervasive that it is consistent only with a positive model of the allocation of education spending in which teacher welfare directly influences spending, over and above its impact on school quality.

We want to stress that although we argue that the evidence is consistent with enormous inefficiencies in education spending due to relatively high spending on teacher inputs, this should not be construed as an attack on teachers, who are the backbone of any educational system. “Government” actions are neither deus ex machina nor satanas ex machina, but the result of the interaction of differing interests. Suggesting that educators defend their interests is a methodological assumption common to all economics, not an ad hominem attack. Just as there are “market failures” intrinsic to the incentives of allocation by decentralized, uncoordinated exchanges, there are “government failures” which are equally inherent in the structure of incentives created by allocation through the political process. The political pressures for the misallocation of expenditures across functional categories detailed here for education are almost certainly present in other sectors: health clinics with personnel but without drugs or clean needles, highways without maintenance, or agricultural extension workers without transport are all depressingly common in developing countries.

When performance is more a function of incentives than of expenditures the policy implications are less obvious. While in some cases merely increasing the budget is the appropriate educational policy, it is increasingly recognized that in many other situations more fundamental reforms that enhance the importance of outputs of education in spending decisions are necessary (Inter-American Development Bank, 1996).

2. Theories of the allocation of expenditures

This section first explains why some behavioral theory of expenditure allocation is necessary for understanding the results of empirical examinations of the determinants of educational outcomes. We then present a series of simple illustrative positive models of educational expenditure allocation that aid in the interpretation of the empirical literature.

2.1. Why it takes a theory

Possibilities are always limited by technical constraints. A “production function” is an expression for the maximum amount of output possible for an amount of inputs. By applying this metaphor to education one can talk of an “educational production function” which is determined by an underlying, undoubtedly extremely complex, pedagogical processes. But even though the production function is derived from technical, not behavioral relationships, one needs a behavioral theory to understand the results of empirically estimating a production function for two reasons.

First, a second algebra book will likely help a student less than the first, and a tenth book much much less. Since the learning gain from additional inputs is not constant the contribution of an input depends on the rate of input utilization at which it is assessed. This is particularly important in educational research, as there is a crucial distinction between testing whether inputs have low productivity at their current rate of application versus testing whether they are “inputs” at all. Teachers, classrooms, and instructional materials are clearly inputs into education. If we’re to understand statistical tests of whether the effect of class sizes or teacher wages are different from zero as tests of whether these inputs ever matter the whole endeavor appears silly and pointless (which has often been the reaction of the education community to this line of empirical research). However, as the example described above suggests, a failure to reject zero marginal product of higher teacher wages or reduced class sizes may simply mean that the rate of

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4 Although it should be pointed out this trend in considering market and government failures is a renewed, not new, interest for economists. For example, Pigou (1912) states: It is not sufficient to contrast the imperfect adjustments of unfettered private enterprise with the best adjustment economists in their studies can imagine. For, we cannot expect that any State authority will attain, or will even whole heartedly seek, that ideal. Such authorities are liable alike to ignorance, to sectional pressures, and to personal corruption by private interest.

5 By analogy one could study the effect of either fertilizer or Mozart piano concertos on corn production. Even though at sufficiently high rates of application the size of fertilizer’s impact could be small (or even negative), fertilizer is clearly an input, and the only question is the magnitude of the impact at various rates of application. On the other hand, Mozart’s music, while delightful, might not be an input into corn production, in the sense that it has zero corn production impact across all soil conditions, concerti, and volumes.
application is so high that the marginal product at the chosen rate of application (conditional on other inputs) is statistically indistinguishable from zero. Nearly all education production function studies are not experiments in which input use is chosen by a researcher, but rather use data drawn from reality. In reality, the spending on inputs (e.g., teacher wages, class sizes, buildings, textbook use) is chosen for reasons other than research and since input use is chosen, the theory of the input choice predicts the rate of application and thereby observed input productivity.

Second, the policy implications of empirical findings depend critically on the theory of input choice. If the input use is the result of a political process, changes may not be feasible without changes in the underlying process that determines budgets. Recommendations like “spend more on input x” are not necessarily valuable contributions to policy making when policies are endogenously politically determined.

2.2. Elements of a theory behind education production functions

In this section we sketch several possible positive theories of expenditure allocation that might underlie production function estimates (a more detailed derivation of the relationships discussed can be found in Appendix A). The most simple positive model of spending is that inputs are chosen to maximize the production of a single educational output (call it “S” for scores). This theory implies that the marginal product per dollar will be equalized across all inputs. That is

\[ \frac{f_i^S}{p_i} - \frac{f_j^S}{p_j} = 0, \forall i, j \] (1)

where \( i \) and \( j \) are any two inputs, \( f_i^S \) is the marginal product of input \( i \) in producing output \( S \), and \( p_i \) is the price of input \( i \). Basic schooling transmits more than information: it is also intended to convey acceptable patterns of social interaction, and cultural norms and beliefs, and political ideals (Picciotto, 1996). A simple positive model of input choice in the production of multiple, say for simplicity two, outputs \( S \) (scores) and \( C \) (“citizenship”) would imply that the marginal value product of both inputs is equalized:

\[ \frac{f_i^S + p_c \cdot \frac{hC_i}{p_j}}{p_i} = \frac{f_j^S + p_c \cdot \frac{hC_j}{p_j}}{p_j}, \forall i, j \]

or equivalently,

\[ \frac{f_i^S}{p_i} - \frac{f_j^S}{p_j} = p_c \cdot \frac{hC_i}{p_j} - \frac{hC_j}{p_i}, \forall i, j \] (2)

where \( h \) is the production function for output \( C \), and \( p_c \) is the relative value placed on \( S \) and \( C \).

This introduction of multiple outputs implies if one input is relatively more productive in the production of one type of output (“citizenship”) then its input use will be higher (and hence marginal product per dollar in scores production lower) than if one were optimizing over scores alone.

A very simple alternative positive model embeds (potentially both of) the output optimizing models as a special case. Suppose expenditure allocations are made to optimize over both educational production (assume just a single output \( S \) for simplicity) but also teacher utility (\( T \)) directly. This implies that inputs would be chosen so that:

\[ \frac{f_i^S}{p_i} - \frac{f_i^T}{p_i} = \frac{\alpha(1 - \delta)}{(1 - \alpha) + \alpha\delta} \cdot U_x \cdot \frac{\gamma_i - \gamma_j}{p_i} \] (3)

where \( \alpha \) is the weight given to teacher’s utility relative to “scores”, \( \delta \) is the degree to which teachers derive utility from the test scores of their students, \( U \) is the utility that teachers derive directly from spending on inputs, and \( \gamma \) is the relative value of input \( I \) to teachers’ utility (see appendix 1 for details of this derivation). The implication of (3) is that the more directly an input enters teacher utility (a higher \( \gamma \)) the lower the marginal product per dollar of that input in producing the educational output (\( S \)) will be at the chosen allocation of inputs, relative to

\[ \text{...} \]

6 The “meta-analysis” approach to creating statistical significance is problematic, in that it ignores the actual underlying policy question and the asymmetry in what the two sides of the “money matters” debate are actually asserting. That is, if the impact of teacher inputs (e.g., salaries, class size) is not demonstrably different from zero this is not to say it is everywhere and always zero, but that it is unlikely to be optimal, in the normative sense of maximizing production. However, showing only that teacher inputs, or budgetary inputs generally, have some impact (which can be done with meta-analysis, e.g., Hedges, Laine, & Greenwald, 1994) does not address the question of whether input levels are optimal.

