A multiple-unit, multiple-period auction in the British electricity spot market

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Abstract

This paper examines the electricity pool in England and Wales. The approach followed in this paper builds on the auction approach of Von der Fehr and Harbord (Von der Fehr, N.H.M., Harbord, D., 1993. Spot market competition in the UK electricity industry. Econ. J. 103, 531–546), but adds realism by allowing explicitly for multiple-unit firms and multiple periods. In a formal setting, a bidding range will be derived, by characterizing the polar cases. The more interesting lower bidding rule will be characterized in detail, providing insights in the performance of the British electricity pool. Four aspects are examined more closely: (1) the mark-up; (2) the number of firms; (3) the auction frequency (bidding flexibility); and (4) load-profile competition. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

In 1990, the electricity supply industry in the United Kingdom has been re-
formed thoroughly, setting an example for various other countries. One of the major elements was the introduction of a (centralized) electricity pool with the purpose to coordinate the (competitive) generation of electricity. It has been expressed that the generation firms ‘play a game’ in determining who will be producing how much and against which price. The idea is that the generation firms offer willingness-to-produce against an offered price. A central institution — the electricity pool — schedules which firms with which generation units will actually produce how much and against which price. The procedure to determine generation-quantities and -prices is accurately specified in the so-called pool rules.

The ‘game’ which is played by the generation firms has attracted academic attention. Two approaches have been developed to model the pool and thereby the behavior of the generation firms. First, and the most influential, stems from Bolle (1992) and Green and Newbery (1992), with a recent and powerful extension by Newbery (1998). They have modeled the pool by means of so-called supply-function equilibria. The advantage of this approach is that it is tractable to a large extent; the functions are continuous and differentiable, which allows mathematical manipulation. As the framework stands it turns out that it is fairly easy to extend analysis of the pool without using high-powered mathematics. The other side of the same coin, however, is exactly the assumed continuity; it is an abstraction. In reality, the generation units are not atomistically small. The marginal costs of a firm are, due to multiple generation units, rather discontinuous or stepwise. Another problem is that in the supply-function approach demand is taken to be elastic, which does not entirely conform to the pool rules.

These criticisms triggered an alternative approach to model the pool: the auction approach by Von der Fehr and Harbord (1993). They note with regard to the supply-functions approach (Von der Fehr and Harbord, 1993, p. 532): ‘as we demonstrate [...] the particular types of [supply functions] equilibria [...] do not generalize to a model where sets are of positive size’. Its main element is stepwise increasing bids, which is inherently discontinuous. Whereas this might be realistic, it contains the major difficulty as well. It hardly allows mathematical manipulation. Nevertheless, Von der Fehr and Harbord come up with some insightful conclusions, which invite further research in this direction. The auction approach offered by Von der Fehr and Harbord seems to have been somewhat short of attention in comparison to the supply-function approach. However, the auction approach does offer further potential to extract insights in the pool behavior.

It is the purpose of this paper to extend the auction approach of Von der Fehr and Harbord and extract further insights in the performance of the British
electricity pool. In particular, Von der Fehr and Harbord’s Proposition 2 will be specified in a multiple-unit context. It explicitly allows for multiple-unit firms and multiple periods. The paper derives a tractable mathematical expression, which appears to characterize the real-world bidding curve quite well. Consequently, analysis of the bidding curve provides further insights in the performance of the pool. The topicality remains high, since whereas the electricity pool in England and Wales has been pioneering, by now, various other countries have installed similar pooling systems.4

Recently, Wolfram (1998) examined the pool empirically, assuming a multiple-unit auction explicitly, thereby staying on the trail set out by Von der Fehr and Harbord (1993). In particular, Wolfram (1998) concentrates on the size (i.e. the number of generation units) of the firms. In a consultancy document, Culy and Read (1995) come to interesting conclusions for the Victorian power pool by means of simulations. A conclusion is that increasing the frequency of bidding increases efficiency. The same has been conjectured for the Norwegian power pool by Knøsflå and Rud (1995). The model in this paper will provide theoretical arguments for several of the empirical observations.

Section 2 will first briefly describe the basics of the pool rules. Section 3 will then summarize the basics of the approach of Von der Fehr and Harbord. The main contribution of this paper is in Section 4. It will specify the bidding behavior, relying heavily on the Propositions 4 and 5 in Von der Fehr and Harbord (1993, p. 536). Section 5 analyses the bidding curve by means of a simulation.

2. The pool rules

The main purpose of the pool is to schedule generation of electricity and determine a spot price, the so-called System Marginal Price (SMP).5 The generation is scheduled a day in advance. The schedule day is divided up into 48 periods of 30 min each. For each period of 30 min a SMP is determined. The generators (i.e. the generation firms) generally have a set of generation units. For each unit they offer the capacity which they are willing to give available and a price which they would like to receive for actually producing with this unit. In practice, the price bids are the more important strategic variables. For this reason the capacity offers will be neglected in the following and attention will be directed to the price bids. It will be assumed throughout that the units are offered at full capacity. The pool rules describe an extensive set of price bids, reflecting the cost structure of generation units. For this paper only the most important price bid will be considered; marginal

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4At the moment of writing, there are renewed proposals to thoroughly reform the British electricity pool. These highly topical developments are still preliminary, however, and will not be discussed in this paper.

