Pareto-optimal electricity tariff rates in the Republic of Armenia

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Abstract

The economic impact of electricity tariff rates on the residential sector of Yerevan, Armenia, is examined. The effect of tariff design on revenue generation and equity measures is considered, and the combination of energy pricing and compensatory social policies which provides the best mix of efficiency and protection for poor households is examined. An equity measure is defined in terms of a cumulative distribution function which describes the percent of the population that spends \(x\%\) or less of their income on electricity consumption. An optimal (Pareto-efficient) tariff is designed based on the analysis of survey data and an econometric model, and the Armenian tariff rate effective 1 January 1997 to 15 September 1997 is shown to be non-optimal relative to this rate. © 2000 Elsevier Science B.V. All rights reserved.

JEL classifications: D11; L94

Keywords: Econometric methods; Efficient frontier; Pareto-optimal tariffs; Tariff design strategies

1. Introduction

In order to cushion social problems and provide basic standards for subsistence, many governments of Eastern Europe and the Republics of the Former Soviet Union subsidize public services, essential commodities, construction, and urban transportation. As governments reduce subsidies for essential commodities such as...
bread prices and utilities, however, the problem of protecting vulnerable people is aggravated.

The purpose of this paper is to analyze the impact of electricity tariff rate structures on a cross-section of household groups in Yerevan, Armenia. Who in Armenia would be hurt, and by how much, if electricity were to rise to efficient prices? How would a change in the tariff rate affect the distribution of consumption expenditures of the population? As tariff rates change and it costs more money to purchase the same amount of electricity, how will the increased cost effect the consumer? What combination of energy pricing and compensatory social policies provides the best mix of efficiency and protection for poor households? And how should the tariff structure change as international pressure from lending agencies such as the EBRD, IMF, World Bank, USAid, etc., encourage tariff rates that parallel market prices and reflect the cost of production? Generally speaking, these are difficult questions to answer that involve the confluence of financial, historical, political, social, and technical issues. In this paper, therefore, the scope of analysis is more narrowly defined to assess the simultaneous effect of tariff rate structures on revenue generation and induced equity measures.

The ability and readiness of consumers to pay for energy is a central problem for the government of the Republic of Armenia. Although the price of electricity has been liberalized, it is still below the cost of production, and as the price of electricity increases (Table 1), there is a two-fold effect on the population:

1. It has been difficult to collect the fees from consumers, and during the early years of transition (1993–1996) there was much illegal activity and ‘left lines’ were abundant. As system security and control improved, however, much of this activity has been reduced, and there is evidence to suggest consumers are (reluctantly) adjusting to the new rates. Estimates indicate that 60–80% of the population pay their electricity bill (White, 1997).

Table 1
Residential electricity tariff rates in Armenia

<table>
<thead>
<tr>
<th>Time period</th>
<th>Electricity price (ADR/kWh)</th>
<th>Energy cost (ADR/MJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2/93–12/93</td>
<td>8</td>
<td>2.22</td>
</tr>
<tr>
<td>1/94–12/94</td>
<td>8.5</td>
<td>2.36</td>
</tr>
<tr>
<td>1/95–9/95</td>
<td>10</td>
<td>2.78</td>
</tr>
<tr>
<td>10/95–3/96</td>
<td>12</td>
<td>3.33</td>
</tr>
<tr>
<td>4/96–12/96</td>
<td>14</td>
<td>3.89</td>
</tr>
<tr>
<td>1/97–9/97</td>
<td>13, $E_i \leq 100$</td>
<td>4.72</td>
</tr>
<tr>
<td></td>
<td>20, $E_i &gt; 100$</td>
<td></td>
</tr>
<tr>
<td>9/97–12/98</td>
<td>15, $E_i \leq 100$</td>
<td>5.74</td>
</tr>
<tr>
<td></td>
<td>22, 100 \leq E_i &lt; 250</td>
<td></td>
</tr>
<tr>
<td></td>
<td>25, E_i &gt; 250</td>
<td></td>
</tr>
<tr>
<td>1/99</td>
<td>25</td>
<td>6.94</td>
</tr>
</tbody>
</table>
2. More importantly, the cost of electricity prices place an uncertain and unknown burden on the socio-economically disadvantaged portion of the population. It is not generally known what impact, if any, the increased rates have had on the population.

Ideally, reforms would accelerate price increases in conjunction with compensatory payments to the poor (Gray, 1995) but this may not be feasible in Armenia especially when social infrastructures are still in their infancy (Human Development Report, 1995). A tariff can be designed to distribute the burden of payment in a controlled manner, but to understand the impact of various structures a model of the tariff rates must be developed in conjunction with empirical data describing the income structure and consumption patterns of the population in review.

A number of special characteristics and problems of transition economies complicate the picture. The main factors to consider include:

1. income decline and poverty;
2. fixed income of population subgroups;
3. non-marginal price increase of electricity; and
4. socialist legacy.

The impact of these factors depends primarily upon the structural characteristics of the country, and the case of Poland for example has been examined in Meyers et al. (1994) and Freund and Wallich (1997). A large fraction of the population of transition economies are on fixed incomes and pensions which are vulnerable to inflation and other cost of living increases (Milanovic, 1995). In Armenia, incomes have fallen considerably since 1989 and poverty has increased (Human Development Report, 1995; Bakhshian et al., 1997). It is thus difficult, from either a policy or political perspective, to impose the full economic impact of ‘market’ conditions on the population; i.e. shock therapy. Electricity prices in Armenia (as well as prices of other public-sector goods) remain well below market values (although substantially higher than in Soviet times), and the price increases to attain efficiency are non-marginal in nature; e.g. the average level of Armenian electricity tariff in 1997 (3.63 US cents/kWh) is less than half the average financial cost (8.7 US cents/kWh). Consumers thus face a tariff-level that is too low to encourage efficient use of electricity, and the power sector is unable to raise sufficient funds from revenues to finance maintenance, overhaul, and expansion work. This is a common problem in transition economies (Bacon, 1995; Gray, 1995). Finally, because of the socialist legacy and subsidized prices of goods and services over two generations, many people have the feeling of entitlement to goods and are not willing (or, at least, are quite reluctant) to pay for such services. The socialist legacy is especially strong among the older generation. The combination of these factors, increased cost of living, as well as social pressures has led to interesting migration patterns within the Former Soviet Union of both the nationalist and economic variety (Heleniak, 1997).
1.1. Electricity tariff structure

The Armenian energy sector does not have an official tariff schedule, nor are there well defined terms and conditions for electrical service. Tariff changes are normally announced through government press releases and are met with much anxiety on the part of the population. Wholesale power tariffs are based on calculations which proceed in a ‘backwards’ fashion, starting with revenues from retail tariffs and subtracting unit operating and maintenance costs for each company in order to obtain its power purchase rate from Armenergo (Kasprowicz, 1998). In regulated energy environments this is a common and standard procedure and has historically dominated the US electric and natural gas industry. The natural gas industry was federally regulated until the 1980s (Tippee and Tussing, 1995) and electricity generation is currently enjoying a deregulated market in a few states. The rate design process in transition economies, however, is not only dependent on the level of costs to be recovered and market pressures, but more importantly, political and social pressures which exhibit their own volatility.