7 While this is generally being presented as a normative model, what a person would do if in fact they wanted to optimize, the simplest behavioral model is to assume the normative model is the positive model. That is, just assume that the chooser actually behaves as if he/she were optimizing. While this assumption is heroic, it is implicit (and sometimes explicit) in nearly all education production function research. For example, to estimate and interpret the cost function as the dual of the production function requires the assumption of maximization (Jimenez, 1986; Jimenez, & Paqueo, 1996; James, King, & Suryadi, 1996).
the output maximizing level. The intuition is clear: since the choosers of inputs cares directly about teacher utility this will lead to a higher level of spending on inputs that teachers care about and if marginal products are declining with higher rates of use this higher level of utilization will lead to a lower marginal product per dollar of that input. In this model, the marginal product per dollar of inputs that teachers value directly (e.g. wages) will be smaller than the marginal product per dollar of inputs which teachers value indirectly (e.g. smaller class sizes, better physical facilities) which, in turn, will be smaller than those that teachers value only because of their effect on achievement (e.g. books). The difference in marginal products will be larger, the larger the weight on teacher utility ($\alpha$) and the smaller the teacher valuation of educational outputs ($\delta$).

Fig. 1 illustrates the case with only two hypothetical inputs, call them “books” and “wages” for concreteness. The educational output optimizing choice would be to choose point A, where the educational output isoquant,\(^9\) whose slope is the relative marginal product of books to teachers, is tangent to the budget line, whose slope is the relative price of books to teachers with the same budget. At this point the first order condition (1) for an (interior) maximum are reached. With the same budget, adding teacher’s utility directly to the chooser’s objectives implies a new equilibrium with more teacher and less book spending (point B). This higher level of spending reduces the marginal product per dollar of teachers (illustrated in the southeast quadrant) and increases the marginal product per dollar of books (illustrated in the northwest quadrant), which were equal when the objective function included only achievement. Starting from a production optimizing allocation, if teacher’s utility is added to the objective function, relatively less is spent on “books” and relatively more is spent on “wages” so that at this new rate of input utilization the marginal product per dollar of “books” is high relative to the marginal product per dollar of “wages.”

2.3. Three possible models underlying the model

The last positive model simply assumed only that the decision maker cared about teacher utility, hence it is more of a description of a broad class of models. We have not articulated a complete model which specifies which person or entity controls the allocation of spending (a school board? a legislature? the ministry of education?) nor why exactly that person or entity cares directly about teacher welfare. While for many of our conclusions the exact model is not important, predictions about the impact of specific reform need a more complete model. There are (at least) three prototype models that could generate an objective function that produces something like (3): principal-agent, teacher power, and patronage, each of which would have different implications for policy.

2.3.1. Principal-agent

Suppose the decision maker is the parents (or equivalently a manager that faithfully represents parent interests) but is uninformed about the production function. In this case, as with other professional services such as medical practitioners, teachers will systematically misrepresent the production function so as to lead the chooser of budgets to choose greater spending on those inputs that teachers prefer. The source of teacher “weight” in the objective function ($\alpha$) is superior knowledge by teachers about the production function. This principal agent effect would extend to the private sector as well if parents are misinformed about the true production function, although potentially mitigated by competition among providers.

2.3.2. Teacher power

The second possibility is that the decision maker is a politician (e.g. minister of education) or political institution (e.g. legislature or school board) that controls budgetary allocations. Given that on issues like teacher wages educators have a very clear and direct interest, they may be more successful at organizing to influence public decisions than are parents. In this case, the problem is not that parents are systematically under-informed about the production function, but that parents have greater difficulty overcoming the free rider problem inherent in collective action than do teachers (Olson, 1965).

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\(^9\) The combinations of books and teachers that produce constant amounts of educational output.
2.3.3. Patronage

Another possible model with political decision makers is that teachers are not particularly powerful but that the decision maker is powerful. Suppose that the Minister of Education can allocate all jobs and each job creates a favor. In this case teachers may well want high salaries but the chooser may prefer teacher salaries to be relatively low (but at a slight premium so that the job is a “favor”) but would want to maximize the employment of teachers. In this case teachers would be individually poorly paid but there would far too many teachers and an inflated wage bill for teachers. Before moving to the empirical evidence we should point out that overspending on teacher inputs need not imply that teachers are individually “overpaid”, a point we return to in the final section.10

Nor are we arguing “too much” is spent on education as we do not model the determination of the overall budget for education, which could be low. In many cases spending on teachers (or the budget overall) is “too low” in any absolute sense, even if teacher spending is “too high” relative to the productivity of other inputs.

3. (Our reading of) the evidence

How do the predictions of the two classes of models, output maximizing and teacher influence, fare when faced with the evidence? Together the empirical evidence overwhelmingly rejects both variants of the educational output optimizing models (single and multiple output) in favor of a model which incorporates teacher utility into decision making. We review three types of evidence in the literature: studies of cost effectiveness, education production function estimates, and direct evidence from differences in parental control within educational systems. To preview the results, the failure of the equalization of marginal product per dollar definitively rejects single output optimizing. Multiple output models are not consistent with the systematic pattern of the rejection of equal marginal product per dollar across inputs (that educator related inputs tend to be overused relative to non-teaching outputs) or with the comparisons in input use under different incentives (e.g., between public and private schools, or schools with more or less parental control, or comparisons of outcomes with teacher unions of varying power).

3.1. Empirical estimates of the cost effectiveness of various inputs

The key hypothesis to test the single output optimizing model is whether the marginal products per dollar of all educational inputs are equalized. Table 1 reports the cost effectiveness estimates derived from a large scale evaluation of an education project in Northeast Brazil (Harbison, & Hanushek, 1992). These estimates line up exactly as we would have expected if there is a substantial amount of teacher influence. The cost effectiveness of teacher salaries (normalized to 1) is by far the lowest. Material inputs which provide amenities to the school and teachers, such as teacher tables, toilets, bookcases, have a cost effectiveness on average 7.7 times larger than teacher salaries. Instructional materials, which provide benefits to teachers only insofar as they increase scores,

<table>
<thead>
<tr>
<th>Material inputs</th>
<th>Estimated marginal effects averaged over years, subjects, and grades</th>
</tr>
</thead>
<tbody>
<tr>
<td>Textbook usage</td>
<td>17.7</td>
</tr>
<tr>
<td>Writing materials</td>
<td>34.9</td>
</tr>
<tr>
<td>Software*</td>
<td>19.4</td>
</tr>
<tr>
<td>Infrastructure inputs</td>
<td></td>
</tr>
<tr>
<td>Hardware*</td>
<td>7.7</td>
</tr>
<tr>
<td>Alternative teacher</td>
<td></td>
</tr>
<tr>
<td>education strategies</td>
<td></td>
</tr>
<tr>
<td>Curso de</td>
<td>5.0</td>
</tr>
<tr>
<td>Qualificacao</td>
<td></td>
</tr>
<tr>
<td>Logos</td>
<td>8.3</td>
</tr>
<tr>
<td>4 years primary school</td>
<td>6.7</td>
</tr>
<tr>
<td>3 years secondary school</td>
<td>1.9</td>
</tr>
<tr>
<td>Teacher salary</td>
<td>1.0</td>
</tr>
</tbody>
</table>

* Hardware: water, bookcase, teacher table, pupil chair, pupil desk, two classrooms, large room, director’s room, kitchen, toilet, store cupboard. Software: writing material, chalk, notebook, pencil, eraser, crayons, textbook usage.

