Dynamic energy-demand models:  
a comparison

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Abstract

This paper compares two second-generation dynamic energy demand models, a translog (TL) and a general Leontief (GL), in the study of price elasticities and factor substitutions of nine Swedish manufacturing industries: food, textiles, wood, paper, printing, chemicals, non-metallic minerals, base metals and machinery. Several model specifications are tested with likelihood ratio test. There is a disagreement on short-run adjustments: the TL model accepts putty–putty production technology of immediate adjustments, implying equal short- and long-run price elasticities of factors, while the GL model rejects immediate adjustments, giving out short-run elasticities quite different from the long-run. The two models also disagree in substitutability in many cases. © 2000 Elsevier Science B.V. All rights reserved.

JEL classifications: D21; D24; D29

Keywords: Energy demand model; Price elasticity; Factor substitution; Dynamic factor adjustment

1. Introduction

The purpose of this paper is to compare the empirical results from two energy-demand models, the translog (TL) and the general Leontief (GL), when applying to nine Swedish manufacturing industries.

The first generation of energy-demand models consists of traditional single-equation models that do not accommodate interactions among factor demands in...
Koyck partial-adjustment. The second generation, introduced by Nadiri and Rosen (1969) and Rosen and Nadiri (1974), allows for factor interactions with simultaneous equations by generalizing the first. The second-generation models thus concern interrelated disequilibriums in which a factor adjustment is proportional to the disequilibriums in all factors. The third-generation models, cost of adjustment (COA) models developed by Berndt et al. (1977), are often considered dominant. However, Walfridson (1987) suggested that the COA models misspecify the role of capital; Watkins (1991) pointed out that under certain assumptions the COA model collapses into the first-generation type, that is believed primary to the second-generation.

The models adopted in this study are second-generation type. One of the reasons to choose the second-generation models instead of the third is that they are easier to handle. Three adjustment mechanisms, the factor–input adjustments, cost share adjustments, and input/output ratio adjustments can be specified for the models. However, the second-generation models based on cost share adjustments have been criticized by Berndt et al. (1977) for not satisfying the basic principle that short-run own-price elasticities should be smaller in absolute value than the long-run own-price elasticities. Hogan (1989) also points out that cost share adjustment mechanism is misspecified at least in cases of rapid price changes for not meeting the slow adjustment requirement. Thus, the specification of cost share adjustments is not considered in this study. Instead, we assume both factor and input/output adjustments for the TL model and only input/output adjustments for the GL model. For simplicity, these models employ diagonal matrices of adjustments. Using such adjustments, Berndt et al. (1977) found that adjustment rate could be unreasonably low with unequal diagonal-elements, while Walfridson (1985, 1987) argued that equal diagonal-elements were tenable with input/output ratios, assuming that capital embodies the other factors. We will test the equal diagonal-element specification.

The configuration of this paper is as follows: Section 2 presents the data; Section 3 formulates the models; Section 4 tests model specifications; Section 5 reports the results of comparison and Section 6 summarizes and draws conclusions.

2. Data

The data sets, compiled from Statistics Sweden and National Accounts, contain one output, four inputs (electricity, fuels, labor and capital), and all the prices of nine Swedish manufacturing industries: food (including beverages and tobacco); textiles (including wearing apparel and leather); wood (including wood products, such as furniture); paper (including paper products); printing (including publishing); chemicals (including petroleum, coal, rubber and plastic products); non-metallic minerals (except for petroleum and coal); base metals and machinery (including machinery, other fabricated metal products and equipment). The sample period is from 1965 to 1989.
Electricity is measured in kilowatt hours (kwh); its price is calculated as the purchase cost divided by the quantity. Fuel is measured in kwh-equivalents, and its price is calculated similarly. Labor is measured in hours, and its price is calculated as the total annual compensation to labor divided by the total working hours in the year. The quantity of capital stock is measured as the net capital stock of buildings, construction and machinery, and its price is the user’s cost. All the prices are adjusted to 1980 levels.

On average, paper, non-metallic minerals, and base metals each consumed about twice the energy per unit of output as the chemical industry did, the latter using about twice the energy per unit of output as the other industries. Of all the industries, printing used the least energy. Two-thirds of energy input are fuels in non-metallic minerals but one-third in paper, printing, and chemicals. Inputs of fuels and labor per unit of output were generally decreasing in the period, whereas the inputs of electricity and capital were generally increasing; exceptions were the electricity/output ratios in chemicals and base metals. The data suggest a substitution of electricity and capital for fuels and labor; the substantial increases in capital in all the industries, even when output decreased sharply or consistently, also suggest a general substitution of capital for other factors.