There are many factors that influence tariffs (Wilson, 1993), and in particular, electric tariffs are dependent upon variables such as the cost of service; source of supply and types of generation (hydroelectric, thermal, nuclear); type of customers (residential, industrial, commercial) and customer mix; age of the sector system; rate of growth of the sector system; sources of funds and methods of financing; and price elasticity (Tariff Structure and Design, 1997). The Energy Regulatory Commission (ERC) of the Republic of Armenia specifically states that the tariff rates are set to ‘provide compensation of expenditures for safe and efficient exploitation, maintenance, and accumulated provisions of depreciation of fixed assets, capital repair, investments, as well as debt servicing expenditures of the energy system’ (Energy Commission of the Republic of Armenia, 1997).

The retail tariffs in Armenia for ‘residential’ electricity consumption has been in flux for the past few years (Table 1). The present analysis is based on the tariff rate effective 1 January 1997 to 1 September 1997, which is a two-tier system based on a cut-off energy usage of 100 kWh/month:

\[
\tau = \begin{cases} 
13 \text{ (ADR/kWh)} & \text{if } E_i \leq 100 \text{ (kWh)} \\
20 \text{ (ADR/kWh)} & \text{if } E_i > 100 \text{ (kWh)} 
\end{cases}
\]

In other words, for the first 100 kWh electricity consumed, the price is $13 \text{ ADR/kWh}$, while for the balance of consumption (in excess of 100 kWh) the price is $20 \text{ ADR/kWh}$. The tariff rate is a form of ‘lifeline’ pricing and serves to

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1Residential demand is defined by the ERC to include the 400 V service group which includes both residential and non-residential consumers such as schools, offices, hospitals, and governmental buildings.

2At the time of the study, the Armenian dram currency (ADR) exchange rate was 500 ADR = $1. The exchange rate has remained fairly stable since 1997.
subsidize energy prices for the poor. Lifeline pricing involves charging a low, subsidized price for a fixed energy quota and a higher price for consumption above that level. Survey data indicate that a disproportionate burden of costs already falls on the most economically disadvantaged portion of the population (Kaiser), and thus it makes sense to incorporate a tier-tariff system to lessen the burden on those consumers unable to pay. In lifeline pricing all consumers of course receive the subsidy (regardless of need), and consumers who use less than the minimum quota have little or no incentive to conserve which in economic terminology is said to represent an inefficient use of resources. On the other hand, consumers who use more than the base rate are also provided with a quantity of electricity at subsidized rates, and so the benefit thus accrues to all consumers regardless of income.

1.2. Minimal energy requirements

Minimal energy requirements per month for an ‘average’ Armenian household is approximately 300 kWh electricity usage. This amounts to a cost ADR 5300 (US $11), which represents 55% of the average salary (US $20/month) and 110% of the average pension (US $10/month) (Human Development Report, 1995). In 1996 special groups of residential customers (e.g. the poor, elderly, pensioners, and energy sector state employees) received a 50% cost reduction on their energy bill, but many of these exemptions have now been phased out. Generally speaking, families cannot be expected to spend more than a quarter of their total income on energy expenditures, and if they do, then an undue hardship will fall on that segment of the population.

The percentage of income spent on energy usage can also be viewed from a public health perspective (Armenian Monthly Public Health Report, 1996). A ‘market basket’ is the monthly cost of food and services, including apartment rent, electricity and other utilities, and public transport for a family of four. In February 1996, the market basket was ADR 62864 (US $156), with food expenses accounting for 83% of the market basket and non-food expenses comprising the remaining 17% (ADR 10687 or US $26). With the privatization of the housing market, apartment rent (including maintenance fees) is essentially zero (Anlian and Vanian, 1996), while the price of public transportation (if used) can easily amount to ADR 8000/month. The estimates of electricity expenditure from both a nominal usage and public health perspective are thus comparable in a broad sense.

1.3. Cost of energy sources

From basic engineering calculations, the amount of energy (heat) that fuel types

3As a comparison, in the West energy expenditures typically range from 3 to 7% total income. In Armenia the cost of energy can be considered prohibitive since among the working class it is already common practice in households to unplug refrigerators during the winter, install one-out-of-four bulbs in light fixtures, be clothed in outdoor winter apparel inside households during the fall–winter season, etc.
Table 2
Energy of fuel consumption

<table>
<thead>
<tr>
<th>Fuel</th>
<th>Energy after combustion (kCal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas (Propane, Butane)</td>
<td>11 500</td>
</tr>
<tr>
<td>Kerosene</td>
<td>11 000</td>
</tr>
<tr>
<td>Coal</td>
<td>10 000</td>
</tr>
<tr>
<td>Wood</td>
<td>4100</td>
</tr>
</tbody>
</table>

yield after combustion is readily computed. The energy of fuel consumption is shown in Table 2. The units of kWh are used as the base reference. One kCal is approximately 4.19 kJ, and a kWh is equal to 3600 kJ. The price and cost comparison based on 1997 data are given in Table 3. Electricity is certainly the most convenient energy source to use, and it is also (locally) one of the ‘cleanest’ and safest sources since it does not involve generation of the noxious fumes of other sources; e.g. ‘stinkpots’ which use ‘skunk oil’ (kerosene). The cost of electricity, however, is also one of the most expensive energy types as is commonly known.

1.4. Problem definition

The purpose of this paper is to assess the simultaneous effect of tariff rate structures on the revenue generation capacity of the government and induced equity parameters on the population. In particular, we wish to develop a methodological procedure to study the economic impact of energy policy on the residential consumer.

1. How should tariff rates be adjusted to minimize the hardship on the low-income portion of the population?
2. What effect will the tariff structure have on the revenue generation of the government and the distribution of the percentage of income the population spends on electricity usage?