Source: Derived from Harbison, & Hanushek (1992), Table 6-2.

10 Several recent papers have examined whether teachers are “overpaid” by comparing wages or incomes of teachers either with cross-national norms (Cox-Edwards, 1989; Carnoy, & Welmond, 1996) or with wages of other individuals, controlling for individual characteristics, like age and education (e.g Psacharopoulos, 1987; Grootaert, & Komenan, 1990; Piras, & Davedoff, 1998). To see why this literature is distinct, imagine that receiving a teaching position was like a sinecure, for which the teacher received a fixed annual payment completely independent of performance, and imagine that therefore teachers did no teaching at all. In this hypothetical case the “wages” of teachers could be arbitrarily low and yet the ratio of marginal product of teacher to non-teacher expenditures still be quite high.
have cost effectiveness ratios between 17 and 34 times as large as the impact of additional spending on teacher salary increases. The prediction of the simple optimizing model, that cost effectiveness is equalized, is wrong by at least an order of magnitude.

Table 2 provides similar estimates from a large scale study of student achievement from districts in eight states of India (World Bank, 1996a). The cost effectiveness of increasing teacher salaries is again normalized to 1. The cost effectiveness of spending on improving physical facilities is higher than that of teacher salaries (1.2 times higher), however that for increasing just classroom size is between 1.7 and 4 times higher. The cost effectiveness of spending on instructional inputs is between 4 and 14 times higher than that of increasing teacher salaries. Again the actual ratio between the marginal product per dollar of salary inputs and instructional materials is different from the level predicted by optimizing model by more than an order of magnitude, 14 to 1.

What is somewhat puzzling is that class size reductions are even less effective than teacher salaries. However both of these results are consistent with recent estimates by Kingdon (1996) using sample survey data for students from 30 schools in urban Uttar Pradesh, who finds that if the cost effectiveness of teacher salary is normalized to 1, the cost effectiveness of class size reductions is 0.029 but the cost effectiveness of increasing school physical resources is 3.2.11

3.2. Education production function estimates

Do these few cost effectiveness studies represent the typical case across a variety of conditions? While there are very few studies that calculate cost effectiveness for different types of inputs, there are hundreds studies which relate student achievement to school characteristics, such as inputs, while controlling for student’s background characteristics (e.g. parental income, education) and there have been several prominent reviews of this literature. These reviews face enormous difficulties in summarizing the empirical results of the different studies as the estimated coefficients, which are reported in physical quantity units (e.g. test score gains per square foot) are not directly comparable either across inputs or across studies. Therefore the reviews summarize only whether the effect was estimated to be positive or negative and whether it was statistically significant (although some do attempt to report “effect sizes” which, at least normalizes the test scores to standard deviation, Lockheed, & Hanushek, 1988).12

3.2.1. Confirmation percentage evidence

Acknowledging these problems, there are two ways the enormous educational production function literature

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<table>
<thead>
<tr>
<th>Instructional materials</th>
<th>14.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Complete package of instructional materials and aids</td>
<td>4.0</td>
</tr>
<tr>
<td>Coverage under Operation Blackboard, equipment only</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Physical facilities</th>
<th>1.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full package of facilities</td>
<td>4.3</td>
</tr>
<tr>
<td>One additional square foot per student</td>
<td>1.7</td>
</tr>
<tr>
<td>Cost based on school construction in Uttar Pradesh</td>
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<tr>
<td>Cost based on District Primary Education Project average</td>
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</table>

<table>
<thead>
<tr>
<th>Teacher Quality</th>
<th>1.0</th>
</tr>
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<tbody>
<tr>
<td>Increasing average teacher salary by Rs. 100 per month per school</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Opportunity to learn</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reducing the pupil teacher ratio by one</td>
<td>1.0</td>
</tr>
<tr>
<td>Adding one additional school day per academic year</td>
<td>0.6</td>
</tr>
<tr>
<td>Adding one additional teaching hour per academic year</td>
<td>29.0</td>
</tr>
<tr>
<td>Adding one additional hour of language instruction per academic week</td>
<td></td>
</tr>
</tbody>
</table>


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11 Since this study did not report cost data for school resources, we used data from World Bank (1996a) to calculate these ratios.
12 We should note that the reviews discussed here include both published and unpublished studies. While this does raise the issue of the quality of some of the underlying work, it does avoid the problem of publishing bias whereby significant effects (in this case strong teacher effects) might be more likely to be published (Begg, 1994; Vevea, & Hedges, 1995). Moreover, while there are many econometric issues involved in the estimation of production functions, issues which may be exacerbated by the inclusion of unpublished papers, it is important to keep in mind that in order for the issues to be real problems for this assessment they would need to consistently create a pattern of small effects of teacher valued inputs relative to others, which is a more complicated proposition. Further caveats on the estimation of production functions are discussed below.
is relevant to the competing positive models. First, we can compare the “confirmation percentage” of various inputs, the frequency with which an input was found to be statistically significant of the expected sign. The inputs are sorted by our conjectures as to which inputs are of most, and which of least, direct importance to teachers.13,14 Results from the most recent of these compendia, Fuller, & Clarke (1994) in Table 3 show a markedly higher confirmation ratio for the inputs, like facilities and instructional materials, which are less likely to appear directly in teacher’s utility: “Teacher’s salary” has a confirmation rate of 36% for primary and 18% for secondary schools, the confirmation rates of “teacher-pupil” ratio are 35 and 9% respectively. In contrast, the “presence of a school library” has a confirmation rate of 89% and the “availability of textbooks” a confirmation rate of 73% for primary and 54% for secondary schools (two other reviews, Harbison, & Hanushek, 1992; Velez, Schiefelbein, & Valenzuela, 1993 report similar results and are included as Appendix B).15

While this evidence is only corroborative, as it is possible the differences in precision could have affected the pattern of statistical significance, the most straightforward way to read this evidence is that studies that use non-experimental data systematically evaluate the marginal product of inputs at expenditure allocations that are not education output maximizing because, at the spending allocations typically observed which are the result of choice, the marginal product of inputs is lower for those inputs more highly valued by teachers and hence they are less likely to be found statistically significant.

