Non-axisymmetric matrix cracking and interface debonding in unidirectional brittle matrix composites

F. S. JI* and L. R. DHARANI * 1

Abstract. – A discrete fiber model is developed in this paper and applied to study the problem of non-axisymmetric matrix cracking and interface debonding in unidirectional brittle-matrix fiber-reinforced composite materials subjected to an axial tensile load in the fiber direction. In this model, it is assumed that the interface debond remains open. Considering the non-axisymmetry of the matrix crack and interface debond, a three-dimensional stress analysis is introduced into the above model. The variational approach based on the principle of minimum potential energy is employed to obtain the numerical results for stress and displacement fields. On the basis of Irwin–Kies compliance calibration formulation, the strain energy release rates associated with the matrix crack and the interface debond are calculated. The competition between these two modes is then assessed by comparing the corresponding toughnesses. It is shown that the magnitude of the toughness of the fiber/matrix interface plays a predominant role on the debond initiation and extension. The stabilities of the debond and the matrix crack extension are investigated. © Elsevier, Paris

1. Introduction

At present, brittle matrix composite materials such as ceramic matrix composites are being considered for aerospace and energy related structural applications as a results of their high strength and damage tolerance at high temperatures. In such materials, prefailure damage initiates with the formation of multiple, regularly spaced cracks in the matrix, as observed by Sambell et al. (1972) in carbon fiber composites with ceramic and glass matrices, by Philips (1972) in glass reinforced by carbon-fiber, by Prewo and Brennan (1982) in glass matrix composites reinforced by silicon carbide and by Marshall and Evans (1985) in glass ceramics reinforced by silicon carbide fibers. It is essential to investigate and understand the formation and growth of the damage in order to utilize brittle matrix materials safely. Because the failure mechanism in these materials is very complex, it is very difficult to analyze the problem by directly using classical linear fracture mechanics or the existing micromechanics theories. Some assumptions, therefore, must be made in an effort to simplify the analysis and make the problem mathematically tractable. A typical assumption is that the composite material is composed of the fiber-matrix assemblies, each of which can be regarded as a separate entity, and its damage

* Department of Mechanical and Aerospace Engineering and Engineering Mechanics, University of Missouri-Rolla, Rolla, MO 65409-0050, USA.

1 Author to whom correspondence should be addressed.

EUROPEAN JOURNAL OF MECHANICS, A/SOLIDS, VOL. 17, N° 2, 1998
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characteristic represents that of the composite material. The model based on the above assumption is called discrete fiber model. Generally, the model is a composite cylinder in which a round fiber is surrounded by an annular matrix region. In terms of geometry, the discrete fiber model can be presented as: a) two dimensional, the damage configuration is either axisymmetric or planar; b) three dimensional, the damage configuration is non-axisymmetric or non-planar. The employed micromechanics theories include classic shear-lag analysis presented by Cox (1952) and Hedgepeth (1961), in which the effect of transverse stress on the axial strain is neglected, consistent shear-lag theory presented by Jones (1985), and variational approach presented by Zhao and Ji (1990).

Aveston, Cooper and Kelly (1971) first used the above two-dimensional model and the classical shear lag analysis (Cox, 1952) to relate the interfacial debond length to the applied load and the interfacial friction to study the matrix cracking and interface debond in unidirectional brittle matrix composites. Avenston (1971) and Aveston and Kelly (1973) then extended and improved the above work. Their work were collectively referred to as ACK model. Marshall, Cox and Evans (1985) established qualitatively the relation between the ACK's mechanics of materials model and fracture mechanics as it pertains to brittle matrix composites. McCartney (1987) further extended the work of Marshall et al. (1985), and established the relation between the matrix-cracking stress and the size of a pre-existing defect. On the basis of fracture mechanics theory, Budiansky, Hutchinson and Evans (1987) used the above two-dimensional model and the classical shear lag analysis to present a more rigorous and general approach than that of the ACK model. Hutchinson and Jensen (1990) used this in the study of models of fiber debonding and pull-out in brittle composites with friction and Wijewickrema and Keer (1991) used this model for the axisymmetric analysis of a fiber embedded in a matrix with annular matrix crack. Dharani, Chai and Pagano (1990) employed the two-dimensional model and consistent shear-lag theory to investigate the matrix cracking in ceramic matrix composites. Among these studies, the common theme is that the friction over the debonded interface plays a key role in growing the interface debond stably. When the friction vanishes, it is predicted that the interface debond may propagate unstably until the fiber is entirely separated from the matrix under very low load.