5It is beyond the scope of this paper to be extensive on the pool rules. The interested reader may be referred to, e.g. NGC Settlements Limited (1991) or Brunekreeft (1997, ch. 9).
costs (or in terms of the pool rules ‘incremental cost bid’). These costs are mainly the costs of fuel. Although the pool rules do actually allow some more flexibility, it is not uncommon to assume these marginal costs to be constant over the entire capacity range of a generation unit. It should be stressed, that marginal costs and (marginal) price bids are two different things. The basic idea is that the bid reflects marginal cost, but if the firms bid strategically the bids will actually tend to deviate from marginal costs. Every firm makes one price bid for every generation unit for the entire day; thus, one price bid is valid for all 48 periods of 30 min. This may seem rather restrictive, but it conforms to the pool rules.

For every period of 30 min of the schedule day, demand is estimated centrally, and is taken to be price-inelastic for this estimate. Based on the price- and capacity-bids by the generation firms, the pool calculates a cost-minimizing schedule for the entire day, specifying for every 30 min and every generation unit, how much and when it will produce. In general terms, the generation units are scheduled in increasing order of the price bids until capacity is sufficient to meet demand for the corresponding 30 min. The last unit which is scheduled (the unit in the schedule with the highest price bid) is called the marginal unit. For every 30 min the pool calculates SMP. It suffices for this paper to assume that SMP is equal to the price bid of the marginal unit of that 30-min period. The revenue of the firms is assumed to be derived solely from SMP. Every firm gets paid SMP times its entire production. This too reflects neo-classical theory. Irrespective of the bids, after the schedule has been determined and thereby how much the firm will produce, each firm receives SMP for its production (rather than its own bid), irrespective of the ownership of the marginal unit.

To summarize, the basic aspects of the pool rules which suffice for the auction approach to be presented below are: the price bid, actual marginal costs, the system marginal price. It is further important to note the difference between firms and their respective (generation-) units. Demand is fluctuating and broken down into 48 periods of 30 min. There is only one price bid for each unit, valid for all periods of 30 min; in contrast, SMP is calculated for every period of 30 min. This is not trivial. It will be shown in Section 5, that the resulting rather inflexible bidding causes a rigidity, which in turn causes a reduction in competitive pressure. These aspects provide the ingredients for the analysis below.

3. The auction approach of Von der Fehr and Harbord

Von der Fehr and Harbord (1993, p. 533) call their approach ‘a first-price,
sealed-bid, multiple-unit, private-value auction with a random number of units. It is a static analysis, neglecting long-term entry, concentrating on the current market structure. Furthermore, there is no uncertainty about the costs of other firms; only demand is taken randomly from a probability distribution.

Von der Fehr and Harbord (1993, p. 532) assume $N$ independent generators (firms), each having constant marginal costs $c_n \geq 0$, $n = 1, 2, \ldots, N$. The index $n$ ranks the generators in increasing order of their marginal costs, $c_n \leq c_{n+1}$. The capacity of firm $n$, $q_n$, is the sum of the capacities of the units $i = 1, m_n$, belonging to this firm $n$; thus $\sum_{i=1}^{m_n} q_{ni} = q_n$. Although this assumption is realistic, because indeed in practice the firms do have several units, in the model it is not used anymore, which is peculiar. It might as well be assumed instead that all firms have only one unit. This peculiarity can be seen directly from the assumption that a firm has constant marginal costs, although this firm is supposed to have different units. More realistic would be to assume that firms have several units, with different (constant) marginal costs. This will be assumed in this paper, resulting in a multiple-unit auction; mind that multiple in this sense means that a firm is allowed to have multiple units. This extension is not trivial. Multiple-unit firms will have to consider their already scheduled capacity, while bidding with their marginal unit; this makes a non-trivial difference in the bidding behavior.

The firms are to make price bids for their units; output (or capacity) is implicitly assumed to be bid at full capacity. The bids are denoted $g_{ni}$ for all $i$ and $n$. The bids are ranked in increasing order, and the units are scheduled subsequently until total scheduled output suffices to meet demand. The last unit which is scheduled (somewhere between zero and its maximum capacity) is the marginal unit and sets the equilibrium price SMP, which is equal to the bid of the marginal unit. The firms receive for all their scheduled units, SMP times their respective outputs.

The main contribution of Von der Fehr and Harbord (1993, p. 536) is to argue that for sufficiently uncertain demand there exists no pure strategy equilibrium. To come to this result the authors first derive results for insufficiently uncertain demand (i.e., the uncertain range of demand is so small that it stays within the capacity range of one unambiguously determined unit). Suppose a two-generator, one-unit each, setting. Suppose furthermore that demand is low (with certainty), such that the capacity of the most efficient unit suffices. Then the bid of the low-cost unit will unambiguously be just below the marginal cost of the high-cost unit (Von der Fehr and Harbord's Proposition 2). Otherwise the high-cost unit will immediately undercut. The marginal costs of the non-scheduled high-cost unit serves as a limit-price for the low-cost unit. The underlying principle of ‘undercutting’ is typical for the auction approach. It contrasts sharply to the supply-functions approach, as noted by Von der Fehr and Harbord (1993, p. 534). Because of the positively sloped supply functions, ‘undercutting’ would necessarily imply a price

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8The reader may be referred to Bolle (1997) for a more technical treatment of multiple-unit auctions, applied to the electricity pool as well.
drop, which affects all units below the marginal unit. In the auction approach, due to the positively-sized units, which are to be bid with constant price for the unit’s entire capacity, the ‘slope of a unit’s supply function’ is zero (and undetermined for stepping from one unit to another). Thus, it is principally possible to undercut another unit with an infinitely small step, thereby increasing one’s output substantially, while hardly lowering SMP. In effect, the relevant units play a kind of Bertrand game, because for these units the residual demand is perfectly elastic. The discontinuity between units, however, allows strategic bidding. Von der Fehr and Harbord’s Proposition 3, on the other hand, examines the case of high demand, such that it is certain that both units will be scheduled. The bid of the high-cost unit will unambiguously be the highest admissible price (an artificial limit); the bid of the low-cost unit is largely irrelevant and may be equal to zero.