Table 3
A comparison of energy costs in Armenia (1997)

<table>
<thead>
<tr>
<th>Energy source</th>
<th>Energy (kJ)</th>
<th>Average price (ADR)</th>
<th>Energy cost (ADR/MJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electricity (1 kWh)</td>
<td>3600</td>
<td>17</td>
<td>4.72</td>
</tr>
<tr>
<td>Gas (1 kg)</td>
<td>48 200</td>
<td>325</td>
<td>6.74</td>
</tr>
<tr>
<td>Kerosene (1 kg)</td>
<td>46 100</td>
<td>135</td>
<td>2.93</td>
</tr>
<tr>
<td>Coal (1 kg)</td>
<td>41 900</td>
<td>40</td>
<td>0.95</td>
</tr>
<tr>
<td>Wood (1 kg)</td>
<td>17 200</td>
<td>20</td>
<td>1.16</td>
</tr>
</tbody>
</table>
To understand the extent of the hardship and to provide a first-order analysis of this problem, a model which incorporates revenue and equity factors is developed. The main task of this paper is to design an ‘optimal’ tariff that maximizes the revenue generation of the government while simultaneously maximizing the ‘equity’ of the designed tariff. This will lead us to precisely define the notion of an equity measure.

2. Survey methodology

A survey covering 250 households from Yerevan, Armenia, was obtained in personal interviews during July 1997 and provides the database for this study. One major difficulty facing research on expenditure patterns is the availability of reliable data. The personal interviews were conducted by students, and although time-consuming, provided for a small but accurate data set. Since the students tended to interview friends, relatives, and neighbors, however, the sampling can not be considered ‘random.’ The trade-off between collecting accurate data (from a biased sample set) or deleting inaccurate data (from unwilling participants) is believed to be evenly balanced in this study. The collected data included:

- household income, size, and education;
- type of energy carriers used by the household; and
- consumption patterns of energy carriers.

There are essentially two seasons in Armenia, summer and winter, and although monthly data collected over an extended duration, aggregated, and averaged would provide an ideal statistical sample, for the purpose of this study data for 2 months was considered sufficient. Data for the months of July (summer) and January (winter) were collected. For a complete analysis of the data, including Engel curve computations and energy carrier patterns, the reader is referred to Kaiser. In the present work, only the data necessary to develop the tariff model is discussed.

2.1. Percentage of household income spent on electricity

Histograms that represent the percentage of household income spent on electricity is depicted for the summer and winter season in Figs. 1 and 2. The distributions are Lognormally distributed with parameters \(8.8, 6.7\) and \(10.7, 7.3\), respectively, i.e. an average of 8.8 and 10% of income is spent on electricity consumption. The discrepancy between these values and the estimated ‘average’ described in Section 1.2 can be attributed to a general decrease in electricity consumption, under-reported income, or more likely, a combination of these factors. Note that other energy sources are available to the consumer (Table 3), and in Armenia, the cross-price elasticity of substitute fuels play a significant role in the mix of fuel types used by the population. The variability of the data is relatively high, and the absolute values of usage may seem quite comparable to Western standards (as
Fig. 1. Histogram representing the percentage of household income spent on electricity during the summer season.
Fig. 2. Histogram representing the percentage of household income spent on electricity during the winter season.
Fig. 3. Histogram representing the average consumption pattern of electricity.
discussed earlier, where expenditures typically range from 3 to 7% total income, but the situation is really of a completely different nature due to the wide disparity in average incomes, the distribution type and large standard deviation. Since the distributions are Lognormally distributed with a large standard deviation, the area to the right of the average of these curves represent households who use a large percentage of their income on electricity use.

2.2. Electricity consumption histogram

The electricity consumption histogram describes the quantity of electricity (in kWh) consumed and serves as an estimate for the population density function. Survey data describes the amount of money spent on electricity consumption (under the two-tier tariff rate in effect during data collection), and thus the data is processed (in a reverse manner using the economic data coupled with the tariff structure) to determine the quantity consumption pattern as shown in Fig. 3. The average amount of energy consumed during the summer and winter season is 246 kWh with a standard deviation of 200 kWh.

3. The design of optimal tariff rates

The problem in tariff planning is to design rates which are simultaneously efficient, feasible, and equitable for consumers while covering the cost of production for the government (in regulated environments) or the mark to market valuation (in deregulated environments). To determine the effect of tariff rate structures on equity measures of the population and revenue streams for the government, a model of tariff rates is developed.

The primary input data to this model includes:

1. electricity consumption density function;
2. tariff rate structures; and
3. model assumptions.

The electricity consumption density function was determined from the sample survey data and is a primary input parameter. The tariff rate structures and model assumptions are now discussed. A determination of tariff rates that are Pareto-efficient is the main output of the model.

3.1. Tariff rate structures

The tariff rate structures considered in this paper include the flat tariff rate

\[ \text{Utilizing data from the state concern Armenergo, it is possible in principle to avoid sampling altogether and obtain the 'exact' consumption density function. Unfortunately, there remain some difficulties in collecting this information due to the 'proprietary' nature of the data.} \]
\[ \tau = \tau_1 (\text{ADR}/\text{kWh}), \]

where regardless of the amount of electricity consumed (kWh), a charge of \( \tau_1 \) (\text{ADR}/\text{kWh}) per unit kWh is imposed. A flat tariff rate is the simplest and most common structure and provides a benchmark for comparison with non-linear designs. Flat tariff rates are functions of one parameter, \( \tau_1 \).

A two-tier tariff rate is defined by:

\[
\tau = \begin{cases} 
\tau_1 \text{ (ADR}/\text{kWh)} & \text{if } E_i \leq E_0 \text{ (kWh)} \\
\tau_2 \text{ (ADR}/\text{kWh)} & \text{if } E_i > E_0 \text{ (kWh)} 
\end{cases}
\]

where for the first \( E_0 \) (kWh) electricity consumed, the price is \( \tau_1 \) (\text{ADR}/\text{kWh}), and for the balance of the monthly consumption \( (E_i - E_0) \), the price is \( \tau_2 \) (\text{ADR}/\text{kWh}). The two-tier tariff is a function of three parameters \( (\tau_1, \tau_2, E_0) \) and thus allows for a considerable degree of flexibility (i.e. optimization) in the parameter selection. The selection of the parameters \( (\tau_1, \tau_2, E_0) \) determine the impact of the tariff on the consumer, and the ‘optimization’ is dependent upon the characteristics of income and energy consumption and the relationships between these two factors. The design of an ‘optimal’ tariff is possible due of the degree-of-freedom inherent in the selection of \( (\tau_1, \tau_2, E_0) \).

### 3.2. Model assumptions

The tariff model is constrained by four primary assumptions.