In the next section, we formulate the energy-demand models.

3. Models

This section formulates TL and GL models. For each model, we have a number of specifications to test.

3.1. TL model

TL cost function is often used for empirical analyses; a few examples are Moroney and Trapani (1981), Gollop and Roberts (1983), Holly and Smith (1986), Vlachou and Samouilidis (1986), and Tsai and Norsworthy (1991) among others. We first assume that the production is represented by a TL cost function

\[
\ln C = \alpha_0 + \sum_i \alpha_i \ln p_{ij} + \alpha_j \ln y_i + \alpha_T T + \alpha_D \ln D_i \\
+ (\sum_i \beta_i \ln p_{ij} + \beta_j \ln y_i + \beta_{TT} TT + \beta_{DD} \ln D_i \ln D_i)/2 \\
+ \sum_j \gamma_{ij} \ln p_{ij} + \sum_j \gamma_{ij} T \ln p_{ij} + \sum_j \gamma_{ij} \ln D_i \ln p_{ij} + \mu_{ij} T \ln y_i \\
+ \mu_{ij} \ln p_{ij} \ln y_i + \mu_{ij} D_i \ln D_i + \mu_{ij} T \ln D_i
\]

(1)

in which, \( C, p_{ij}, y, T, \) and \( D_i \) are total costs, exogenous factor price, output, time-index to catch technical change, and degree days, respectively, while \( \alpha_0, \alpha_i, \alpha_j, \alpha_T, \alpha_D, \beta_i, \beta_j, \beta_{TT}, \beta_{DD}, \gamma_{ij}, \gamma_{ij} T, \gamma_{ij} D, \mu_{ij}, \mu_{ij} T, \mu_{ij} D, \) and \( \mu_{ij} D \) are parameters in which subscript \( t \) stands for time, while \( i = 1,2,3,4 \) and \( j = 1,2,3,4 \) stand for inputs: 1 = electricity, 2 = fuels, 3 = labor, and 4 = capital. Considering symmetry and
linear homogeneity of cost function in factor prices, we constrain the parameters so that $\beta_i = \beta_{ij}$, $\Sigma_{ij} \alpha_j = 1$, $\Sigma_i \beta_{ij} = \Sigma_i \beta_{ij} = 0$, $\Sigma_i \gamma_{ij} = 0$, $\Sigma_i \gamma_{ij} = 0$, and $\Sigma_i \gamma_{ij} = 0$. According to Shephard’s lemma, the $i$th cost share is

$$S_{it} = \frac{\partial \ln C_{it}}{\partial \ln p_{it}} = \alpha_i + \Sigma_j \beta_{ij} \ln p_{it} + \gamma_{ii} \ln y_t + \gamma_{ij} T + \gamma_{ij} \ln D_t$$

in which, the asterisk marks long-run. Here the adding-up condition $\Sigma_i S_{it}^* = 1$ holds. The long-run factor demand is $x_{it}^* = S_{it}^* C_{it}/p_{it}$ and the short-run factor demand is adjusted from the long-run’s

$$x_i = Bx_i + (I - B)x_{i-1}(\forall t)$$

in which, $t$ denotes time and $I$ is an identity matrix, while $B$ is an adjustment matrix for factor vector $x_i$ with $0 \leq |B| \leq 1$. For vintage or putty–clay production technology (see Forsund and Hjalmarsson, 1987), $B_i = 1 - (1 - \delta)/(1 + g)$, in which $\delta$ is the depreciation rate and $g$ the capacity growth. The null value of $B$ implies no adjustments, whereas an identity adjustment matrix corresponds to instantaneous adjustments.

The long-run input/output ratio is obtained as $a_{it}^* = x_{it}^*/y_t$ and the short-run adjustment is

$$a_i = \beta a_{it}^* + (1 - B)a_{i-1}$$

in which $a_i$ is a vector of input/output ratios. Eq. (2) or Eq. (3) together with Eq. (1) constitutes the two variations of basic TL dynamic model with either factor–input adjustments or input/output ratio adjustments.