The fundamental insight provided by Von der Fehr and Harbord is stated in their Propositions 4 and 5. Suppose that demand is uncertain on a scale sufficiently large such that either generator has a positive probability of setting SMP, irrespective of the bids. In this case, no pure strategy equilibrium exists. Should demand be high, SMP will be set by the highest bidder and the lowest bid will be irrelevant. However, the lower bidder is not indifferent about its bid, because of the positive probability that its bid will be the marginal bid, i.e. should demand turn out to be low. Thus, given the bid of the high bidder, the low bidder has an incentive to increase its bid to secure a high bid in case of low demand. As a result, both bids will be the same — the upper bound. However, given a high bid of the low bidder, the high bidder will have an incentive to undercut. Thereby, it increases its output in case of low demand, while in case of high demand nothing changes. This, of course, tends to reduce the bids, until the low bid is exactly such that the high bidder will not have an incentive to undercut — the lower bound. There is no pure strategy equilibrium; a generator always has an incentive to change its bid, given the bid of the other.

For the purpose of this paper, Proposition 4 in Von der Fehr and Harbord is essential. It may be noted that the assumption that demand is uncertain on a scale large enough to support positive probabilities of being scheduled for either generator is analytically equivalent to assuming fluctuating demand in a multiple-period context. The main contribution of this paper, thus is not so much the introduction of multiple periods, but rather multiple units per firm. The latter, however, is only interesting in case of multiple periods or alternatively as in Von der Fehr and Harbord in case of sufficiently large demand uncertainty. Otherwise, no matter how many units the firms may have, the Propositions 1–3 of Von der Fehr and Harbord apply.

4. The model

The following notation will be used. Denote:

\[ k = 1.. K, \text{ number of firms;} \]
u = 1.. U, number of units per firm; and
i \in \{1.. (U \cdot K)\}, any unit (to be defined below).

Note the symmetry assumption; each firm has U units. For the bidding rule to be derived below it is convenient to transform the indices u and k into one index i, which identifies and ranks all units uniquely. The ranking will be such that \( c_i > c_{i+1} \) for all i. Three underlying assumptions concerning u and k determine the transformation. These are:

1. \( c_{u,k} > c_{u,k+1} \)
2. \( c_{u,k} > c_{u+1,k} \)
3. \( c_{u,k} > c_{u+1,1} \)

The first assumption ranks the firms for a given order of units. For example, each firm has a most expensive unit; \( u = 1 \) for all k. For a given unit, the firms are ranked in decreasing order of marginal costs. Thus unit 1 of firm 1 has higher marginal costs than unit 1 of firm 2, etc. The second assumption ranks the units for a given firm. Each firm has U different units with different marginal costs; these are ranked in decreasing order of the marginal costs. Thus the marginal costs of unit 1 of firm 1 are higher than the marginal costs of unit 2 of firm 1. The third assumption makes a step to secure monotonicity in going from unit u to unit u + 1. It is only meaningful in combination with the two other assumptions. It says that the marginal costs of unit u of the last firm in the ranking (\( k = K \)) are higher than the marginal costs of unit u + 1 of the first firm in the ranking (\( k = 1 \)).

A formal way to transform u and k into i, accounting for the assumptions made above, is as follows. For the paper, this transformation is merely a technical detail. The paper will continue with the index i; it is important, however, to realize the underlying assumptions. Take any \( i \in \{1.. (U \cdot K)\} \). Define \( S_i := \{s \in \mathbb{N} | s = u \cdot K, u = 1..U\} \) and \( s_{\text{inf}} := \{\inf S | s \geq i\} \) and define \( u := (s_{\text{inf}} / K) \) and \( k := (i - s_{\text{inf}} + K) \). From i, the unit u belonging to firm k can be derived unambiguously. Then the assumption \( c_i > c_{i+1} \) for all i, secures the three assumptions made above. An illustration as in Table 1 may be helpful. The firms \( k \) are represented horizontally, the units \( u \) vertically, and the corresponding \( i \) is given in the cells. For this illustration, let \( U = 4 \) and \( K = 3 \).

Table 1
Illustration of the transformation

<table>
<thead>
<tr>
<th>Unit u</th>
<th>Firm k</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>i = 1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
</tr>
</tbody>
</table>
Note two aspects of the ranking. No firm has two (or more) units next to each other in the ranking. In fact, the firms are ranked such that unit 2 of firm 1 is ranked after unit 1 of firm K and so on. This ensures that there is no load-profile monopoly; i.e., there are several candidates for producing for each particular part of the load. The order of the units is such that the units of one firm take maximum distance from each other in the ranking. The assumption sets the polar case. With respect to the ranking of the units, competition will be strongest with this ranking. Note furthermore that this transformation has been made desirable by allowing multiple units\(^9\). Mind that \(C_i > C_{i+1}\) is the reversed order as compared to the Von der Fehr and Harbord approach; for the purpose of this paper this ranking is more convenient. The last unit to be scheduled (i.e., the one with the highest marginal costs) will be used by the firms as the reference in the bidding rule to be derived. Consequently, it seems quite natural to denote the reference unit, ‘unit 1’ and count backwards from thereon. Note that within the setting of the model, the highest demand is known and fixed and thereby unit 1 is known and fixed.