1. The electricity consumption density function \( f(x) \) describes the distribution of the quantity of electricity consumed by the population (Fig. 3). The density function is normally estimated through sampling data and is seasonally dependent with the absolute amount of electricity usage (typically) dependent on a number of socio-economic factors such as the size and income level of the household, and environmental parameters such as average temperature, heating degree days, etc. Sample data is used to estimate the population density function and the data represents paying customers from given districts of Yerevan; the resulting consumption function constructed histogram may thus be slightly biased since non-paying customers who receive electricity illegally (through ‘left’ lines) and those customers who do not pay for the electricity they receive (approximately 20–40% of the population) are not included in the analysis.\(^5\) In addition, the data set in this paper represents a relatively small sample of urban consumption patterns, and it is difficult to extrapolate the current estimates to rural settings. Electricity consumption patterns are seasonal and can fluctuate considerably, and so monthly averages of the density function should be computed and then aggregated into seasonal categories and used in the evaluation of the tariff measure. For the purpose of this study,

\(^5\)This bias is not considered a major source of error.
Table 4
Sample data and computed parameters for the tariff $\tau$

<table>
<thead>
<tr>
<th>$E_i$ (kWh)</th>
<th>$I_i$ (ADR)</th>
<th>$C_i$ (ADR)</th>
<th>$\gamma_i = C_i/I_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>$I_1$</td>
<td>$C_1$</td>
<td>$\gamma_1$</td>
</tr>
<tr>
<td>$E_2$</td>
<td>$I_2$</td>
<td>$C_2$</td>
<td>$\gamma_2$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$E_n$</td>
<td>$I_n$</td>
<td>$C_n$</td>
<td>$\gamma_n$</td>
</tr>
</tbody>
</table>

however, and considering the viability of such an endeavor, data were collected for a summer and winter month in 1997. Since the temperate climate in Armenia comes essentially in two types (summer and winter), this is considered to be an acceptable first-order approximation. Although the density function will change shape over time, its structure is assumed static for this analysis.

2. Revenue and equity measures are computed based on the estimated density function $f$ and sample data concerning the income and energy expenses of individual households. These data take the form as shown in Table 4. The (monthly) demand for electric energy by household $i$ is denoted by $E_i$ and is measured in kilowatt-hours (kWh), and total (monthly) household income $I_i$ is reported in Armenian drams (ADR). The parameters computed from the sample data include the cost $C_i$ of the electricity (ADR), and the percentage of income household $i$ spends on electricity, $\gamma_i = C_i/I_i$.

3. From the government’s perspective it is assumed that:
   - more revenue is preferred to less revenue; and
   - greater equity is preferred to lower equity.
From the consumer’s perspective it is assumed that:
   - lower cost is preferred to higher cost; and
   - greater equity is preferred to lower equity.
Revenue and equity represent conflicting goals which require a trade-off in the design process. It is clearly not possible to increase government revenue while simultaneously decreasing household expenses on electricity, however, with the availability of the two-tier tariff it is possible to design a tariff that meets certain a priori conditions within the constraints imposed on the system.

4. The analysis is performed in the short run (1–3 months), meaning that consumers do not self-select (Räsänen et al., 1997) or change their consumption pattern in response to the tariff rate changes. This is a strong assumption but it can be justified to some extent due to the inelastic nature of the demand–income relationship common in electricity demand studies (Pindyck, 1979; Dahl, 1994). In the long run (> 3 months), consumer behavior tends to be more elastic, but precisely how much more is not known due to the lack of available data and particular structural characteristics of the utility environment in Armenia. In the absence of realistic data, therefore, the inelastic assumption is not only convenient but appropriate. Parametric analysis can be incorporated within the model to account for elasticity values.
3.3. Model statistics

The demand for electric energy $E_i$ and total income $I_i$ for household $i$ is evaluated over the period of length $k$ (which is selected as 1 month). The tariff structure $\tau$ determines the cost of the energy consumed by household $i$ and is denoted by $C_i$ (ADR). The ratio $C_i / I_i = \gamma_i$ represents the percentage of the household's total income that is spent on electricity during the time period. Household $i$ is thus characterized by the vector $(E_i, I_i, C_i, \gamma_i)$:

$$\text{Household } i \leftrightarrow (E_i, I_i, C_i, \gamma_i)$$

Sampling from the population $\mathcal{P}$ of consumers yields the data

$$(E_i, I_i), \quad i = 1, \ldots, n,$$

which under the given tariff $\tau$ determines $C_i$ and $\gamma_i = C_i / I_i$:

$$\mathcal{P} = \{(E_i, I_i, C_i, \gamma_i), \quad i = 1, \ldots, n\},$$

as shown in Table 4. The terms $\Sigma E_i$, $\Sigma I_i$, $\Sigma C_i$ represent the total energy consumption (kWh), total income (ADR), and total revenue (ADR), respectively, for the sample $\mathcal{P}$ generated by the tariff $\tau$. Histograms are computed directly from the data set $\mathcal{P}$ and are used to infer the corresponding continuous (population) density functions for the random variables $E$, $I$, $C$, and $\gamma$.

The value of electricity consumption $E$, income $I$, cost of consumption $C$, and percentage of income spent on electricity $\gamma = C / I$, are continuous random variables and are assumed to follow the density functions $f$, $g$, $j$, and $h$, respectively. The cumulative distribution functions are denoted by $F$, $G$, $J$, $H$, and are defined, for example, as

$$F(x) = \int_0^x f(y) dy.$$  

In this case the lower value of the integrand is zero since electricity consumption is non-negative in value. The statistical characteristics of distributions are described through their moments, namely, the mean $(E(X))$, standard deviation $(\sigma(X))$, skewness $(sk(X))$, and kurtosis $(ku(X))$. The moment vector $(E(X), \sigma(X), sk(X), ku(X))$ is thus used to characterize and distinguish distribution types under variable tariff structure.

3.4. Revenue and equity measures

The two most important measures used to assess the impact of tariff rates are the revenue and equity functionals. For a given tariff rate $\tau$ and energy consumption density function $f$, revenue $R(\tau, f)$ and equity $E(\tau, f)$ are real-valued functionals dependent upon $\tau$ and $f$:  

$$\text{Revenue } R(\tau, f) = \int f(x) dx \quad \text{and} \quad \text{Equity } E(\tau, f) = \int g(x) dx.$$
The total revenue generated from tariff $\tau$ is easy to compute when based on the sample set $S$:

$$R(\tau) = \sum C_i.$$

Revenue is easy to understand, and from the government’s perspective revenue induces a preference structure among tariff rates such that tariff $\tau_1$ is preferred to tariff $\tau_2$ ($\tau_1 \geq \tau_2$) if it generates more revenue ($R(\tau_1) > R(\tau_2)$).