Dharani and Ji (1996) presented a three-dimensional model and variational approach to investigate the non-axisymmetric matrix cracking in brittle matrix composites. It is contemplated that a pre-existing surface crack within the matrix may extend unstably up to the fiber/matrix interface in the radial direction, once the applied load reaches its critical value. Subsequently, the crack extension may take place in two ways: (a) the crack continues to extend within the matrix region circumferentially around the fiber, because the matrix/fiber interface is relatively tough and (b) the crack deflects into the matrix/fiber interface and causes the interface debonding, because the interface has a lower toughness. Case (a) has been analyzed by Dharani and Ji (1996) and will now be extended to analyze case (b). In this paper, a variational approach based on the principle of minimum potential energy presented by Zhao and Ji (1990) is employed to obtain the numerical results for stress and displacement fields. On the basis of the Irwin-Kies compliance calibration formulation (Irwin and Kies, 1954), the strain energy release
rates associated with the matrix cracking and the interface debonding are computed. The competition between these two modes is then assessed by comparing the respective strain energy release rates with the corresponding toughnesses, in order to determine the fracture mode of the composites. The results indicate that the interfacial debond grows unstably until the interface is entirely debonded, once the applied load reaches a critical value. However, this critical value is relatively high rather than negligible. This conclusion could not be obtained by using the model adopted in the previous references.

2. Formulation

Consider a composite cylinder with a transverse matrix crack and an interfacial debond, in which an elastic fiber of length $2L$ with radius $R_f$ is surrounded by the annular matrix region which has an outer radius $R_e$, as shown in Figure 1. A tensile load parallel to the fiber axis is applied at the composite edges. A cylindrical coordinate system is set up as shown in Figure 1. The crack is located in the $r\theta$-plane in the shape of a fan, impinging on the matrix/fiber interface. Its size is defined by the crack angle $\alpha$. The interfacial debond is $2L_0$ long and forms an arc of $2\alpha R_f$. Due to symmetry of the composite cylinder with respect to both plane $z = 0$ and plane $\theta = 0^\circ / 180^\circ$, only one quarter of the composite ($z \geq 0$, $0^\circ \leq \theta \leq 180^\circ$) need to be considered in the analysis for stress and displacement, while the half ($0^\circ \leq \theta \leq 180^\circ$) being considered for the fracture analysis.

![Diagram of a single fiber composite with matrix crack and interface debonding.](image)

The composite cylinder is first sub-divided into several elements in the shape of bar or strip in order to carry out the numerical analysis. A typical element mesh is shown in Figure 2, and the element is identified by indices $m$ and $n$. The geometry of a typical element $(m, n)$ is shown in Figure 3.
Fig. 2. A schematic of discretization for numerical analysis.

Fig. 3. A typical element \((m, n)\) for finite difference analysis.
2.1. **TOTAL POTENTIAL ENERGY**

Let the displacement components of element \((m, n)\) be \(U_{m,n}(z), V_{m,n}(z)\) and \(W_{m,n}(z)\) along the \(r, \theta\) and \(z\)-directions, respectively. Using the finite-difference method, the strain-displacement relations for element \((m, n)\) in the cylindrical coordinate system can be approximated as follows:

1. \[
\varepsilon^z_{m,n} = \frac{dW_{m,n}}{dz}
\]

2. \[
\varepsilon^r_{m,n} = \frac{1}{\Delta r_m} (U_{m,n} - U_{m-1,n}) \delta^*_{m1}
\]