Let unit \(i\)’s capacity be \(Q_i\). Define \(Q'_i = \sum_{j=u+1}^{U} Q_{k,j}\). This denotes the ‘rest’ capacity of firm \(k\); i.e., the capacity of firm \(k\) which is already scheduled if unit \(i\) is the marginal unit.\(^{10}\) This is important because the pool rules prescribe that SMP is paid to all scheduled units and not only to the marginal unit.

Let there be \(t = 1 \ldots (U \cdot K)\) periods of different demand. Thus, the number of relevant periods is exactly equal to the number of units. Notably, demand in the different periods is known with certainty, or is uncertain only within the capacity range of a unit. As has been explained in Section 3, the focus is on fluctuating, rather than on uncertain demand. Assuming both \((U \cdot K)\) periods of demand and \((U \cdot K)\) units reduces notational effort considerably.\(^{11}\) It effectively allows dropping notation for time. It is assumed that demand fluctuates such that if all firms bid exactly marginal costs, all units would set SMP exactly once. Implicitly thus it is assumed that demand fluctuates exactly in the capacity of the units. The order of fluctuation is irrelevant. This simplifies the analysis substantially and should be seen as a polar case.

In order to abolish any capacity-constraint problems, it is assumed that there is at least one more firm with a unit having higher marginal costs than \(c_i\); these marginal costs will be denoted by \(\gamma\). Under marginal-costs bidding of all firms, this unit thus will not be used. It serves merely as an artificial cap, to forestall bids which are infinitely high, and might in principle as well be replaced by the threat

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9 Furthermore, stepping from unit 1 of firm K to unit 2 of firm 1 causes a discontinuity with respect to the number of units which are already running for each firm; in casu, this number is larger for firm K than for firm 1. This will turn out to cause a relatively unimportant, but remarkable effect in the bidding.

10 In case unit \(i\) belongs to firm \(k\), the capacity of unit \(i\) is not included in the ‘rest’ capacity.

11 One can assume more units than exactly the number of periods. But they will either always be scheduled or never. This would actually lead to Propositions 1–3 in Von der Fehr and Harbord (1993) for these ‘irrelevant’ units. It suffices to constrain the model to the ‘relevant’ units.
that OFFER intervenes.\footnote{See for a discussion on such longer-term upper bounds also Vickers and Yarrow (1994, p. 48 ff.).} It is the pendant to the highest admissible price in Von der Fehr and Harbord (1993).

The firms are called upon to submit bids for their units; these bids are constant for the entire unit-capacity and the same for all periods. The latter may seem restrictive, but the pool rules prescribe this rather inflexible bidding. The bids are denoted by $g_i$, and are ranked in increasing order and the units are scheduled accordingly. Unit $i$ is the marginal unit and will set SMP. All units which are scheduled will get SMP times their production.

If firms undercut, they might do so by bidding $\delta$ lower than the unit they undercut. However, introducing $\delta$ everytime undercutting is examined would make notation unnecessarily cumbersome. Therefore, it is simply assumed that $\delta$ is (very close to) zero and will be dropped from the notation where possible. If two firms have the same bid, actually one of these bids is slightly lower than the other, which then has taken over production from the other. Taking over production means being scheduled one more period. It will be clear from the context which firm has the $\delta$-lower bid. To clarify, the notation is illustrated in Fig. 1.

These preliminaries sum up to the following propositions.

**Proposition 1.** The upper bound in the bidding is the following rule (corrected for $\delta$):

$$g_i = g_{i-1} = \gamma, \quad \text{for } i = 1,2,...,(U \cdot K)$$

This proposition is the pendant of Proposition 3 in Von der Fehr and Harbord (1993). The proof of this proposition is trivial. The bid of ‘unit 0’ is $\gamma$ by
assumption. The bid of unit 1 will be adjusted to the highest admissible price. Unit 2 will adjust to this, given that unit 1 will not undercut, and so on for all other units. This results in maximum profits for all units, for all periods, because in each period SMP is equal to the highest admissible price. The assumption, given that undercutting does not take place is crucial, however. It illustrates the tendency to increase one’s bid, given the bid of the next expensive unit. The limit is determined by not being undercut automatically by the dispatch; i.e. the next higher bid, corrected for \( \delta \). Proposition 1 would be the Nash-equilibrium if bidding were sequential, such that first unit 1 commits to a bid, then given this information, unit 2 commits to a bid and so on. An alternative interpretation would point to tacit collusion: if the players can credibly commit not to undercut then Proposition 1 would be the result.

**Proposition 2.** (for symmetry) the lower bound in the bidding is the following rule:

\[
g_i = g_{i-1}(Q'_{i-1} + Q_{i-1}) + c_{i-1}Q_{i-1} \over Q'_{i-1} + 2Q_{i-1}, \quad \text{for } i = 1,2,...,(U \cdot K) .
\]

The difference-Eq. (2) can be rewritten in terms of marginal costs only. First, denote \( x_i = Q_i + Q'_{i} \) and \( z_i = Q_i / (2Q_i + Q'_{i}) \).

Using this, then gives:

\[
g_i = \gamma \prod_{j=0}^{i-1} x_j + \sum_{l=0}^{i-2} c_l z_{i-l} \prod_{j=l+1}^{i-1} x_j + c_{i-1} \cdot z_{i-1} .
\]

The formal proof of Proposition 2 is given in Appendix A. Proposition 2 is the multiple-unit pendant of Proposition 2 in Von der Fehr and Harbord (1993). In the lower bidding rule, the bid of unit \( i \) is a weighted average of the bid of unit \( i - 1 \) and the marginal costs of unit \( i - 1 \). The weights are determined by the capacity of unit \( i - 1 \) and the rest capacity of the firm to which unit \( i - 1 \) belongs. What thus actually happens is that the firms derive their bids from the bids above in order not to induce undercutting. However, due to multiple demand, this effect is strengthened, because all firms do this. Proposition 2 would be the Nash-equilibrium, if bidding were sequential, such that first the lowest-cost unit would commit to a bid, then given this bid the next-expensive unit commits to a bid and so on. In this case, the incentive to undercut determines the limit. This behavior expresses fierce price competition.