We follow ad-hoc adjustments for simplicity and assign $B_{i*} = 0$. The long-run elasticity of factor $i$ with respect to price $j$ is

$$\varepsilon_{ij}^* = \frac{\partial \ln x_{it}^*}{\partial \ln p_{it}} = S_{it}^* + \beta_{ij}/S_{it}^*$$

and with respect to its own price is

$$\varepsilon_{ii}^* = \frac{\partial \ln x_{it}^*}{\partial \ln p_{it}} = S_{it}^* + \beta_{ii}/S_{it}^* - 1 = -\Sigma_{j*} \varepsilon_{ij}^*$$

(see Berndt and Wood (1975), Berndt et al. (1977), Fuss (1977), Siddayao et al. (1987), and Hogan (1989)). The short-run price elasticity is

$$\varepsilon_{ij} = \frac{\partial \ln x_{it}^*}{\partial \ln p_{it}} = (x_{it}^*/x_{it}) B_i \varepsilon_{ij}^*$$

in which $B_i = B_{ij}$, and Allen partial-elasticity of substitution between inputs $i$ and $j$ is calculated as $\sigma_{ij}^* = \varepsilon_{ij}^*/S_{it}^*$ (see Chamberg, 1988).

3.2. GL model

Following Walfridson (1992), we now assume that the short-run costs are jointly determined both by capacity and actual output. With the defined output ($y_t$),
inputs \((x_i)\) and prices \((p_{ij})\) above, we now formulate a GL model beginning with a short-run GL cost function

\[
C_i = y_i^{\omega}Q_i^{1-\omega}\sum_j \beta_i(p_{ij}/p_{ij})^{1/2}\exp(dT)
\]

in which, \(Q_i\) is a proxy for long-run output and is here defined as capacity; \(\omega\) is cost flexibility; \(\beta_i\) \((\beta_{ij} = \beta_{ji})\) is a parameter; \(d\) is an indicator of disembodied technical change and \(T\) is a time index. In the long-run, \(y_i = Q_i\), the production is constant returns to scale. In the formulation, \(y_i^{\omega}Q_i^{1-\omega}\) may be replaced by \(y_i U_i^{(\omega-1)}\), in which, \(U_i\) is capacity utilization. The inclusion of capacity utilization is meaningful since it affects the costs through factor hoarding and returns to scale. As capacity data are not directly available, we expect it to be given by the capital stock of the previous period divided by the short-run optimal capital input coefficient of the same period, as Walfridson (1992) did. Using Shephard’s lemma, short-run optimal factor demand is obtained as

\[
x_{it} = \frac{\partial C_i}{\partial p_{it}} = y_i^{\omega}Q_i^{1-\omega}a_{it}^\ast
\]

in which \(x_{it}\) is jointly determined both by \(y_i\) and \(Q_i\) (alternatively by \(U_i\)) \(a_{it}^\ast = \sum_j \beta_i(p_{ij}/p_{ij})^{1/2}\exp(dT)\) is long-run input coefficient of factor \(i\). We assume different elasticities \(\omega_i\) of factor \(i\) with respect to output:

\[
x_{it} = y_i^{\omega_i}Q_i^{1-\omega_i}a_{it}^\ast.
\]

Together with Eq. (4), it constitutes the basic GL model. Moreover, we can allow for non-neutral technical change by replacing \(d\) with \(d_i^\ast:\)

\[
a_{it}^\ast = \sum_j \beta_i(p_{ij}/p_{ij})^{1/2}\exp(d_i^\ast T).
\]

The specification of neutral technical change will be tested against non-neutral in Section 4. Since long-run output equals capacity, the long-run factor demand is obtained as

\[
x_{it}^\ast = Q_i^\ast a_{it}^\ast.
\]

The adjustment in input/output ratio is

\[
a_{it} = B_i a_{it} + (1 - B_i)a_{it-1}.
\]

The interpretation of \(B_i\) is the same as that for the TL model. The long-run elasticity of factor \(i\) with respect to price \(j\) is

\[
\varepsilon_{ij}^\ast = \frac{\partial \ln x_{it}^\ast}{\partial \ln p_{ij}} = \beta_{ij}(p_{ij}/p_{ij})^{1/2}/(2a_{it}^\ast)
\]

and with respect to its own price is

\[
\varepsilon_{ii}^\ast = -\sum_k \varepsilon_{kii}^\ast.
\]
The short-run cross-price elasticity of factor \(i\) with respect to price \(j\) is

\[ \varepsilon_{ijt} = \left( \frac{a^*_i}{a^*_j} \right) B_i \varepsilon_{ijt} \]

and the Allen partial elasticity of substitution between inputs \(i\) and \(j\) is

\[ \sigma_{ij} = \varepsilon_{ijt} / S_{ijt}^* . \]

The next section tests some specifications for the models.