The second way production function studies can be brought to bear on the competing positive models is to calculate estimates of the marginal product per dollar from the reported results.16 Unfortunately, after scouring this literature we found very few studies that report the necessary information, principally because they lack data on the relative costs of various inputs. Even among those that do report some estimates of cost effectiveness, standard practice is to calculate cost effectiveness only for those estimates that happen to be statistically significant (and in the right direction).17

| Table 3 | Confirmation percentages of various educational inputs sorted by direct importance to teacher utility |
|-----------------|-----------------|-----------------|-----------------|-----------------|
|               | Number of studies | Positive and significant relation | Confirmation Percentage |
| Primary Schools: | Teacher’s salary | 11 | 4 | 36.4 |
|                 | level          | 26 | 9 | 34.6 |
|                 | School teacher pupil ratio | 18 | 9 | 50.0 |
|                 | Teacher’s years of schooling | 23 | 13 | 56.5 |
|                 | Teacher’s experience | 17 | 15 | 88.2 |
|                 | Class instructional time | 11 | 9 | 81.8 |
|                 | Class frequency of homework | 18 | 16 | 88.9 |
|                 | School library | 26 | 19 | 73.1 |
| Secondary schools : | Teacher’s salary | 11 | 2 | 18.2 |
|                 | level          | 22 | 2 | 9.1 |
|                 | School teacher pupil ratio | 12 | 1 | 8.3 |
|                 | Teacher’s experience | 16 | 12 | 75.0 |
|                 | Class instructional time | 13 | 7 | 53.8 |

Source: Fuller, & Clarke (1994).

13 We include only inputs reported in the surveys of the literature that were subject to 10 or more studies.

14 This ranking is problematic as the interpretation of a coefficient depends critically the other variables included in the regression. For instance, some regressions include “instructional time,” which means that if teacher salaries are included in the same regression (say as a proxy for teacher input quality) then more time for the same salary would be a real wage reduction which teachers should oppose so that the marginal product should be very high. If on the other hand teacher salaries are not included then the sign of “instructional time” is ambiguous. Similarly, the coefficients on “teacher training” or “teacher education” all depend on what else is included in the regression.

15 An additional point of interest from Velez, Schiefelbein, & Valenzuela (1993) is that their review of 43 studies (Appendix B Table 8) found that only 4 of those found a significant and positive relationship between “teacher satisfaction” and student achievement.

16 There is a distressingly common practice of reporting the production function estimates of known (as opposed to pedagogical innovations such as radio instruction, computer assisted learning, alternative teaching strategies) educational inputs in quantity units (e.g. class size, text books, physical facilities) and their statistical significance without comparative information on costs. We would claim this is nearly devoid of policy interest.

17 Calculating cost effectiveness for only those estimates that are statistically significant confuses the magnitude with the precision of the estimate as an insignificant result can result either from a small, precisely measured, effect or a large, imprecisely measured, effect.
Table 4 reports the educational production function estimates in the Philippines by Tan, Lane, & Cousître (1997). They calculate cost effectiveness only for those few inputs, particularly workbooks and furniture, that were statistically significant but they do not compare cost effectiveness per dollar with other inputs, such as reductions in class size. Using their estimates and information about teacher salaries and class sizes from other sources we calculate that, at their reported estimates, providing workbooks was 800 to 1600 times, and providing furniture 700 to 1200 times, more cost effective than reducing class sizes. One might object we are using the statistically insignificant, and hence potentially imprecise, coefficient on class size in this calculation. However, even if we add two standard deviations to the point estimate for class size to increase it to the upper range of its 95% confidence interval, the marginal product per dollar of workbooks or furniture was still roughly 100 times larger than of reducing class size. Even at the outer bounds of precision the ratio predicted by an optimizing model is off by two orders of magnitude.

We should not overstate our case as to the reliability of the educational production function literature in supporting any hypothesis. Many of the studies find nothing to be statistically significant and often find perverse patterns in the point estimates (for example negative signs on inputs like class size or the availability of books). There are several additional econometric difficulties with the literature and we should mention at least five: inadequate controls for students’ background, simplistic and differing specifications, multi-collinearity, insufficient variation in input use, and different sensitivities among inputs.18

3.3. Direct evidence on the impact of institutional reform

The 10 to 100 fold differences in marginal product per dollar across inputs, and particularly the consistent pattern across inputs, rules out the simple single output optimizing model. However, one could still return to the multiple output model and conjecture that the observed pattern of marginal products in producing scores corresponds to the inverse pattern in the marginal products in producing some other output. In this section we first argue this objection is unfounded, and then show additional evidence that leans in favor of a positive model with teacher influence versus an optimizing model with multiple outputs.

It is possible to claim that the unobservable output of education requires the enormously higher teacher salaries, more teachers, but much less physical facilities and less instructional materials than the observable output that would be consistent with this pattern, but we do not find that plausible. Moreover, someone with an interest in rejecting a positive model of expenditures with teacher influence could always assert that the patterns of the marginal product of inputs for some unobserved (or even unobservable) component of educational output are in fact exactly what they would have to be in order to rationalize any observed pattern of marginal product per dollar without fear of contradiction (or confirmation).

Fortunately, there are sources of direct evidence which distinguish the single and multiple output maximizing from the other models. If any of the optimizing models were correct then one should not see systematic patterns

18 One point on the last two concerns. Suppose that there were a number of potential non-teaching inputs with high, but steeply declining, marginal products (like algebra books). A perfectly random allocation of resources among these non-teaching inputs would produce a pattern of some very large and some small estimated marginal products with the high productivity of some of these factors due to the very low utilization. Moreover, there may be inputs for which the marginal product is very high at low levels but declines very rapidly. Since the estimated effect is the average at low and high levels of use, one could see a high marginal product for that input, but also observe that the input was rarely in range of low use, and hence the empirical finding is of limited usefulness. For instance, Glewwe (1998), a study of the determinants of school quality in Ghana using a large scale data set, finds the only statistically significant school level effects on math and reading test scores to be the fraction of classrooms that have blackboards, and the fraction of classrooms that cannot be used when it rains. However, even in Ghana, 90% of schools have blackboards and only 15% of middle schools have leaking classrooms.

Table 4
Calculating relative cost effectiveness of teacher to non-teacher inputs while taking precision of estimation into account

<table>
<thead>
<tr>
<th>Input:</th>
<th>Subject:</th>
<th>Ratio of cost effectiveness of inputs to increasing class size:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>At point estimates of all variables</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Adding two std. dev. to class size point estimate</td>
</tr>
<tr>
<td>Furniture</td>
<td>Math</td>
<td>689</td>
</tr>
<tr>
<td></td>
<td>Filipino</td>
<td>1243</td>
</tr>
<tr>
<td>Workbooks</td>
<td>Math</td>
<td>842</td>
</tr>
<tr>
<td></td>
<td>Filipino</td>
<td>1592</td>
</tr>
</tbody>
</table>

Notes: In order to calculate the cost of decreasing class size, we use the fact that average teacher salary is 2.5 times GNP per capita (World Bank, 1996b) and assume that each teacher teaches an average of two classes, and that the average class size is 40.19 as reported in Tan, Lane, & Cousître (1997).
of input use and efficiency across incentive regimes. Greater local financing or control of schools, differing degrees of competition, or differing amounts of union power should not affect technical efficiency. But the evidence suggests that these factors do have an important effect both on the allocation of expenditures across inputs and on expenditure efficiency. The following section discusses three sources of evidence relating to various sources of differences in parental power: teacher unions, the degree of local control, and private schools.