**Proposition 3.** Under the assumptions in the model, there exists no pure strategy equilibrium.

This is the pendant of Proposition 4 in Von der Fehr and Harbord (1993, p. 536),
and it can be shown verbally. Given that all other units bid according to Proposition 2, unit $i$ will have an incentive to increase its bid to just below the bid of unit $i - 1$. Thus, bidding according to Proposition 2 cannot be Nash equilibrium. Bidding according to Proposition 1 is not a Nash equilibrium either. Given that all units are bid according to Proposition 1, unit $i$ will have an incentive to undercut unit $i + 1$. Neither Proposition 1, nor Proposition 2 is Nash-equilibrium, because the bidding is simultaneous and not sequential. Further assumptions or refinements are needed to determine equilibrium.

The intuitive extension is to look for a mixed-strategy equilibrium. However, a closer look reveals that a mixed-strategy equilibrium is far from obvious. Unit $i$ should play a mixed strategy in the bidding game with unit $i - 1$; this would result in equilibrium expected values of playing either strategy. However, unit $i$ simultaneously plays a mixed strategy game with unit $i + 1$; this too would result in expected values for playing either strategy. These expected values of unit $i$ would be equal only coincidentally. If they do not coincide, the result is still indeterminate and one should take the analysis one step further and look for something like a ‘mixed-mixed’ strategy, which should be solved for all units simultaneously. I must admit that I have not been able to derive an expression for such an equilibrium and I must confess to have some reservation about the rather heroic assumptions which would underlie the equilibrium. It may be preferable to state that the equilibrium is indeterminate, or perhaps does not exist. Alternatively, equilibrium conditions may be derived by additional assumptions on behavior, which is not done here. Deriving a (mixed-strategy) equilibrium will be left for further research. In the following, the lower bound in the bidding (Proposition 2) will be examined in detail.

5. Analysis of the lower bidding rule

5.1. An analysis of the mark-ups

Fig. 2a gives a simulation of the bidding rule in Proposition 2. In order to compare with real-world data, the cost data from Von der Fehr and Harbord’s Fig. 2 have been approximated. Note that the most expensive unit — unit 1 — is placed at the right side; at unit 1, cumulative capacity and thereby output is at its maximum; it allows comparison with the figures in Green and Newbery 1992, p. 943 and Von der Fehr and Harbord 1993. Fig. 2b plots the (relative) mark-ups,

\[13\]This is actually the pendant to Von der Fehr and Harbord (1993, p. 536) who state with respect to a unique mixed-strategy equilibrium, that they ‘have not been able to find an algebraic expression for the probability that the high-cost generator submits a bid below that of the low-cost generator, but a lower bound can be established.’ Moreover, indeterminacy somehow conforms to the supply-function approach (Green and Newbery, 1992, p. 938); in the latter approach, there are many Nash-equilibria, and consequently, the general result is indeterminate as well. Mind, however, that this may be convincing, rather than a handicap.
which have been defined as the difference between the bid and marginal costs divided through the bid.

The similarity of Fig. 2a with Fig. 2 in Von der Fehr and Harbord (1993, p. 542/543) suggests that the expression of Proposition 2 may be a good approximation of the bidding behavior. Typical bidding behavior (see Green and Newbery, 1992; Von der Fehr and Harbord, 1993) is revealed from especially Fig. 2b: relatively large deviations of the bids as compared to marginal costs are found for high-cost units, which of course corresponds to high-demand periods. It can be seen that, starting with unit 1, the mark-up increases rather sharply, reaches a maximum and then decreases. This pattern depends on $\gamma$ and marginal costs, but

Fig. 2. (a) The bidding curve of Proposition 2. (b) The (relative) mark-up for the bidding in (a).
the maximum is generally biased towards high-cost units. Two effects counter-
balance each other. First, the effect that the bids adjust to the preceding bids. A 
bid, $g_i$, will be somewhere between the bid and the marginal costs of the preceding 
unit. If the bids refer relatively strongly to the preceding bids rather than the 
preceding marginal costs, then the mark-up will increase. On the other hand, if the 
bids refer to the preceding marginal costs rather than to the preceding bids, then 
the development of the mark-ups will follow the rate of change of the marginal 
costs, which is assumed constant in this simulation. This can be seen from the 
bidding rule in Eq. 3. For periods of high demand, i.e. for low $i$, the rest capacity 
$Q_r$ is relatively large. This implies that $x_i$ tends to 1 and $z_i$ tends to 0. Stepping to 
periods of lower demand, i.e. stepping to higher $i$ values, the rest capacity 
decreases; $x_i$ goes to one-half and $z_i$ goes to one-half. In Eq. (3), this means that 
for low $i$ (high demand) the first term will be relatively strong, and for high $i$ (low 
demand) the second term will be relatively strong. In other words, recalling that $\gamma$ 
is larger than $c_i$ for all $i$, in periods of high demand, the bids refer relatively 
strongly to the preceding bids. On the other hand, in periods of low demand, the 
bids refer relatively strongly to the preceding marginal costs.