4. Hypotheses and tests

In this section, we make and test some hypotheses of model specifications. For the TL models, we have six hypotheses to test:

**H1:** Equal adjustment rates with \( B_i = B_j = \text{const.} \)

**H2:** Vintage or putty–clay technology, \( B_i = 1 - (1 - \delta)/(1 + g) \).

**H3:** Instantaneous adjustments or \( B_i = 1 \).

**H4:** Hicks neutrality of technical change \( \gamma_{ij} = 0 \).

**H5:** No time effect or \( \alpha_t = 0, \beta_{it} = 0, \gamma_{ij} = 0, \mu_{jy} = 0 \) and \( \mu_{yt} = 0 \).

**H6:** Constant returns to scale CRTS with \( \alpha_y = 1, \beta_{yy} = 0, \gamma_{ij} = 0, \mu_{jy} = 0, \mu_{yt} = 0 \) and \( \mu_{y^2} = 0 \).

For the GL model, we will test the following seven hypotheses:

**H1:** Equal adjustment rate, \( B_i = B_j = \text{const.} \)

**H2:** Vintage or putty–clay technology, \( B_i = 1 - (1 - \delta)/(1 + g) \).

**H3:** Putty–putty technology or immediate adjustment, \( B_i = 1 \).

**H4:** Neutrality of technical change, \( d_i = d_j \).

**H5:** No technical change or time trend, \( d_i = 0 \).

**H6:** Constant returns to scale with \( \omega_t = 1 \).

**H7:** Quasi-fixed capital with \( \omega_{capital} = 0 \).

For each model, the alternative hypotheses are nested in the basic model and are tested by using the likelihood ratio (LR) test presented in Conrad and Unger (1987) and Berndt (1991). The LR test-statistic is \( n(\ln |V_R| - \ln |V_U|) \), that is asymptotically chi-square distributed with a number of degrees of freedom (d.f.) equal to the number of restrictions in the null hypotheses being tested, in which, \( V_R \) and \( V_U \) are the restricted and unrestricted estimators of the variance–covariance matrix, respectively, while \( n \) is the number of observations. An LR-statistic, for the restricted hypothesis, if greater than the critical value at the conventional significance level (SL) of 0.05 as shown in Table 1, implies a rejection of the restricted hypothesis (in favor of the unrestricted); otherwise, the value implies an acceptance of the restricted hypothesis.
Table 1
The critical values for chi-squared statistic

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<tr>
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<th>1</th>
<th>3</th>
<th>4</th>
<th>8</th>
<th>9</th>
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<tbody>
<tr>
<td>SL = 0.05</td>
<td>3.84</td>
<td>7.81</td>
<td>9.49</td>
<td>15.51</td>
<td>16.92</td>
</tr>
<tr>
<td>SL = 0.01</td>
<td>6.63</td>
<td>11.34</td>
<td>13.28</td>
<td>20.09</td>
<td>21.67</td>
</tr>
</tbody>
</table>

4.1. Tests of hypotheses for TL model

LR-statistics for the alternative hypotheses for the TL model are shown in Table 2 (the estimated parameters are not reported here to save space); the asterisked numbers are less than the relevant critical values given in Table 1, and therefore the corresponding hypotheses cannot be rejected at SL = 0.05. The hypotheses corresponding to the underlined and unmarked numbers are rejected at SL = 0.05 and SL = 0.01, respectively.

Both \( H_1 \) and \( H_3 \) cannot be rejected (at SL = 0.05) with factor-adjustments for all the industries, while they are in a few cases rejected (at SL = 0.05) with input/output adjustments. \( H_2, H_4, \) and \( H_5 \) are rejected (at SL = 0.01) in all the cases for input/output adjustments and most cases for factor adjustments. \( H_6 \) is rejected in five cases (at SL = 0.01) for the factor adjustment specification and in six cases for input/output adjustments.