3.3.1. Unions

Caroline Minter-Hoxby (1996) shows that, even after controlling for the endogeneity of union formation, school districts in the US that are unionized are less effective than the non-union school districts. In districts with unions, spending on schools is higher but a larger proportion of spending goes to teachers. This type of variation across states or provinces within a country is a promising line of research, but we are unaware of any appropriate data for developing countries.

3.3.2. Local and parental contributions

Consider the impact of direct parental participation and contributions. Modest amounts of direct contributions of parents should have large effects on performance only to the extent that some inputs (infrastructure, equipment, instructional materials) are so dramatically under-funded by the publicly allocated budget that their marginal products per dollar are very high. If marginal products were equalized (even according to an optimizing model with multiple outputs) then source of funding should be irrelevant and the relatively small parental contributions should only have a proportionately small effect. Evidence of substantial impact by parent’s groups, or of local control, or of the proportion of resources from local sources is itself a sign of a failure to maximize outputs in the overall system of expenditure allocation.

Jimenez, & Paqueo (1996) using data from the Philippines find that schools with less than 5% of local financing are incredibly less productive than schools with over 25% of local finance: their achievement score per peso is only a third as high, 0.06 versus 0.18 (Table 5). More importantly, the cost savings that produce this greater cost effectiveness come disproportionately from reducing expenditures on personnel whose cost per student is 52% lower, with much smaller cost savings for other inputs.

Similarly, James, King, & Suryadi (1996) find that in Indonesia a greater share of funds that are locally raised compared to centrally allocated increases significantly educational output (in their model, decreases cost for a given output) a finding that is true even controlling for whether schools are public or private.\(^\text{19}\)

Results on local financing are consistent with more centrally controlled resources being allocated by an optimizer with a high concern for teacher welfare (high \(\alpha\)) principally to teacher inputs, while locally raised funds being subject to a greater degree of parental control and hence high concern for educational output (low \(\alpha\)) and are allocated principally to non-wage inputs with high marginal product.

3.3.3. Competition and private schools

While the ability to control for selection effects is always problematic, the evidence from five developing countries marshaled by Jimenez, & Lockheed (1995) shows that, even after their attempts to control for selection effects, private schools are significantly more cost effective than public schools. This fact alone is inconsistent with the output optimizing models of the allocation of the public budget. However, it is possible that the improvement in efficiency is due to pure X-inefficiency of public schools, that is public and private schools do not differ in the allocation of inputs but simply that private schools are uniformly more productive for all inputs.\(^\text{20}\) An alternative explanation is that private schools, if they are more subject to competition for enrollments, are more directly controlled by parents.\(^\text{21}\)

By being more subject to parental pressure and less susceptible to teacher power the managers of private schools (or private school systems) will have a lower “\(\alpha\)” than public school system managers and these differences in parental control should produce both higher effectiveness of the budget overall and also different ratios of inputs. Importantly, since evidence from private schools is based on parental willingness to pay for schooling it does not depend on test scores as the outcome measure, as it incorporates parent’s concerns about

\(\text{19}\) Both of these papers use econometric estimation of the dual, that is, cost per student controlling for quality, rather than estimate a production function. However, their results on local financing indicate that the primal problem is not maximization of output subject to a budget constraint, but rather some more complicated objective function involving teacher utility of the type we explore here.

\(\text{20}\) However, Jimenez, & Lockheed (1995) does contain some evidence on the allocation as well. The differences in the fraction of private versus public schools paying teacher salaries above the median were public 56%, private 3.4% in Colombia and public 48%, private 12.9% in Tanzania.

\(\text{21}\) This should also be true of public schools where managers face more competition. Minter-Hoxby (1994) shows that the larger the number of school districts within a given metropolitan area the better the performance of the schools. She argues this is evidence that greater competition among public school districts substitutes for school choice and increases performance.
Table 5
Relative cost effectiveness and expenditures composition of locally versus non-locally financed schools

<table>
<thead>
<tr>
<th>Proportion of funds from local sources:</th>
<th>Percentage difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 5%</td>
<td>&gt; 25%</td>
</tr>
<tr>
<td>Overall score</td>
<td>42.46 48.25</td>
</tr>
<tr>
<td>Total expenditures</td>
<td>675 265</td>
</tr>
<tr>
<td>Achievement score per peso</td>
<td>0.06 0.18</td>
</tr>
<tr>
<td>Cost per student (Philippines pesos)</td>
<td>469 226</td>
</tr>
<tr>
<td>Personnel</td>
<td>42 35</td>
</tr>
<tr>
<td>Maintenance and operating</td>
<td>7 5</td>
</tr>
</tbody>
</table>

Source: Adapted from Jimenez, & Paequeo (1996), Table 1.

the all schooling outputs. If the observed difference in marginal products per dollar of inputs were the result of optimizing over multiple outputs and public and private schools were optimizing over the same set of outputs, then one should not observe systematic differences in the ratios of marginal product per dollar across various inputs between private and public schools.

Fortunately a recent study provides almost exactly this test. Alderman, Orazem, & Paterno (1996) examine how the characteristics of schools (like instructional expenditure per student and class sizes) affect parental decisions between private and public schools in Lahore, Pakistan, an urban area where 58% of students are enrolled in private schools even though private school fees are 5 times higher than public ones. They find, controlling for class sizes and instructional expenditures, that achievement scores were 60% higher in private schools.

Since school level instructional expenditure represents almost entirely teacher salaries we can compare how much parents are willing to pay to increase salaries versus decrease class sizes. As shown in Table 6, parental willingness to pay to increase the salaries of teachers in the private sector was 5.7 times larger than in the public sector. This suggests the marginal product of additional teacher wages in producing the entire bundle of parent valued outputs (not just scores) in the public sector was already quite low. Conversely, class sizes were so high in the public sector that parents were willing to pay substantial amounts to reduce them, and were substantially more willing to pay for class size reductions than wage increases. This was the exact opposite of the private sector.

4. Implications of a positive theory for educational expenditures

Educational output optimizing models of expenditure allocation are inconsistent with the evidence and a simple model in which teacher utility receives direct weight in expenditure choices is more consistent with the accumulated facts. Having a positive model of educational spending is important for the analysis of policy options, especially between the two types of educational policies frequently proposed: more of the same (higher enrollments, higher expenditures, better teacher training, etc.) and fundamental reform of incentives (decentralization, community involvement, school

Table 6
Parental willingness to pay for teacher salary increases versus class size reductions in private and public schools in Pakistan

<table>
<thead>
<tr>
<th>Private school</th>
<th>Public school</th>
<th>Ratio private to public</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenditures per teacher (rupees per month)</td>
<td>7171</td>
<td>8406</td>
</tr>
<tr>
<td>Parental willingness to pay to increase instructional expenditure per pupil (a proxy for teacher salaries of Rp 200)</td>
<td>40.4</td>
<td>7.05</td>
</tr>
<tr>
<td>Average class size</td>
<td>25.2</td>
<td>42.5</td>
</tr>
<tr>
<td>Parental willingness to pay to decrease class size by 10 students</td>
<td>− 35.7</td>
<td>7.10</td>
</tr>
<tr>
<td>Ratio of willingness to pay an additional rupee to teacher salaries versus class size reductions</td>
<td>4</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Notes: Adapted from Tables 2 and 5, using willingness to pay of parents with monthly household income of Rp. 3000 (Alderman, Orazem, & Paterno, 1996).
choice, etc.). Guidance on the choice between these requires modeling the underlying public sector decision and control mechanisms.