3. \[
\varepsilon^\theta_{m,n} = \frac{1}{r_m \Delta \theta_n} (V_{m,n} - V_{m,n-1}) \delta^*_{n1} + \frac{U_{m,n}}{r_m}
\]

4. \[
\gamma^{rz}_{m,n} = \frac{dU_{m,n}}{dz} + \frac{1}{\Delta r_m} (W_{m,n} - W_{m-1,n}) \delta^*_{m1}
\]

5. \[
\gamma^{r\theta}_{m,n} = \frac{dV_{m,n}}{dz} + \frac{1}{r_m \Delta \theta_n} (W_{m,n} - W_{m,n-1}) \delta^*_{n1}
\]

6. \[
\gamma^{\theta z}_{m,n} = \frac{U_{m,n} - U_{m,n-1}}{r_m \Delta \theta_n} \delta^*_{n1} + \frac{\delta^*_{n1}}{\Delta r_m} (V_{m,n} - V_{m,n-1}) - \frac{V_{m,n}}{r_m}
\]

where \(\delta^*_{ij} = 1\) for \(i \neq j\) and \(\delta^*_{ij} = 0\) otherwise.

For this problem, the displacement boundary condition is

7. \(W_{m,n}(0) = 0\) for uncracked elements

and the force boundary conditions are

8. \(P_{m,n}(L) = P_{m,n}^0\) for all elements

9. \(P_{m,n}(0) = 0\) for cracked elements

where \(P_{m,n}(z)\) is the axial load on element \((m, n)\), and \(P_{m,n}^0\) is the applied axial load acting on element \((m, n)\) in \(z\)-direction. In general, \(P_{m,n}^0\) can be determined by the following equation

10. \[
P_{m,n}^0 = E_{m,n} \left( r_m - \frac{\Delta r_m}{2} \right) \Delta r_m \Delta \theta_n \varepsilon_0
\]

where \(\varepsilon_0\) is the remote axial strain in \(z\)-direction, and \(E_{m,n}\) is the Young's modulus of element \((m, n)\).
In order to deal with displacement and force boundary conditions at the debonded interface, it is assumed that there exists a very thin layer (interlayer) between the fiber and the matrix (Popejoy and Dharani, 1992), which may be regarded as an interface ply and has the same material properties as the matrix. The interfacial debond means that the interlayer has failed to support any load over the zone where the debonding has taken place. This assumption makes the problem mathematically more tractable. The strain energy over that zone is simply taken to be zero. The effect of the thickness of the interlayer will be assessed later and the proper thickness of the interlayer will be chosen to minimize the loss of accuracy resulting from this assumption.

For the configuration considered here, the potential energy \( \pi_p \) for a composite cylinder with damage subjected to axial loading in the \( z \)-direction on the ends can be written as

\[
(11) \quad \pi_p = \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \int_{L}^{L'} \frac{E_{m,n} (r_m - \frac{\Delta r_m}{2}) \Delta r_m \Delta \theta_n}{2(1 + \nu_{m,n})} \left[ (\epsilon_r^{m,n})^2 + (\epsilon_z^{m,n})^2 \right] dz \right. \\
+ (\epsilon_r^{m,n})^2 + \frac{\nu_{m,n}}{1 - 2\nu_{m,n}} (\epsilon_r^{m,n} + \epsilon_z^{m,n})^2 + \frac{1}{2} (\gamma^{r\theta}_{m,n})^2 \\
\left. + \frac{1}{2} (\gamma^{\theta r}_{m,n})^2 + \frac{1}{2} (\gamma^{\theta z}_{m,n})^2 \right\} dz \}
\]

where \( \nu_{m,n} \) is the Poisson’s ratio of element \((m,n)\), \( \eta_{m,n} = 1 \) for debonded elements and \( \eta_{m,n} = 0 \) otherwise.