According to the tests above, we consider \( H_1 \) and \( H_3 \) as candidate hypotheses for TL model with both factor-adjustment and input/output-adjustment except in

Table 2
The LR-statistics of TL specifications against the basic model

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</tr>
</thead>
<tbody>
<tr>
<td>( H_1 )</td>
<td>3</td>
<td>2.31*</td>
<td>4.05*</td>
<td>117.25</td>
<td>2.88*</td>
<td>98.18*</td>
<td>0.00*</td>
<td>15.74*</td>
<td>5.80*</td>
</tr>
<tr>
<td>( H_2 )</td>
<td>4</td>
<td>163.32</td>
<td>187.35</td>
<td>83.57</td>
<td>77.71</td>
<td>118.12</td>
<td>126.82</td>
<td>134.13</td>
<td>94.97*</td>
</tr>
<tr>
<td>( H_3 )</td>
<td>4</td>
<td>3.82*</td>
<td>5.56*</td>
<td>121.22*</td>
<td>1.20*</td>
<td>105.54*</td>
<td>9.99*</td>
<td>25.08*</td>
<td>14.43*</td>
</tr>
<tr>
<td>( H_4 )</td>
<td>4</td>
<td>61.26</td>
<td>89.62</td>
<td>43.07</td>
<td>52.26</td>
<td>55.61</td>
<td>41.90</td>
<td>2.66*</td>
<td>64.48</td>
</tr>
<tr>
<td>( H_5 )</td>
<td>8</td>
<td>56.78</td>
<td>118.23</td>
<td>57.22</td>
<td>46.62</td>
<td>55.06</td>
<td>42.82</td>
<td>7.31*</td>
<td>61.38</td>
</tr>
<tr>
<td>( H_6 )</td>
<td>8</td>
<td>0.93*</td>
<td>45.12</td>
<td>61.46*</td>
<td>105.48</td>
<td>86.40*</td>
<td>58.19</td>
<td>27.60</td>
<td>53.14</td>
</tr>
</tbody>
</table>

\( \text{Factor adjustments:} \)

\( \text{Input / output adjustments:} \)

\( * \)Notes. The asterisked values cannot be rejected at the 5% level of significance; values underlined are rejected at the 5% levels of significance; the others are rejected at the 1% level of significance; `-' means that the model is not convergent and d.f. is the degrees of freedom.
the case of machinery industry, for which, we accept $H_6$. We thus have to test $H_3$ against $H_1$ (with d.f. = 1) for all the industries except machinery with input/output adjustments. For the tests, we control the overall conventional SL (5%) and uniformly assigned a significance level of 0.0125 to each d.f. for the testing of $H_3$ against $H_1$ obtaining an interpolated critical value (chi-squared) of 6.33 (d.f. = 1, SL = 0.0125). The statistics for testing of $H_3$ against $H_1$ are shown in Table 3; all the statistics are less than the critical value of 6.33 (d.f. = 1) at the assigned significance level of 0.0125, thus we cannot reject $H_3$ but accept it. Because a large number of coefficients of auto-correlation in the input/output-adjustments are larger than unity, we discard this adjustment mechanism in favor of factor-adjustments.

### 4.2. Tests of alternative GL model hypotheses

For the GL model, we test all the seven hypotheses listed above against the basic GL model at the conventional SL = 0.05. The LR-statistics for the alternative hypotheses of restricted GL models are shown in Table 4 (again the estimated parameters are not reported to save space), in which, a rejection is either underlined (rejected at SL = 0.05) or unmarked (rejected at SL = 0.01), while an acceptance is asterisked. According to this table, no hypotheses are accepted in all the cases; only $H_3$ and $H_6$ are rejected in all the cases, while the other hypotheses may be acceptable in some cases.