### 4.1. Budgets and outcomes

As mentioned above, one of the puzzles in the literature on educational production functions is that it is often difficult to demonstrate a positive impact of increased spending on educational outcomes. The literature is criticized precisely because the results seem so implausible to educators and educated alike: of course “money matters” and on some level everyone knows that. But the implication of our positive model is that while “money matters” to educational outcomes, money also matters to those who receive it as income and that this “mattering” has implications for publicly determined outcomes, like the allocation of expenditures.

If non-experimental data are taken from jurisdictions (or time periods within the same jurisdiction) that differ in either the degree of educational output concern \((1-\alpha)\) or in the degree of concern teachers have for output \((\delta)\) then equal budgets could produce very unequal outcomes due to the allocation across inputs. Any estimation of the relationship between expenditures and budgets using non-experimental data across units that differ in which do not control for these differences can produce any relationship at all: positive, zero, or negative. Fig. 2 shows the relationship between budgets and outcomes for the hypothetical jurisdictions A, B, and C. For any given level of \(\alpha\), lower budgets lead to worse outcomes (the sense in which money matters) as illustrated in the move from C to C’ to C”. But a higher \(\alpha\) implies worse outcomes for any given budget so that in the move from C” to C’ to C, the budget is constant but output falls as the inefficiency in allocation grows.

Given that both budgets and their allocation matter, what should be the focus of improving school performance? The answer that will be guaranteed vocal support is “increase budgets” as this generates support among parents, the general public who have an interest in promoting better education, and teachers (for both professional and self-interested reasons). However, if the existing allocation of inputs is not optimal, school performance can also be improved by a reallocation of expenditures without increases in the overall budget. These proposals will tend to split the above coalition and pit educators against all others. Which type of reform will be the most effective if implemented? The answer is, not surprisingly (and even reassuringly), it all depends. If the budget is high and concern for output is low \((1-\alpha)\) then fundamental reforms are more pressing. If on the other hand the budget is low and \(1-\alpha\) is high then budgetary reform should receive priority.

---

22 The two best arguments against an argument the expenditures never matter this are the meta-analysis of studies in the US (Hedges, Laine, & Greenwald, 1994) and the studies of the impact on subsequent wages of budgetary resources while in school (Card, & Krueger, 1996), also done exclusively in the US. Both of these are reasonably convincing that there is some, but small, connection between expenditures and outcomes. But neither of these methods speak to the question of either the efficiency or the allocation of the expenditure pattern chosen.

23 Differences in the relative weight of teachers also happen over time within a given jurisdiction as across jurisdictions. For instance, in Paraguay after the end of a long period of dictatorship teacher wages more than tripled in a six year period, with an almost complete lack of complementary reforms either in the school system or even in the structure of teacher pay. Given the total lack of objective comparable testing it is impossible to say for sure, but many doubt this increase led to commensurate increases in student performance.

24 For instance, Hanushek, & Kim (1996) use the internationally comparable test scores of achievement in math and science derived from the various IEA and IEAP studies and expenditures and find that both public expenditure per student and the ratio of total educational expenditure to GDP are significantly negatively related to achievement as illustrated in Fig. 2 with a line fitted to hypothetical outcomes A, B, and C.
4.2. Implications of the positive model for school reform

A positive model of educational expenditures will provide guidance to the actions or reforms that are likely to improve outcomes.

4.2.1. Ignorance or incentives?

Even though, as in all fields that involve human behavior, there are enormous areas of ignorance, the fundamental problem is typically not teacher or technocrat ignorance that could be resolved by more research. Our positive model implies several things about education research. First, since some types of research are inimical to furthering certain interests one should expect there to be relatively little research into cost effectiveness of various inputs. The principal agent model would predict a systematic tendency to resist evaluation. The amount of ignorance is endogenous, and sometimes it pays to be ignorant.26

Second, an implication of using the normative optimizing production function model as a positive model of behavior in an empirical study is that if educational policy makers are assumed to be attempting to optimize, then choosing too much teacher expenditure relative to spending on physical facility, equipment or books could only have been a “mistake” about the “true” production function. But then the study predicts its own impact: if believed the study should fix the “mistake” and cause a reallocation of resources towards the more productive resources. Although we acknowledge that it is always difficult to do so, we conjecture it would be hard to document a single instance in which an empirical study, or even a series of studies, actually affected the distribution of expenditures, unless the causal path of the effect was that the cumulative weight of the evidence was to change not just decisions, but the system of accountabilities and incentives.

The third, political, positive model is consistent with no response to new information as incentives, not ignorance, are the problem. The weight of international evidence of too little spending on some recurrent non-wage inputs has always been impressive, and has been accumulating over time. Fuller, & Clarke (1994) point out that the 37 new studies since the review of the evidence in Fuller (1986) mostly confirmed previous patterns, particularly the relatively greater importance of books and instructional materials than class size and teacher inputs. In contrast, while recent data are very difficult to come by, the share of educational expenditures devoted has, if anything, shifted towards an even greater share of teacher compensation (although this may be due in part to a fiscal contraction). Only well disseminated and widely understood studies that forment deep public dissatisfaction with the schools would be likely to change the incentives of the actors within the system and hence have any influence on outcomes. Merely documenting the differences, as has been done in many times and many places, will typically have no effect.

4.2.2. School choice with and without competition

Since teacher welfare increases directly with their “weight” in the decision maker’s objectives, teachers as a group will have reason to oppose reform efforts that reduce that “weight”. Changes that provide more school choice but do not create true competition in supply or change incentives on input use will be unlikely to change input expenditure outcomes significantly.

Exceptions to the widespread predominance of public schools perhaps prove the rule. In Argentina private schools are subsidized by direct government payments and private schools account for a large fraction of enrollment. However, these payments are structured such that they create no incentive at all for the private schools to economize on teaching inputs. First, the teacher’s union covers both private and public teachers and private schools hire only union teachers. Second, the government subsidy is not a voucher or a per student payment that can be used for any input, but rather a subsidy to pay teacher wages. This means that private schools compete with public schools for enrollment, but have little incentive to economize on the number of teachers and have no control over wages. A simple model of teacher power would predict that a teacher input subsidy to private schools is the political outcome in the situation where teachers are powerful and parents demand school heterogeneity (on “taste” grounds, such as ideological or religious) as this allows schools to compete in parental tastes without jeopardizing teacher’s welfare by competing in costs or input mix (James, 1987). This highlights the role of choice as complex, as “choice” is never a matter of all or none, but how much and with what limits so the Argentine case was choice of schools but not choice of input mix.
4.2.3. Decentralization/localization/school autonomy

Recognition of the incentive problems inherent in centrally controlled public provision of schooling has led to moves to increase local control over schools. These range from “decentralization” type reforms which shift the governmental authority with control over the provision of schooling to a lower level of government (e.g. federal to state or provincial, or provincial to municipal) to “localization” or “school autonomy” initiatives to move more control over schools from whatever level of government to the schools themselves.