The latter effect has a straightforward and important interpretation. Basically, 
undercutting lowers SMP, which reduces revenue for all scheduled capacity. Ergo, 
if the rest capacity is relatively large, a firm has much to lose by undercutting. 
Since in periods of low demand, the rest capacity gets smaller, undercutting gets 
more attractive, thereby increasing competitive pressure.

5.2. Effect of the number of firms

The effect of the number of firms is unambiguous. A decrease of the number of 
firms, unambiguously increases SMP, ceteris paribus. If for given total capacity and 
total demand the number of firms is decreased, $x_i$ goes to 1 and $z_i$ goes to 0. In 
the bidding rule of Eq. (3), this implies that bidding according to the preceding bids 
(the first term) gets relatively stronger as compared to bidding to the preceding 
marginal costs (the second term). As a result, the bids will be higher and 
consequently, ceteris paribus, SMP will be higher. The reverse, of course, holds as 
well.

The simulation results are illustrated in Fig. 3a,b, where the marginal-costs curve 
is assumed linear. It can readily be seen that a decreasing number of firms 
increases both the bids and the mark-ups over the entire range.

To be sure, the intuition is not the very fact that there are less firms. Within the 
setting, it is the ceteris paribus clause that less firms implies that these firms are 
larger. Within the multiple-unit auction of the pool, larger firms have more to lose. 
As explained above, undercutting basically lowers SMP. If firms have a long tail of 
scheduled capacity they will forego much revenue, due to the quantity effect. In 
contrast, small firms have little to lose by undercutting. The lesson to be drawn 
from this is that large firms, which already have much scheduled capacity, will have 
little incentive to undercut. Small firms are more competitive. This is in essence 
what has been empirically observed by Wolfram (1998) in the British pool.
Flexible bidding (i.e. a higher auction frequency)

The pool rules prescribe that firms are to make one price bid for every unit for the entire schedule day, irrespective of fluctuating demand. From these bids then an SMP is derived for every 30 min. This is a matter of institutional design, not a law of nature. It might as well have been chosen to allow firms to make a bid (for every unit) for every 30 min. In a simulation for the Victorian power pool, Culy and Read (1995) came to the conclusion that more flexible bidding increases competitive pressure. Knivsfjå and Rud (1995) recommend a higher frequency of the auction for the Nordic power pool; I suppose that a higher auction frequency is equivalent to more flexible bidding. Allowing, say four different bids instead of only one, is equivalent to breaking up the schedule day into four parts; conceptually there would be four different auctions. In effect the frequency of the auction would be increased.

A simulation of the model, which compares four (short) periods of different bidding to only one (long) period, is graphically illustrated in Fig. 4.
It can be seen that increasing the auction frequency generally lowers the bids. As explained above, there is an effect in the bidding that the bids refer to the preceding bids, pushing up the bids overall. The counterbalance effect of this is that undercutting lowers SMP for at least one period. This lowers revenue for all already scheduled units. The lower SMP, however, is not the bid with which undercutting takes place (with \( \delta = 0 \)); instead, it is the bid which is undercut, now determining SMP for another period. Effectively, without undercutting, SMP was \( g_i \) and \( g_{i+1} \), whereas undercutting (with \( \delta = 0 \)), causes SMP to be \( g_{i+1} \) and again \( g_{i+1} \). The latter effect does not apply for a unit which is not scheduled at all; in the model, the unit with marginal costs \( \gamma \). If this unit undercut, it shifts out the neighboring unit completely; the unit which is undercut (unit 1) will not set SMP anymore, simply because period ‘0’ is not a schedule period. Ergo, undercutting with the unit, which is not scheduled at all, does not lower SMP. As long as the bid remains above marginal costs \( \gamma \), it is profitable to undercut, irrespective of the rest capacity. This causes the competitive pressure in the unit-1 region. The very same principle translates to increasing the frequency of the auctions. Making four auction periods rather than one, causes four times a unit to be just not scheduled at all in the relevant period. Thus four times, the bidding curve as in Fig. 1 starts at the marginal costs of the unit which is just not scheduled in the relevant period, rather than bidding according to the preceding bid of this unit.

5.4. Load-profile competition

In the model the units are ranked such that perfect load-profile competition
exists; i.e. they are ranked such that the units of one and the same firm take maximum distance of each other in the ranking. Every unit is surrounded by units of other firms only. In effect, two units belonging to the same firm cannot be ranked next to each other. But suppose, the latter would be the case. Suppose there are two units belonging to one and the same firm ranked next to each other. Suppose that units $i$ and $i + 1$ belong to the same firm. This leads to the following conjecture: imperfect load-profile competition decreases competitive pressure. This is only a conjecture, because within the model the bidding rule can no longer be applied to prove this conjecture. The bidding rule has been proven to hold for symmetry. If units $i$ and $i + 1$ belong to the same firm, symmetry no longer holds and ergo, the bidding rule need not apply. It may, but need not.

An intuitive explanation is straightforward. The firm will not undercut its own unit $i + 1$ with unit $i$. Thus one may expect the bid $g_{i+1}$ to be close to $g_i$ according to Proposition 1, rather than take the distance described by Proposition 2. All bids $g_{i+1,j}$ for $j = 2, (U:K)$, will be according to Proposition 2, whereas their reference $g_{i+1}$ is now higher; consequently, all these bids are higher. Roughly, one might consider the unit $i$ and $i + 1$ as one unit with relevant marginal costs $c_i$ and a corresponding bid $g_i$ for both units; it is as if the capacity of unit $i + 1$ simply belongs to unit $i$.