There are a number of measures of equity for income distribution, e.g. the Gini coefficient and Theil index are popular measures (Sadolet and deJanvry, 1995; Trade Development Report, 1997), but to assess the impact of a tariff on a population more general measures need to be employed. Consumer and producer surplus provide a measure that is commonly employed in microeconomic studies (Wilson, 1993; Bacon, 1995; Sadolet and deJanvry, 1995; Freund and Wallich, 1997), while in this paper a more direct modeling procedure is developed. To examine the effect of tariff rates on revenue generation and the ‘fairness’ of those rates, one choice to quantify equity is to examine the distribution of the random variable $\gamma$. The random variable $\gamma = C / I$ describes the percentage of a household’s income spent on electricity consumption and is described by the density function $h_{\gamma}(x)$ and cumulative distribution function:

$$H_{\gamma}(x) = \int_0^x h_{\gamma}(y)dy.$$

The average value of $\gamma$, $\overline{\gamma}$, is a potential equity measure. Given tariff $\tau_1$ and $\tau_2$, if the average value of a household’s income spent on electricity under $\tau_1$ is smaller as compared to $\tau_2$, then $\tau$ is preferred to tariff $\tau_2$; i.e. if:

$$E(\tau) = \overline{\gamma} = \int_0^\infty yh_{\gamma}(y)dy$$

and $E(\tau_1) < E(\tau_2)$, then $\tau_1 \geq \tau_2$.

A more precise means to quantify equity is to examine the distributional effects on the percentage of income households spend on electricity after imposition of the tariff. For example, if the percentage of the population that spends less than, say 10%, of their income on electricity increases as the tariff rate is changed from $\tau_1$ to $\tau_2$, then the tariff rate change can be considered a desirable one since more people now spend less than 10% of their income on energy costs.

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6 If the number of households in the sample is $n$, and the number of households in the population is estimated to be $N$, then the total revenue generated by the tariff can be estimated as $(N/n)R(\tau)$. 
The cumulative distribution function $H_x(x)$ describes the percent of the population that spends $x\%$ or less of their income on electricity consumption. The equity measure $E_x(\tau,f)$ is defined in terms of this quantity:

$$E_x(\tau,f) = H_x(x) = \int_0^x h_x(y) dy,$$

and is a function of the evaluation parameter $x$. This is a more accurate and comprehensive assessment of equity as compared to the moment method. Tariff $\tau_1$ is thus preferred to tariff $\tau_2$ and is said to be more equitable if $E_x(\tau_1,f) > E_x(\tau_2,f)$; i.e. $H_{\tau_1}(0.1) > H_{\tau_2}(0.1)$.

4. The flat-rate tariff

A flat tariff rate is a one-parameter tariff that provides a benchmark for comparison with more complicated tariff designs. A flat tariff rate is the simplest possible tariff and is still the most widely used throughout the world, especially for residential rates. A flat tariff is given by

$$\tau = \tau_1(ADR/kWh),$$

or in other words, a flat rate of $\tau_1$ is imposed for any quantity of electricity consumed. This neither encourages nor discourages consumption. A consumer who uses $E_i$ kWh electricity per month is charged $\tau_1 E_i$ (ADR). The total revenue $R(\tau,f)$ generated by the tariff is thus given by:

$$R(\tau,f) = \sum \tau_1 E_i = \tau_1 \sum E_i = \tau_1 E_T,$$

where $E_T$ represents the total electricity consumption of the sample $S$. It is clear that under a flat-rate tariff, total revenue does not depend explicitly on the shape of the distribution curve, but only on the total amount of consumption in the sample. A plot of revenue vs. quantity electricity consumed for a typical residential consumer is a linear function that passes through the origin with slope $\tau_1$.

A number of conclusions result from the application of the flat-rate tariff. The first two results hold in general and follow from the revenue expression.

1. $R(\tau,f)$ is an increasing linear function of $\tau$ and is independent of the form of $f$.
2. The average value $E(\gamma)$ is an increasing linear function of $\tau$.

For a given sample $S$, $E_T$ is fixed and revenue is a linear function of $\tau$, and so

---

7 The choice of the value $x = 10\%$ is arbitrary but appeared to be a natural cut-off when examining the output data of the analysis (although in principle any value can be used). In fact, due to the complicated nature of the functional and its dependence on the underlying distribution $h_x(x)$, $H_x(x)$ may induce ambiguous preferences as the cut-off value $x$ changes.
Table 5
Flat tariff rate and related parameters

<table>
<thead>
<tr>
<th>Tariff $\tau$ (ADR/kWh)</th>
<th>$\mathcal{R}(\tau; f)$ (ADR)</th>
<th>$E(\gamma)$</th>
<th>$H_0(0.10)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>596,957</td>
<td>5.4</td>
<td>91.7</td>
</tr>
<tr>
<td>12</td>
<td>756,349</td>
<td>6.5</td>
<td>87.2</td>
</tr>
<tr>
<td>14</td>
<td>855,741</td>
<td>7.6</td>
<td>80.5</td>
</tr>
<tr>
<td>16</td>
<td>955,132</td>
<td>8.7</td>
<td>70.3</td>
</tr>
<tr>
<td>18</td>
<td>1,074,524</td>
<td>9.7</td>
<td>64.2</td>
</tr>
<tr>
<td>20</td>
<td>1,193,915</td>
<td>10.8</td>
<td>58.1</td>
</tr>
<tr>
<td>22</td>
<td>1,313,307</td>
<td>11.9</td>
<td>50.6</td>
</tr>
<tr>
<td>24</td>
<td>1,432,693</td>
<td>13.0</td>
<td>40.7</td>
</tr>
</tbody>
</table>

as the magnitude of the flat-rate increases (decreases), the revenue generated will increase (decrease) in proportion to the tariff. In the short-run, therefore, and assuming no change in the consumer response, total revenues will vary linearly with the tariff change. In the long-run, however, as individuals adjust their consumption patterns in accord with the new tariff, the revenue change is dependent on the price elasticity of demand of the population. In the short-run, the percentage of a household’s income spent on electricity will rise (fall) with a tariff increase (decrease). This can be seen as follows. The random variable $\gamma = C / I$ is described by the density function $h$, and in the discrete case, the expected value of $\gamma$ is given by:

$$
\gamma = \sum_{i=1}^{n} \frac{C_i}{I_i} = \frac{\sum_{i=1}^{n}(E_i\tau_i/I_i)}{n} = \frac{\tau_1\sum_{i=1}^{n}(E_i/I_i)}{n},
$$

a linear function of $\tau$ since $E_i$ and $I_i$ are fixed in the sample data.

The next two conclusions are empirical results based on the specific form of the collected data (refer to Table 5):

3. The percentage of the population that spends 10% or less of their total income on electricity consumption ($H_0(0.1)$) is a decreasing (approximately) linear function of $\tau$.