The impact of jurisdictional “decentralization” depends partially on how it alters power between educators and parents. Merely moving from federal to state control seems unlikely to change much as moving to a quite local level seems to be necessary to bring about real shifts in relative strengths. Moreover, much depends on how, and how well, teachers are organized. If teachers are effectively organized at a geographic level higher than that of the school administration (e.g. national unions versus local school boards) some of the most crucial decisions (e.g. level and structure of teacher wages, work conditions, even class size norms) may be taken outside the scope of control of the local school authorities.

The impact of school “localization” or direct parental participation will depend to a large extent on the degree to which this shift fundamentally changes the balance between teacher and parent interests. Many “localization” reforms merely created parent groups which were given specific responsibilities (like mobilizing additional expenditures or maintaining buildings) but did not shift managerial control, particularly over monitoring teachers, teacher hiring and firing, teacher wages, etc. This is the type of “localization” most often observed, perhaps precisely because teachers naturally resist coming under the authority of parents, either in instruction, which is perhaps rightly thought to be their professional prerogative, or in wages and work conditions, where they would prefer to negotiate a centrally bargained solution. In this case one would expect the experience of decentralization to have mixed results, as it will have some, but likely modest, impacts on school performance. This might explain those cases in which the marginal product of teacher salary was greater than that of class size as the total wage bill to the chooser is more important than the wage.

There is another kind of reform, “school autonomy” is more about the autonomy of the individual school from a larger administrative or political unit (to confuse matters this is often also referred to as decentralization). Some versions even propose “teacher autonomy” implying a greater control of individual teachers over classroom practices and instructional method. Here it is obvious that “control” over different aspects of the overall educational process are being discussed. Suggesting that individuals should be able to choose their own health care provider to increase patient power and thereby discipline health markets does not imply a loss of professional autonomy of surgeons over what happens on the operating room. In fact, in an environment with greater consumer power, institutions which provide greater autonomy at the organizational (e.g. school, hospital) and professional (e.g. doctor, teacher) level may do even better than institutions with centralized controlling bureaucracies. Greater parental power might not increase or decrease teacher and school power, but rather change its dimensions. In our model greater teacher professionalism helps output. However, the mechanisms that foster greater teacher professionalism are precisely those that tend to create greater teacher power by making cooperation in lobbying easier. A common recommendation of education professionals is to get teachers to work together, to share experience, give support, etc. However, teachers are no different than any other group, and Adam Smith’s adage likely applies.

5. Conclusion

There are advantages to thinking through the implications of positive models of the allocation of expenditures. Both the “expected” results and interpretation of estimating an educational production function are determined by researchers’ (usually implicit) positive models. Any optimizing model has the implication that, at the input utilization observed, the marginal product per dollar of all inputs in increasing the value of output should be equalized. However, no study that we know has ever tested that hypothesis and did not reject it, moreover, nearly every study finds that the “cost effectiveness” of educational inputs differs by an order of magnitude or more. These results are too strong and the pattern of the relative cost-effectiveness among inputs too consistent to be happenstance. In addition, there is strong evidence that the incentive structure affects the composition of expenditures and technical efficiency.

Given the evidence, we think the correct model that describes the actual allocation of educational expenditures for most, if not all, developing countries must include the fact that educators have enormous influence over the allocation of spending and that spending is biased towards those educational inputs that also directly increase the welfare of teachers. Crudely put, teachers lobby (and form unions, and strike, and write) and books and desks do not. Parents generally have been insuf-

27 “People of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices.”
ficiently strong on behalf of books. This implies that perhaps the main role of the estimation of educational production functions is not to inform an idealized output maximizing decision maker of the “true” technical production function and “optimal” composition of inputs, but rather to provide the information necessary to encourage the deeper educational reforms that change the very structure of decision making power.

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Appendix A

The theory behind educational production function estimates

A plausible (at least to economists) theory, with variations

Variation 1: Simple output maximizing

If educational output is denoted by $S$ and is related to educational inputs (e.g. books, teachers, desks) denoted as elements $x_i$ of a vector $X$, each of which has a price $p_i$, according to a technically determined production function $f^s$, then the maximization problem subject to a fixed money budget $B$ is:

Maximize $S = f^s(X)$

s.t. $p'X = B$

Given this optimizing model, inputs should be allocated such that the marginal product per dollar (MPPD) is equalized across all used inputs. That is, the first order conditions are:

$$\frac{f^s_i}{p_i} - \frac{f^s_j}{p_j} = 0, \forall i, j$$  \hspace{1cm} (A.1)

which means that the increment to educational output per dollar should be exactly the same for each input.\(^{29}\)

Variation 2: Optimizing with uncertainty

Some might claim that the person allocating the inputs is trying to maximize the test score increment from spending a fixed budget but the “true” educational production function is a very complex thing about which the chooser knows very little. However, no matter how complex the true function, the situation can be formally characterized by assuming the person making the allocation decision believes that on average the true relationship is some production function, $g$. Under some restrictive but not unreasonable assumptions (either risk neutrality or restrictions on the loss function) the optimizing allocation will be such that:

$$\frac{g^s_i}{p_i} - \frac{g^s_j}{p_j} = 0, \forall i, j$$  \hspace{1cm} (A.2)

that is, the person deciding on the allocation believes he or she is equalizing the marginal products per dollar in the production function.

Variation 3: Optimizing with multiple outputs

A second objection is that many would allege that researchers are missing a large part of what schools are all about with their narrow minded obsession with test scores.\(^{30}\) In every society schools are meant to transmit more than information: they also transmit expected and acceptable patterns of social interaction and cultural norms and beliefs (Piccioto, 1996). Moreover, parents, children and society at large care about many more features of a school than just the improvement they provide to test scores: the pleasantness of the environment in general (e.g. personal safety), non-academic cultural opportunities (e.g. music), non-academic recreational opportunities (e.g. sports), equalization of outcomes across students with different innate abilities (e.g. remedial education spending).

In spite of the complexity, this situation can also be

\(^{29}\)This is of course the old (and by now obvious) condition for economic efficiency in any endeavor. As Frank Knight pointed out in 1921 “In the utilization of limited resources... we tend to apportion our resources among the alternative uses that are open in such a way that equal amounts of resource yield equivalent returns in all fields.” (Knight, 1921).

\(^{30}\)Since most researchers have been selected (self or otherwise) to be researchers at least to some extent on the basis on their performance on test scores during their academic career this obsession is perhaps understandable.
formally characterized as a variation on the optimizing model. Suppose that the objective function involves two outputs: test scores, \( S \), and some measure of non-academic output, \( C \) (\( C \) for “Citizenship” or “Culture” or “not being a Criminal”). In this case the optimizing problem is to maximize the total value of all school output subject to the production function for each of the outputs and the budget constraint (expanding this set-up to include more than two outputs or to allow a non-linear objective function would also be straightforward). That is, inputs will be allocated in order to maximize the value of:

\[
S(X) + p_i C(X)
\]

subject to the production functions for \( S \) and \( C \):

\[
S = f^S(X^S, X^C)
\]

\[
C = h^C(X^S, X^C)
\]

and the budget constraint.