6. Conclusions

In the literature, the British electricity pool is modeled in two ways. First, by the supply-functions approach (Bolle, 1992; Green and Newbery, 1992; Newbery, 1998). Second, by the auction approach by Von der Fehr and Harbord (1993). The major difference between the two approaches is that the former takes bids of generation units continuously, whereas the latter takes account of stepwise changes in bids (or, costs) of generation units; the auction approach considers generation units to be of positive size. This paper concentrates on and extends the approach of Von der Fehr and Harbord (1993), mainly by allowing firms to have multiple units.

The main contribution of the paper is the formal derivation and examination of a bidding rule which extends Proposition 2 in Von der Fehr and Harbord (1993) for the multiple-unit context. Basically, the firms use a weighted average of the marginal cost and the bid of the next expensive unit as the reference for the lower bidding rule. The firm sets a ‘limit’ bid, such that there is just no incentive for the next expensive unit to undercut.

In Section 5, the (lower) bidding rule is examined by plotting the mathematical expression. The bidding rule seems to approximate the real-world situation fairly well. A closer examination of the mark-ups reveals the general lesson, that large firms have a lot to lose and therefore tend to be less competitive than small firms. Simulation of the effect of the number of firms reveals that a smaller number of firms increases the bids unambiguously. A smaller number of firms implies, ceteris paribus, larger firms and larger firms are less competitive, because they have more to lose from undercutting. This corresponds to empirical observation for the British pool by Wolfram (1998).
It is argued and shown that a higher auction frequency decreases the bids and mark-ups. A higher auction frequency means that instead of having one bid which is valid for the entire day, the day can be broken up in several periods for which separate bids may be made. For example, one bid for every 30 min, or, as has been done in a simulation, breaking up the day into four periods and thus allowing four separate bids. This provides a theoretical basis for simulation results for the Victorian power pool by Culy and Read (1995) and for a recommendation for the Norwegian power pool by Knivsfål and Rud (1995). Last, it is argued from the bidding rule that a load-profile monopoly decreases competitive pressure.

Appendix A: Proof of Proposition 2

To prove Proposition 2, it is to be shown, first, that given bidding according to Proposition 2, undercutting is profit decreasing. Second, that increasing one’s bid above the bidding in Proposition 2 induces undercutting by the next expensive unit, which in turn must be shown to be profit decreasing.

With lowering the bid two things can happen. Either the scheduling is unaffected, in which case the lower bid will lower SMP in the period in which this unit sets SMP, which will achieve nothing but to decrease profits. Or the unit undercut another (or several other) unit(s), and thus does affect the scheduling. In this case a trade-off exists: a lower SMP vs. more output. It must be shown that, under the bidding rule in the proposition, it will always be profit decreasing to undercut. To proof this, profits without undercutting should be compared with profits after undercutting. For ease of notation, however, only the changes in profits are given; all other drop out in the comparison. If a firm undercut (with its unit $i$) the next unit $i + 1$, it will have unit $i$ be scheduled one more period. By assumption it undercut unit $i + 1$ with $g_{i+1} - \delta$, with, to recall, $\delta = 0$. For this period, unit $i + 1$ will not be scheduled. However, it will be scheduled (and set SMP) in the period $i$; the period in which previously unit $i$ set SMP. This is a relevant change. The bids were the following: in period $i$, SMP was $g_i$ and in period $i + 1$, SMP was $g_{i+1}$. After undercutting, SMP will be $g_{i+1}$ in both periods. Thus undercutting lowers SMP in period $i$, while it gets unit $i$ scheduled one more period. This is the trade-off. In all other periods nothing changes and accordingly the profits resulting from these periods can be dropped from the profit comparison. Moreover, the proof is explicitly constrained to undercutting other firms’ units; this assumption is consistent within the model because it cannot be profitable to undercut one’s own unit. Within the set of units of one and the same firm, the unit with the higher marginal costs would produce more periods than the unit with the lower marginal costs; within the setting of the model this is irrational.

It is assumed that undercutting takes place $\alpha$-times; thus $g_i$ is decreased to $g_{i+\alpha}$. Relevant profit for the firm with unit $i$, in case it bids according to Proposition 2
(without undercutting $\alpha$-times), is:

$$\pi|_a = g_i(Q_i + Q'_i) + \sum_{j=1}^{\alpha} g_{i+j}Q'_j. \quad (4)$$

Its relevant profit for undercutting $\alpha$ times is:

$$\pi^u|_a = g_{i+a}(Q_i + Q'_i) + \sum_{j=1}^{\alpha} g_{i+j}(Q_i + Q'_i) - \alpha c_iQ_i. \quad (5)$$

Eq. (4) describes the relevant part of the profit in case the firm bids according to the proposition; i.e., the part of the profits which would change after undercutting $\alpha$-times. It says that the firm gets its bid $g_i$ times all its scheduled capacity (including the capacity of unit $i$) plus, for the periods $i + 1$ to $i + \alpha$ the respective bids times its scheduled capacity (i.e. without the capacity of unit $i$, because in these periods unit $i$ is not scheduled). Eq. (5) describes the relevant profits if the firm undercut $\alpha$-times. Undercutting lowers the bid for unit $i$ to $g_{i+a}$, and consequently the firm receives this bid times its total scheduled capacity in this period, which is the first term in the right-hand side of Eq. (5). The units which are undercut now determine SMP in the subsequent periods. Thus, in all the periods in which the units which have been undercut set SMP, the undercutting firm receives the respective bids times its total scheduled capacity (i.e. as a consequence of undercutting, including the capacity of unit $i$). This is the second term on the right-hand side in Eq. (5). The third term on the right-hand side in Eq. (5) represents additional marginal costs, incurred by the undercutting. Undercutting $\alpha$-times implies that the unit $i$ runs $\alpha$ more periods, and consequently incurs $\alpha$-times additional marginal costs.