4. Total revenue $\mathcal{R}(\tau; f)$ and $H_0(0.1)$ are (approximately) negatively correlated.

As total revenue increases, the equity $\mathcal{E}(\tau; f) = \mathcal{E}_x(\tau; f)$ as measured by $H_0(0.10)$, decreases. From Fig. 4 it is clear that the equity measure is an approximately linear decreasing function of $\tau$. Since revenue $\mathcal{R}(\tau; f)$ and $\tau$ are proportional, this is equivalent to stating that total revenue is (approximately) a linear decreasing function of $H_0(0.10)$ as shown in Fig. 5. A plot of revenue $\mathcal{R}(\tau; f)$ vs. equity $\mathcal{E}(\tau; f)$ represents an ‘efficient frontier’; i.e. it is not possible to increase revenue without a subsequent decrease in equity, and vice-versa. Note that the shape of the efficient frontier depends on the specific value of $x$ used in $H_0(x)$. 


Fig. 4. Graph of the equity measure $\mathcal{E}(\tau,f)$ as a function of the tariff rate $\tau$ for a flat-rate tariff.
Fig. 5. Graph of the total revenue $R(\tau,f)$ vs. the equity measure $E(\tau,f)$. 
5. The two-tier tariff

A general model for the two-tier tariff rate is given as:

\[
\tau = \begin{cases} 
\tau_1 (\text{ADR/kWh}) & \text{if } E_i \leq E_0 (\text{kWh}) \\
\tau_2 (\text{ADR/kWh}) & \text{if } E_i > E_0 (\text{kWh}) 
\end{cases}
\]

For the first \( E_0 \) kWh electricity consumed per month, the price is \( \tau_1 (\text{ADR/kWh}) \), and for the balance of the monthly consumption \((E_i - E_0)\), the price is \( \tau_2 (\text{ADR/kWh}) \). This is a form of lifeline pricing which involves charging a low, subsidized price for a fixed energy quota and a higher price for consumption above that level. A plot of revenue vs. quantity electricity consumed is a piecewise linear function that changes slope from \( \tau_1 \) to \( \tau_2 \) at the value given by \( E_0 \). The tariff \( \tau \) can be represented by the parameters \( \tau_1, \tau_2 \) and \( E_0 \) written in vector form as \( \tau = (\tau_1, \tau_2, E_0) \), and so the two-tier rate structure is a three-parameter tariff design; e.g. in Armenia from January 1997 to September 1997, \( \tau = (13, 20, 100) \).

Similar to the flat-rate analysis, a class of tariff rates dependent upon the parameters \( \tau_1, \tau_2 \) and \( E_0 \) can be compared, and an optimal tariff in terms of revenue and equity can be determined. The major distinction between the flat-rate and the two-tier rate is that the tariff is no longer a function of a single variable; tariff design is a function of three parameters and by design gives more flexibility in terms of distributional impact.

5.1. Parametric analysis

An approximate range for the tariff parameters was a priori assigned and based on the nominal settings, \( \tau_1 = 13, \tau_2 = 20, E_0 = 100 \):

\( \tau_1 = \{10, 13, 16\} \)

\( \tau_2 = \{17, 20, 23\} \)

\( E_0 = \{75, 100, 125\} \)

Associated with each of the \( 3^3 = 27 \) possible tariff combinations \((10, 17, 75), (10, 17, 100), (10, 17, 125), (13, 17, 75), \ldots \) is a value for the revenue \( R(\tau_j) \) and equity \( E(\tau_j) \) based on the collected sample data. For each tariff rate combination \( \tau \), the revenue \( R(\tau_j) \), moment vector \( E(\gamma) \), skewness \( sk(\gamma) \), and equity measure \( H(\gamma) \) is computed directly from the sample data and is summarized in Table 6. An example illustrates a typical computation.

The population sample provides energy consumption and income data \((E_i, I_i)\), and for a given tariff, \( \tau = (\tau_1, \tau_2, E_0) \) is used to compute the cost of consumption \( C_i \), and \( \gamma_i = C_i/I_i \).

The values of \( C \) and \( \gamma \) are functions of \( \tau \), and the density function of \( \gamma \), \( h_i \), is used to determine the equity measure:
Table 6
Two-tier tariff rates and related parameters