The first order conditions for an interior maximum in this case imply that the marginal product of each input in the production of each output (“scores” and “citizenship”) will not be equalized. The way these vary will depend on both the relative value placed on the two outputs (\( p_i \)) as well as the relative marginal products and prices of the various inputs.

\[
\frac{f_i^S}{p_i} - \frac{f_j^S}{p_j} = p_i \left[ \frac{h_j^C}{p_j} - \frac{h_i^C}{p_i} \right], \forall i,j
\]  

(A.3)

This model implies the marginal product per dollar in producing scores at the optimum will be different if the marginal products of inputs in producing scores and “citizenship” is different. If one input is relatively more productive in the production of “citizenship” then its input use will be higher (and hence marginal product per dollar in scores production lower) than if one were optimizing over scores alone.

### A new positive model of expenditure allocations

Very few people would accept the optimizing model as a complete or accurate description and in fact the production function has had very little empirical success in explaining school performance. However, since the intuitive foundations are so clean and attractive the difficulty in rejecting the optimizing model is proposing a specific, concrete, alternative against which to judge it. Therefore, we propose a very simple alternative model, which embeds the optimizing models as a special case.

Suppose the person allocating inputs accepts a certain budget as fixed and maximizes a weighted average of the utility of school output \( S(X) \) and teacher’s utilities \( T(X) \) (expressed in units of test scores). The implied objective function is:

\[
C(X) = (1 - \alpha)*S(X) + \alpha*T(X)
\]

where \( \alpha \), the weight given to teacher utility, lies between 0 and 1. In this set-up the outcome depends not only on the underlying production function determined by the pedagogical process but also on the structure of the teacher’s utility function.\(^{31}\)

The simplest way to formalize this is to think of the teacher’s utility as the sum of two components: the utility derived directly from spending on educational inputs and that derived from greater educational output. The teacher utility derived directly from educational spending depends on how the spending on those inputs effects their own welfare independently of its effect on scores. For instance, higher teacher wages raise direct teacher utility dollar for dollar, while spending on physical amenities within the school (like larger classrooms) or spending on reducing workload (like smaller classes) would raise teacher welfare, but not by the equivalent of an unrestricted dollar given to the teacher, and spending on items like books or desks for students would have zero (or near zero) impact on direct utility. Teacher utility also depends, through altruism or professionalism, directly on educational output. The total teacher utility function is:

\[
T(X) = (1 - \delta)*U(\gamma'X) + (\delta)*S(X)
\]

where \( U(\cdot) \) is the direct utility derived from inputs, (\( \gamma \) is a vector which gives a weight to each of the inputs (weights which lie between 0 and 1). The individual \( \gamma_i \)'s for each input are the relative prices, in that they are the teacher’s willingness to pay for an additional dollar of spending on input \( i \) relative to their willingness to pay for a dollar of salary and hence are pure numbers. The “professionalism” parameter, \( \delta \), also lies between 0 and 1.

Now the optimization problem in allocating spending to raise this weighted average of teacher utility and test scores is:

Maximize \([(1 - \alpha) + \alpha*\delta]*f^S(X) + \alpha*(1 - \delta)*U(\gamma'X)

s.t. \( p'X = B \)

While this model is in many ways a modest generalization of the previous model, it has very different predictions about the allocation of expenditures. In particular, the ratios of the marginal educational output per dollar of input will not be equalized across inputs. Instead of

\(^{31}\) This is not referring to the direct effect teacher morale or satisfaction or compensation might have on output, as this is already embedded into the production function (so far have said nothing about the structure of the production function).
Table 7
Confirmation ratios of the significance of various inputs sorted by direct impact on educator welfare from 96 studies in developing countries

<table>
<thead>
<tr>
<th>Input</th>
<th>Number of studies</th>
<th>Positive (significant)</th>
<th>Negative (significant)</th>
<th>Insignificant</th>
<th>Confirmation Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher’s salary</td>
<td>13</td>
<td>4</td>
<td>2</td>
<td>7</td>
<td>30.8</td>
</tr>
<tr>
<td>School teacher-pupil ratio</td>
<td>30</td>
<td>8</td>
<td>8</td>
<td>14</td>
<td>26.7</td>
</tr>
<tr>
<td>Teacher’s education</td>
<td>63</td>
<td>35</td>
<td>2</td>
<td>26</td>
<td>55.6</td>
</tr>
<tr>
<td>Teacher’s experience</td>
<td>46</td>
<td>16</td>
<td>2</td>
<td>28</td>
<td>34.8</td>
</tr>
<tr>
<td>School facilities</td>
<td>34</td>
<td>22</td>
<td>3</td>
<td>9</td>
<td>64.7</td>
</tr>
</tbody>
</table>


Table 8
Confirmation ratios of the significance of various educational inputs sorted by their direct utility to educators, from studies in Latin America

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Number of studies</th>
<th>Positive relation</th>
<th>Negative relation</th>
<th>No relation</th>
<th>Confirmation Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher’s satisfaction</td>
<td>43</td>
<td>4</td>
<td>2</td>
<td>37</td>
<td>9.3</td>
</tr>
<tr>
<td>School student/teacher ratio</td>
<td>21</td>
<td>2</td>
<td>9</td>
<td>10</td>
<td>9.5</td>
</tr>
<tr>
<td>Teacher’s years of schooling</td>
<td>68</td>
<td>31</td>
<td>4</td>
<td>33</td>
<td>45.6</td>
</tr>
<tr>
<td>Teacher’s years of experience</td>
<td>62</td>
<td>25</td>
<td>2</td>
<td>35</td>
<td>40.3</td>
</tr>
<tr>
<td>Teacher’s subject knowledge</td>
<td>19</td>
<td>9</td>
<td>1</td>
<td>9</td>
<td>47.4</td>
</tr>
<tr>
<td>School infrastructure</td>
<td>70</td>
<td>23</td>
<td>2</td>
<td>45</td>
<td>32.9</td>
</tr>
<tr>
<td>Access to textbooks and other reading materials</td>
<td>17</td>
<td>13</td>
<td>0</td>
<td>4</td>
<td>76.5</td>
</tr>
<tr>
<td>School access to other instructional materials</td>
<td>34</td>
<td>14</td>
<td>3</td>
<td>17</td>
<td>41.2</td>
</tr>
</tbody>
</table>


implying the difference in marginal utility per dollar of inputs will be zero, as in the simple optimizing model, the first order conditions of this model imply:

$$\frac{f^s_i - f^s_j}{p_i - p_j} = \frac{\alpha(1 - \delta)}{(1 - \alpha) + \alpha \delta} * U^* \left[ \frac{\gamma_j - \gamma_i}{p_j - p_i} \right]$$  \hspace{1cm} (A.4)

That is, as long as the weight given to teacher utility is not exactly zero ($\alpha > 0$) and the degree of professionalism of teachers is not sufficiently high that they are completely selfless ($\delta < 1$) then marginal products per dollar will not be equalized. The more directly an input enters teacher utility (a higher $\gamma$) the lower the marginal product per dollar of that input in producing education will be, relative to another input at the optimum.

Appendix B

Two other reviews of the educational production function literature

Tables 7 and 8

References