### Table 3
The LR test of $H_3$ against $H_1$ for TL model (d.f. = 1)

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<tbody>
<tr>
<td>Factor</td>
<td>−1.51</td>
<td>1.51</td>
<td>−3.96</td>
<td>−1.68</td>
<td>−7.36</td>
<td>−0.99</td>
<td>−4.34</td>
<td>1.31</td>
<td>−1.82</td>
</tr>
<tr>
<td>Input/output</td>
<td>2.57</td>
<td>4.81</td>
<td>4.70</td>
<td>2.97</td>
<td>−36.43</td>
<td>3.51</td>
<td>−15.70</td>
<td>5.20</td>
<td>3.24</td>
</tr>
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### Table 4
Statistics for LR test of GL specifications against the basic model

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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>3</td>
<td>0.09*</td>
<td>10.48</td>
<td>13.39</td>
<td>21.98</td>
<td>11.10</td>
<td>24.91</td>
<td>−2.98*</td>
<td>18.21</td>
<td>14.58</td>
</tr>
<tr>
<td>$H_2$</td>
<td>4</td>
<td>0.10*</td>
<td>9.82</td>
<td>18.57</td>
<td>28.72</td>
<td>25.33</td>
<td>23.34</td>
<td>−3.50*</td>
<td>22.06</td>
<td>42.94</td>
</tr>
<tr>
<td>$H_3$</td>
<td>4</td>
<td>49.04</td>
<td>99.92</td>
<td>63.20</td>
<td>94.40</td>
<td>29.50</td>
<td>92.96</td>
<td>15.62</td>
<td>37.52</td>
<td>48.62</td>
</tr>
<tr>
<td>$H_4$</td>
<td>3</td>
<td>−0.32*</td>
<td>21.96</td>
<td>17.03</td>
<td>11.98</td>
<td>1.69*</td>
<td>23.86</td>
<td>11.66</td>
<td>15.36</td>
<td>34.57</td>
</tr>
<tr>
<td>$H_5$</td>
<td>4</td>
<td>−0.24*</td>
<td>29.71</td>
<td>24.44</td>
<td>21.46</td>
<td>2.00*</td>
<td>36.27</td>
<td>18.07</td>
<td>26.83</td>
<td>38.91</td>
</tr>
<tr>
<td>$H_6$</td>
<td>4</td>
<td>26.32</td>
<td>119.88</td>
<td>75.42</td>
<td>138.74</td>
<td>50.96</td>
<td>94.02</td>
<td>49.68</td>
<td>146.72</td>
<td>135.38</td>
</tr>
<tr>
<td>$H_7$</td>
<td>1</td>
<td>5.14</td>
<td>20.05</td>
<td>1.85*</td>
<td>−3.36*</td>
<td>14.72</td>
<td>11.14</td>
<td>9.15</td>
<td>6.39</td>
<td>32.56</td>
</tr>
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</table>

*Note. The asterisked values cannot be rejected at the 5% level of significance; values underlined are rejected at the 5% levels of significance; the others are rejected at the 1% level of significance and d.f. is the degrees of freedom.
For textiles, chemicals, base metals, and machinery industries, the tests reject all the null hypotheses against the basic model.

For wood and paper industries, $H_7$ is uniquely accepted. For the remainder of the industries, further tests were made as follows.

For the food industry, $H_7$ is nested in $H_1$ and $H_5$ in $H_4$, we might use the LR-test to test $H_2$ against $H_1$ and $H_5$ against $H_4$; because the former two and the latter two hypotheses are not nested in each other, we might use the $J$-test technique Davidson and MacKinnon, 1981 for testing them. However, we found that each of the non-rejected specifications and the basic model produced at least one negative elasticity of factor-input concerning output. Among the remainders, only $H_7$ has produced positive elasticities of factors with respect to output; therefore, we use $H_7$ for further analyses.

In the case of non-metallic minerals, $H_1$ and $H_2$ cannot be rejected. We further tested the latter against the former and obtained the LR-statistics of 0.62 which are smaller than the critical values of 6.33 at the significance level of 5%. In this test, we also control the overall significance level at 5%.

For the printing industry, $H_4$ and $H_5$ cannot be rejected against the basic model as shown in Table 4, $H_5$ is nested in $H_4$, and therefore we can test the former against the latter with an LR-test. We control the overall significance level 5% in the same way. As the LR-statistic of 0.31 is smaller than the critical values of 6.33 at the significance level of 5% we cannot reject $H_5$ but accept it.

By the tests, we find that the two models disagree on production technology, since the TL accepts the assumption of putty–putty production technology, while the GL rejects such an hypothesis.

Table 5
Price elasticities of factors from the TL model

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*Notes. 1, electricity; 2, fuels; 3, labor; 4, capital.
5. Price elasticities of factors and factor substitutions

This section presents the price elasticities of factors and factor substitutabilities from the TL and GL models. As the two models disagree in production technology, we expect different results of price elasticities and factor substitutabilities from them.