<table>
<thead>
<tr>
<th>Tariff $\tau$</th>
<th>$R(\tau, f)$ (ADR)</th>
<th>Moment vector $(E(\gamma), \sigma(\gamma), sk(\gamma), ku(\gamma))$</th>
<th>Histogram $k_\gamma$</th>
<th>$H_t(0.10)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(10, 17, 75)$</td>
<td>911302</td>
<td>$(7.2, 4.9, 2.1, 5.9)$</td>
<td>30</td>
<td>58</td>
</tr>
<tr>
<td>$(10, 17, 100)$</td>
<td>878825</td>
<td>$(6.8, 4.5, 2.2, 7.2)$</td>
<td>33</td>
<td>55</td>
</tr>
<tr>
<td>$(10, 17, 125)$</td>
<td>660202</td>
<td>$(4.7, 3.4, 3.4, 21.5)$</td>
<td>28</td>
<td>54</td>
</tr>
<tr>
<td>$(10, 20, 75)$</td>
<td>1046022</td>
<td>$(8.0, 5.4, 2.0, 4.9)$</td>
<td>28</td>
<td>58</td>
</tr>
<tr>
<td>$(10, 20, 100)$</td>
<td>999625</td>
<td>$(7.4, 4.9, 2.1, 5.2)$</td>
<td>32</td>
<td>57</td>
</tr>
<tr>
<td>$(10, 20, 125)$</td>
<td>956240</td>
<td>$(7.0, 4.5, 2.1, 6.9)$</td>
<td>24</td>
<td>52</td>
</tr>
<tr>
<td>$(10, 23, 75)$</td>
<td>1180741</td>
<td>$(8.7, 5.9, 1.9, 4.2)$</td>
<td>23</td>
<td>57</td>
</tr>
<tr>
<td>$(10, 23, 100)$</td>
<td>1120425</td>
<td>$(8.0, 5.2, 2.0, 5.0)$</td>
<td>27</td>
<td>58</td>
</tr>
<tr>
<td>$(10, 23, 125)$</td>
<td>1064024</td>
<td>$(7.5, 4.7, 2.0, 5.9)$</td>
<td>27</td>
<td>51</td>
</tr>
<tr>
<td>$(13, 17, 75)$</td>
<td>955671</td>
<td>$(8.1, 5.5, 2.4, 8.8)$</td>
<td>27</td>
<td>51</td>
</tr>
<tr>
<td>$(13, 17, 100)$</td>
<td>937112</td>
<td>$(7.8, 5.3, 2.5, 9.8)$</td>
<td>26</td>
<td>51</td>
</tr>
<tr>
<td>$(13, 17, 125)$</td>
<td>91758</td>
<td>$(7.7, 5.2, 2.6, 10.7)$</td>
<td>21</td>
<td>50</td>
</tr>
<tr>
<td>$(13, 20, 75)$</td>
<td>1090390</td>
<td>$(8.6, 6.0, 2.2, 6.9)$</td>
<td>21</td>
<td>50</td>
</tr>
<tr>
<td>$(13, 20, 100)$</td>
<td>1057912</td>
<td>$(8.5, 5.6, 2.3, 8.1)$</td>
<td>26</td>
<td>52</td>
</tr>
<tr>
<td>$(13, 20, 125)$</td>
<td>1027542</td>
<td>$(8.1, 5.4, 2.4, 5.2)$</td>
<td>26</td>
<td>43</td>
</tr>
<tr>
<td>$(13, 23, 75)$</td>
<td>1225109</td>
<td>$(9.6, 6.5, 2.1, 5.6)$</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>$(13, 23, 100)$</td>
<td>1178712</td>
<td>$(9.1, 6.2, 2.2, 6.7)$</td>
<td>20</td>
<td>51</td>
</tr>
<tr>
<td>$(13, 23, 125)$</td>
<td>1135327</td>
<td>$(8.6, 5.6, 2.3, 7.0)$</td>
<td>24</td>
<td>50</td>
</tr>
<tr>
<td>$(16, 17, 75)$</td>
<td>1000039</td>
<td>$(8.9, 6.3, 2.7, 11.8)$</td>
<td>22</td>
<td>50</td>
</tr>
<tr>
<td>$(16, 17, 100)$</td>
<td>995399</td>
<td>$(8.9, 6.2, 2.7, 12.2)$</td>
<td>22</td>
<td>50</td>
</tr>
<tr>
<td>$(16, 17, 125)$</td>
<td>991060</td>
<td>$(8.8, 6.2, 2.7, 12.4)$</td>
<td>18</td>
<td>44</td>
</tr>
<tr>
<td>$(16, 20, 75)$</td>
<td>1134758</td>
<td>$(9.7, 6.7, 2.5, 9.4)$</td>
<td>19</td>
<td>48</td>
</tr>
<tr>
<td>$(16, 20, 100)$</td>
<td>1116199</td>
<td>$(9.5, 6.5, 2.5, 10.3)$</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>$(16, 20, 125)$</td>
<td>1098845</td>
<td>$(9.3, 6.3, 2.6, 11.1)$</td>
<td>15</td>
<td>41</td>
</tr>
<tr>
<td>$(16, 23, 75)$</td>
<td>1269477</td>
<td>$(10.5, 7.1, 2.3, 7.6)$</td>
<td>19</td>
<td>44</td>
</tr>
<tr>
<td>$(16, 23, 100)$</td>
<td>1236999</td>
<td>$(10.1, 6.8, 2.4, 8.8)$</td>
<td>18</td>
<td>45</td>
</tr>
<tr>
<td>$(16, 23, 125)$</td>
<td>1206630</td>
<td>$(9.8, 6.5, 2.5, 9.8)$</td>
<td>18</td>
<td>45</td>
</tr>
</tbody>
</table>
Fig. 6. The histogram $h_{(10,17,75)}(x)$ for the tariff rate $\tau = (10, 17, 75)$. 
Fig. 7. The histogram $h_{\tau}(x)$ for the tariff rate $\tau = (10, 17, 100)$. 
Fig. 8. The histogram $h_{(10,17,125)}(x)$ for the tariff rate $\tau = (10, 17, 125)$. 
\( \mathcal{R}(\tau, f) = H_i(0.1) = \int_{0}^{0.1} h_i(y) \, dy \).

For the tariff \( \tau = (10, 17, 75) \), the revenue is calculated from the consumption sample data and yields \( \mathcal{R}(\tau, f) = 911302 \) ADR and distribution \( h_i \) as shown in Fig. 6. The histogram subsequently yields the moment vector:

\( (E(\gamma), \sigma(\gamma), sk(\gamma), ku(\gamma)) = (7.2, 4.9, 2.1, 5.9), \)

and \( H_i(0.1) = 0.32 + 0.55 = 0.87 \). For the tariff \( \tau = (10, 17, 100) \), the revenue generated is \( \mathcal{R}(\tau, f) = 878825 \) ADR, and the distribution \( h_i \) is shown in Fig 7. The moment vector of the distribution yields

\( (6.8, 4.5, 2.2, 7.2), \)

and \( H_i(0.1) = 0.34 + 0.55 = 0.89 \). Observe that as revenue declines equity increases, while the average amount of income spent on electricity consumption decreases. The value of \( \gamma \) changes in response to the change \( \Delta E_0 = 100 - 75 = 25 \) kW, and the amount of change depends upon the form of \( f \) and the magnitude of \( \Delta E_0 \). The distribution \( h_i \) for the tariff \( \tau = (10, 17, 125) \) is depicted in Fig. 8.

5.2. Model analysis

The main results of the non-linear tariff are summarized as follows:

1. Total revenue \( \mathcal{R}(\tau, f) \) is an increasing function of \( E(\gamma) \).
2. Total revenue \( \mathcal{R}(\tau, f) \) decreases approximately linearly as a function of one parameter (e.g. \( \tau_1, \tau_2 \), or \( E_0 \) while holding the other two parameters fixed).

In terms of the sample data specific to this study:

3. The efficient frontier of the \( \mathcal{R}(\tau, f) - E(\gamma) \) graph is composed of the Pareto-optimal (efficient or non-dominated) tariffs \( \tau^* \):

\( \tau^*_1 = (16, 23, 75) \)

\( \tau^*_2 = (10, 23, 75) \)

\( \tau^*_3 = (10, 23, 100) \)

\( \tau^*_4 = (10, 23, 125) \)

\( \tau^*_5 = (10, 17, 125) \)

4. The tariff rates \( \tau^*_1 \) and \( \tau^*_5 \) represent the extreme ranges of the constraints (\( \tau_1, \tau_2, E_0 \)).
5. The Armenian tariff \( \tau^* = (13, 20, 100) \) is non-Pareto-efficient.

6. The mean and variance decreases within each tariff class \((\tau_1, \tau_2, E_0)\) as \( E_0 \) increases \((\tau_1 \text{ and } \tau_2 \text{ held fixed})\), while the skewness remains essentially constant and the kurtosis measure increases.