If the difference between Eqs. (4) and (5) is always larger than zero, no firm will have an incentive to undercut any other firm, and consequently no firm will have an incentive to lower its bid. To show is thus that the following difference holds:

$$\Delta \pi_i = \pi_i|_a - \pi^u_i|_a > 0. \quad (6)$$

Subtracting Eqs. (4) and (5), writing out and rearranging terms gives:

$$g_i(Q_i + Q'_i) - g_{i+a}(Q_i + Q'_i) - \sum_{j=1}^{\alpha} g_{i+j}Q_i + \alpha c_iQ_i > 0. \quad (7)$$

Noting that by identity:

$$g_1 - g_a \equiv (g_1 - 2g_2) + (2g_2 - 2g_3) + \ldots + (2g_{a-1} - 2g_a) + g_a, \quad (8)$$

it follows from Eq. (7), after writing out and rearranging terms:

$$(g_1 - 2g_{i+1} + 2g_{i+1} - 2g_{i+2} + 2g_{i+2} - \ldots - 2g_{i+a} + g_{i+a})(Q_i + Q'_i)$$

$$- \sum_{j=1}^{\alpha} g_{i+j}Q_i + \alpha c_iQ_i > 0. \quad (9)$$
Noting that:
\[ 2g_{i+1}(Q_i' + Q_i') = g_{i+1}Q_i' + g_{i+1}(Q_i + Q_i') + g_{i+1}Q_i, \]
\[ \text{Eq. 10} \]
it follows from Eq. (9):
\[ g_i(Q_i' + Q_i) - 2g_{i+1}(Q_i' + Q_i') + g_{i+1}Q_i' + c_iQ_i \]
\[ + g_{i+1}(Q_i' + Q_i') \]
\[ - 2g_{i+2}(Q_i' + Q_i) + g_{i+2}Q_i' + c_iQ_i' \]
\[ + g_{i+1}(Q_i' + Q_i) \]
\[ - 2g_{i+a}(Q_i' + Q_i) + g_{i+a}Q_i' + c_iQ_i > 0. \]
\[ \text{Eq. 11} \]

The structure of the lines in Eq. (11) corresponds to the asserted bidding rule (Proposition 2). The first line is equivalent to the bidding rule and is thus equal to zero. The remaining lines look like the bidding rule; they have the same structure, but contain different quantities. If each line is larger than zero individually, then Eq. (11) is true aggregately. To compare is the following:
\[ g_{i+1}(Q_i' + Q_i) - g_{i+2}Q_i' - 2g_{i+2}Q_i + c_iQ_i > g_{i+1}(Q_i'_{i+1} + Q_{i+1}) - g_{i+2}Q_{i+1} \]
\[ - 2g_{i+2}Q_{i+1} + c_{i+1}Q_{i+1}. \]
\[ \text{Eq. 12} \]

The left-hand side is a quasi bidding rule, and the right-hand side is the real bidding rule which is zero. Eq. (12) holds for the symmetrical case: that is, assume that \( Q_i = Q_{i+1} \) and \( Q_i' = Q_{i+1}' \). Using the symmetry, Eq. (12) reduces to \( c_i > c_{i+1}' \), which is true by assumption. The same holds for the other lines and thus Eq. (11) is true.\(^{14,15}\)

The second part of the proof must show that, if all firms use the bidding rule of Proposition 2, even the slightest increase would induce undercutting by another firm, and that this decreases profit for the firm which increased its bid. Assume \( \alpha = 1 \); that is, undercutting would be by the firm directly above the firm which increases its bid. Setting \( \alpha = 1 \) in Eq. (11), reduces Eq. (11) to the bidding rule Eq. (2). Thus, \( \alpha = 1 \) sets exactly the border case, the difference being \( \delta = 0 \). This is no surprise, since the bidding rule says that the firms consider the bid of the firm directly above them as their limit. The ‘limit’ prices for the firms further above are higher than this. This means that, if a firm sets a limit price to forestall the firm directly above to undercut, it automatically sets limit prices for all firms higher than that. This shows that a slight increase in the bids immediately induces undercutting from the firm directly above. It remains to be shown that being undercut would decrease profit. It is important to see that slightly increasing the

\(^{14}\)There is a small technical problem with the bordercase. In going from unit \( i \) of firm \( K \), to unit \( i + 1 \) of firm 1, the ‘rest’ capacity are no longer the same by definition. However, since \( Q_i' > Q_{i+1}' \) and \( g_{i+1} > g_{i+2} \) the proof also holds for the (asymmetrical) bordercase. This inherent asymmetry shows up in the simulation in Section 5.

\(^{15}\)Unfortunately, it cannot be proven for asymmetry; it is only ‘likely’ to be true.
bid and inducing undercutting from the firm directly above would never be profit maximizing. Better would be to further increase the bid, and thereby increase SMP, until further undercutting is just not induced; that is, set the optimal bid of the firm directly above and let this firm, but only this firm, undercut. However, then the only thing that happens is that the firm will produce one period less; a period in which it made a positive profit. In the period in which the firm just above sets SMP, and which will be set by the firm itself now, with exactly the same bid, will thus not change its profit, because SMP remains the same. Consequently, the only thing which happens is that the firm would forego a positive profit for the period in which itself would have set SMP, if it had bid according to the bidding rule. Trying to go even higher would only repeat this profit decreasing procedure, because the next above would undercut. This completes the proof.

References