5.1. Price elasticities of factors from the TL model

From the TL model we obtain the same short- and long-run elasticities of inputs with respect to prices for all the industries (see Table 5), as the model accepts instantaneous adjustments of factors. This unexpected result could either indicate poor data or poor model in capturing a short-run production.

There is one aberration of own-price elasticity; of 36 own-price elasticities, all are negative as expected except elasticity $e_{i1}$ for the paper industry.

5.2. Price elasticities of factors from the GL model

In contrast to the TL model, the GL model has produced quite different short- and long-run elasticities of factors with respect to prices; the absolute values of short-run own-price elasticities of factors obtained with the GL model are relatively smaller than long-runs in all the cases of negative values (see Table 6) as expected. Besides, in most of the cases, the absolute values of the short-run cross-price elasticities of factors are also quite smaller than the long-run counterparts; this is a major difference in the comparison of the GL and TL models. These results indicate that the GL model is better suited to describing short-run production than the TL model, at least for the industries under study.

However, one own-price elasticity (both short- and long-run) for electricity, five for labor and two for capital are positive. Such results are not what we expected. From this point of view, the GL model is not better for estimating own-price elasticities of factors than the TL model.

5.3. Allen elasticities of substitution

Table 7 shows the substitution ($S$) or complement ($C$) between factors according to the Allen elasticities of substitution calculated from the two models. The results from the TL model appear first, with GL results after slashes. Looking first at TL results, electricity is a substitute for fuels in wood, printing, chemicals and non-metallic minerals, but a complement of fuels in the remaining industries. Electricity is a substitute for labor in food, wood, printing, non-metallic minerals and base metals, but a complement of labor in the other industries. There is substitution between electricity and capital in all industries except wood, implying a general increase in electricity demand in the manufacturing sector as a whole when capital price goes up. Fuels and labor substitute for each other in all the industries other than wood and machinery, while fuel and capital substitute for each other in food,
Table 6
Price elasticities of factors from the GL model

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*Notes: 1, electricity; 2, fuels; 3, labor; 4, capital; *, long-run.

We now look at the GL-model results (after slashes in Table 7). Electricity and fuels substitute each other in food, paper, printing, chemicals, and non-metallic minerals; electricity and labor substitute only in printing and chemicals; electricity and capital substitute in all the industries except printing; labor and fuels substitute in all the industries except chemicals; fuels and capital substitute in only food and chemical; labor and capital substitute in textiles, printing, chemicals, and non-metallic minerals.

Comparing the results from the two models (Table 7), we find that the models agree in only half of the cases. Both models show substitution between electricity, textiles, wood, non-metallic minerals, and machinery. Labor and capital substitute for each other in all industries except base metals.
Table 7
The substitutions (S) or complements (C) by TL/GL models*

|-------------|------|----------|------|-------|--------|------|----------|---------|-------|

*Note. A result from the TL model comes first and a result from the GL model comes after the slash.

and capital and between fuel and labor, or between electricity and fuel, in most of the industries. However, they fail to agree in the substitutabilities between electricity and capital in wood and printing, and between fuel and labor in wood, chemicals, and machinery, or between electricity and fuel in food, wood, and paper. For electricity–labor, fuel–capital, and labor–capital these models have produced opposite results in at least four cases. For labor and capital, the TL model suggests substitutions in all the cases except base metals, whereas the GL model shows substitutions in only textiles, printing, chemicals, and non-metallic minerals. For textiles, printing, and base metals, the two models match in substitutabilities in almost all the cases, while for wood, they generate opposite results.

6. Conclusions

The TL and GL models disagree in production technology, since the former accepts putty–putty production technology, while the latter rejects it.
There is no uniform substitution or complement between any two factors except electricity and capital. The models agree in only half of the cases. The GL model shows complement between electricity and labor, fuels and capital, and labor and capital in most of the industries, whereas the TL model shows substitution between labor and capital in most of the industries, and no obvious relationship between electricity and labor or between fuels and capital. However, both models agree that electricity and capital, or fuels and labor, can substitute for each other in most of the industries.
Both models have produced some unreasonable positive own-price factor elasticities: the GL model produced eight and the TL only one.

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References