In the flat-rate tariff, revenue \( R(\tau, f) \) and \( E(\gamma) \) was directly proportional to \( \tau \), and thus, directly proportional to one another. In the two-tier tariff, revenue is now a function of three parameters, but \( R(\tau, f) \) and \( E(\gamma) \) still depict a general increasing trend. This is illustrated in Fig. 9 and can be seen in Table 6 within each tariff class \((\tau_1, \tau_2, E_0)\) as a function of \( E_0 \). Note that as \( E_0 \) increases (from 75 to 100 to 125 kWh), revenue \( R(\tau, f) \) decreases along with the value of \( E(\gamma) \). The slope of the data in each tariff rate class provides a measure of the sensitivity of the total revenue to changes in \( \gamma \); e.g. for \((10, 17, \cdot)\),

\[
\frac{\Delta R(\tau, f)}{\Delta \gamma} = \frac{911302 - 660020}{7.2 - 4.7} = 1.0 \times 10^5
\]

while for \((10, 20, \cdot)\),

\[
\frac{\Delta R(\tau, f)}{\Delta \gamma} = \frac{1046022 - 956240}{8.0 - 7.0} = 8.9 \times 10^4
\]

These sensitivity measures can be used to compare the various tariff rates.

The relationship between \( R(\tau, f) \) and \( E(\gamma) \) for the 27 tariff rate combinations is depicted in Fig. 10.

In Fig. 11 the 27 tariff combinations in Table 6 are plotted relative to the revenue and equity they generate (columns 2 and 5 in Table 6). The efficient frontier is illustrated in Fig. 12 and depicts the Pareto-optimal tariff rates. A Pareto-optimal tariff rate \( \tau^* \) is defined such that no other tariff rate is at least as good as \( \tau^* \) on every objective and strictly better than \( \tau^* \) on at least one objective. In this setting the objectives are revenue and equity, and the tariff rates \( \tau^*_i \), \( i = 1, \ldots, 5 \), are the Pareto-optimal tariffs. The same general trend between revenue \( R(\tau, f) \) and equity \( E(\gamma) \) is observed as the flat rate but the structure is complicated by the non-linear nature of the tariff. As the revenue increases, the equity measure decreases, but there are a number of combinations that are non-efficient (and can be improved either with an increase in the revenue or equity). The Armenian tariff \( \tau^* = (13, 20, 100) \), for example, falls in this classification and is thus a non-optimal tariff. Observe that the tariff \( \tau^*_4 = (10, 23, 125) \) increases the equity measure as compared to \( \tau^* \) by nearly 15% while not changing

\[\tau^*_1 = (\text{high, high, low})\]

\[\tau^*_2 = (\text{low, low, high})\]

and yield, as expected, maximum revenue–minimum equity \((\tau^*_1)\) and minimum revenue–maximum equity \((\tau^*_2)\) measures.
Fig. 9. Graph of the total revenue $R(t, f)$ vs. the mean value $E(\gamma)$ for the 27 tariff combinations $\tau = (\tau_1, \tau_2, E_0) | \tau_1 = 10, 13, 16; \tau_2 = 17, 20, 23; E_0 = 75, 100, 125$. 
Fig. 10. Graph of the total revenue $R(t_1, f)$ vs. the skewness $sk(\gamma)$ for the 27 tariff combinations $\tau = (t_1, \tau_2, E_0); t_1 = 10, 13, 16; \tau_2 = 17, 20, 23; E_0 = 75, 100, 125$. 
Fig. 11. Graph of the total revenue $R(\tau,f)$ vs. the equity $H_{0.1}$ for the 27 tariff combinations $\tau = ((\tau_1, \tau_2, E_0)|\tau_1 = 10, 13, 16; \tau_2 = 17, 20, 23; E_0 = 75, 100, 125)$. 
the revenue generated. The tariff \( \tau_0^T = (10, 23, 75) \) is observed to increase revenue by 12\% and equity by 3\%. This result follows from the fact that \( \tau_0^T \) and \( \tau_0^E \) are elements of the efficient frontier, while \( \tau_0^A \) is not.

5.3. Optimization model formulation

The rates \( \tau_0^T \) and \( \tau_0^E \) represent the extreme ranges of the constraints and can be solved for in terms of an optimization model. In particular, \( \tau_0^T \) and \( \tau_0^E \) tariff designs represent the solution to the constrained optimization problems:

\[
\begin{align*}
\max & \ g(\tau, f) \\
\text{s.t.} & \ 10 \leq \tau_1 \leq 16 \\
& \quad (17 \leq \tau_2 \leq 23) \\
& \quad (75 \leq E_0 \leq 125)
\end{align*}
\]

for \( g(\tau, f) = \mathcal{R}(\tau, f) \) and \( g(\tau, f) = \mathcal{E}(\tau, f) \), respectively. The Pareto-optimal tariffs \( \tau_0^T, \tau_0^E, \tau_0^A \) can also be solved for directly using a linear programming formulation, and for the two objective case, a visual solution is straightforward. The framework of optimization theory, however, allows the problem class to be generalized in significant directions.

6. Conclusions

The purpose of this paper is to quantitatively assess the economic effects of electricity tariff rates on the residential population in Yerevan. The framework developed is a general methodology that can be modified to a number of environments and is a first step to better understand the structure of tariff rates and their impact on the population as a whole. The analysis of tariff rates in a quantitative framework provides policy makers with an objective means to determine the real impact of tariff rate design. Lifeline pricing is not a perfect system because all consumers receive the subsidy, and those consumers who use less than the quota have little incentive to reduce consumption. Lifeline pricing schemes serve to distribute wealth to the poorer segments of the population and thus have desirable social effects and are easy to implement in countries with developing infrastructures.

The structure of the flat rate and two-tier tariff design was examined in a general context and as related to a specific sample data set. The main concern with the Armenian tariff structure is that it is not clear that the tariff was ‘designed’ in any way to achieve stated or desired goals, and in fact, the tariff structure appears somewhat arbitrary. The Armenian tariff rate \( \tau = (13, 20, 100) \) was shown to be non-Pareto-efficient, and thus, for the given electricity consumption function, a
Fig. 12. The efficient frontier and Pareto-optimal tariff rates \((\tau_1^*, \tau_2^*, \tau_3^*, \tau_4^*, \tau_5^*)\) identified in the tariff rate design. The Armenian tariff rate is denoted by \(\tau_A\) and is clearly non-optimal.
A tariff rate can be designed to provide either a higher-revenue (at the same equity as \(\tau^A\)), a greater equity (at the same revenue generated as \(\tau^A\)), or a higher-revenue/greater-equity combination as compared to \(\tau^A\). This is a significant conclusion since it indicates that, by design, a tariff can be optimized with respect to decision parameters that are readily available to the policy maker.

Acknowledgements

Funding for this research was provided in part by the Center for Economic Planning and Research Analysis supported by the US Agency for International Development, and the author acknowledges the support of Dr Greg Gajewski in this endeavor. The opinions expressed herein are those of the author and do not necessarily reflect the views of the US Agency for International Development or the Government of the Republic of Armenia.

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