Substituting Eqs. (1-6) into Eq. (11), the total potential energy \( \pi_p \) can be given as

\[
(12) \quad \pi_p = \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \int_{L}^{L'} F_{m,n} \left[ \left( \frac{dW_{m,n}}{dz} \right)^2 + \delta_{m1}^* (U_{m,n} - U_{m-1,n})^2 \right] dz \right. \\
+ \left( \frac{\delta_{m1}^* (V_{m,n} - V_{m-1,n})}{r_m \Delta \theta_n} + \frac{U_{m,n}}{r_m} \right)^2 + \frac{\nu_{m,n}}{1 - 2\nu_{m,n}} \left( \frac{dW_{m,n}}{dz} \right)^2 \\
+ \frac{\delta_{m1}^* (U_{m,n} - U_{m-1,n})}{r_m \Delta \theta_n} + \frac{\delta_{m1}^* (V_{m,n} - V_{m-1,n})}{r_m} \left( \frac{U_{m,n}}{r_m} \right)^2 \\
+ \frac{1}{2} \left( \frac{dU_{m,n}}{dz} + \left( \frac{\delta_{m1}^* (W_{m,n} - W_{m-1,n})}{r_m \Delta \theta_n} \right)^2 \\
+ \frac{1}{2} \left( \frac{\delta_{m2}^* (U_{m,n} - U_{m-1,n})}{r_m \Delta \theta_n} + \frac{\delta_{m2}^* (V_{m,n} - V_{m-1,n})}{r_m} \right)^2 \\
+ \frac{1}{2} \left( \frac{\delta_{m1}^* (W_{m,n} - W_{m-1,n}) + \left( \frac{dV_{m,n}}{dz} \right)^2}{\Delta \theta} \right) \right\} dz \\
\]

where

\[
F_{m,n} = \frac{E_{m,n} (r_m - \frac{\Delta r_m}{2}) \Delta r_m \Delta \theta_n}{2(1 + \nu_{m,n})}
\]
Then, the principle of minimum potential energy (PMPE) for the composite may be expressed as

\[ \delta \pi_p[U_{m,n}(z), V_{m,n}(z), W_{m,n}(z)] = 0 \]  \hspace{2cm} (13)

with \( W_{m,n}(0) = 0 \) for uncracked elements. If the Lagrange multiplier method is employed, the PMPE may be modified as

\[ \delta \pi_p^*[U_{m,n}(z), V_{m,n}(z), W_{m,n}(z), \lambda_{m,n}] = \delta[\pi_p + \lambda_{m,n} W_{m,n}(0)] = 0 \]  \hspace{2cm} (14)

where \( \lambda_{m,n} \) is the Lagrange multiplier such that \( \lambda_{m,n} = 0 \) for cracked elements.

The constrained displacement boundary condition which manifests itself in the PMPE is not generally included in the modified PMPE. It means that some problems with the displacement boundary conditions which are not easily amenable for solution by the PMPE may now be solved by employing the modified PMPE.

2.2. Numerical Solution

If displacement functions containing unknown parameters are chosen, the unknown parameters are then determined by requiring \( \pi_p \) or \( \pi_p^* \) to be a minimum. The displacement boundary condition is required to be satisfied in advance for the PMPE, but not required for the modified PMPE. In general, if the chosen displacement functions make it easy to satisfy the displacement boundary conditions, the PMPE will be employed, otherwise the modified PMPE will be employed. In this paper, the following set of exponential series are taken for the displacement components and the modified PMPE is employed.

\[ U_{m,n}(z) = \sum_{s=0}^{T} A_{s}^{m,n} e^{-s f(s):/L} \]  \hspace{2cm} (15)

\[ V_{m,n}(z) = \sum_{s=0}^{T} B_{s}^{m,n} e^{-s f(s):/L} \]  \hspace{2cm} (16)

\[ W_{m,n}(z) = \sum_{s=0}^{T} C_{s}^{m,n} e^{-s f(s):/L} \]  \hspace{2cm} (17)

where \( f(s) \) is a decay function, generally determined by trial computations so as to speed up the convergence of solution and \( T \) is the order of the exponential series. The unknown parameters \( A_{s}^{m,n}, B_{s}^{m,n}, C_{s}^{m,n} \) are constants yet to be determined.

Inserting Eqs. (15-17) into energy Eq. (12), completing the integral, performing variation and using the modified PMPE, Eq. (14), a system of linear algebraic equations are obtained. Once this system of equations for the parameters \( A_{s}^{m,n}, B_{s}^{m,n}, C_{s}^{m,n} \) and \( \lambda_{m,n} \) are solved, the complete distributions of the displacement and stress as well as
Lagrange multipliers can be determined. This can then be used in the fracture mechanics formulations such as strain energy release rate.

2.3. Strain energy release rate

An energy analysis of a composite cylinder can be carried out, once the stress and displacement are calculated, and the compliance $C$ of the composite cylinder has been obtained. The Irwin-Kies compliance calibration formulation (Irwin and Kies, 1954) gives a relation between compliance $C$ and strain energy release rate $G$ as follows,

$$ G = \frac{1}{2} P^2 \frac{dC}{dA} $$

(18)

where $P$ is the applied load and $A$ is the area of matrix crack or the interfacial debond. Using Eq. (18) to calculate the strain energy release rate $G$ does not require the stress and displacement fields near the crack tip, since the compliance $C$ depends only on the longitudinal displacement at the edges far away from the crack tip and longitudinal tensile loading which is applied there. Therefore, it is not necessary to specially treat the singular behavior at crack tip and debonding tip. For convenience, the strain energy release rate and the compliance can be normalized as $G = G^* / P^2$, where $G^*$ may be regarded as a strain energy release rate under unit applied load, and $C = C_0 + C^*$, where $C^*$ is the compliance increment, and $C_0$ is the compliance of the undamaged composite. Then, Eq. (18) can be rewritten as

$$ G^* = \frac{1}{2} \frac{dC^*}{dA} $$

(19)

In order to simplify the problem, we assume that if the interfacial debond length $L_0$ held constant while the matrix crack extends circumferentially. With reference to Figure 1, Eq. (19) reduces to

$$ G^*_{\alpha} = \frac{1}{R_f^2 - R_f^2} \left[ \frac{\partial C^*}{\partial \alpha} \right]_{L_0} $$

(20)

where $G^*_{\alpha}$ can be defined as the strain energy release rate under unit applied load when the matrix crack propagates circumferentially around the fiber. Similarly, if the crack angle $\alpha$ held constant while the debond grows, Eq. (19) reduces to

$$ G^*_{d} = \frac{1}{4 \alpha R_f} \left[ \frac{\partial C^*}{\partial L_0} \right]_{\alpha} $$

(21)

where $G^*_{d}$ can be defined as the strain energy release rate at unit applied load when the interfacial debond extends in the fiber direction.
On the basis of the strain energy release rate criterion, the critical load can be related to the strain energy release rate and the toughness of material, as follows

\begin{equation}
    P_{rd} = \sqrt{\frac{G_{dc}}{G^{*}_{d}}}
\end{equation}

\begin{equation}
    P_{r\alpha} = \sqrt{\frac{G_{c}}{G^{*}_{\alpha}}}
\end{equation}

where \( P_{rd} \) and \( P_{r\alpha} \) are the critical loads for the matrix crack extension and the interfacial debond growth, respectively, and \( G_{dc} \) and \( G_{c} \) are the interfacial toughness and the Model I toughness of the matrix material, respectively.

According to the above equations, we can predict the mode of damage propagation, the matrix crack extension in the circumferential direction or the interface debonding in the fiber direction. For the interface debonding to takes place first at an applied load \( P \), we must have \( P_{rd} < P < P_{r\alpha} \), which means

\begin{equation}
    \frac{G_{dc}}{G_{c}} < \frac{G^{*}_{d}}{G^{*}_{\alpha}}
\end{equation}

The above condition is similar to that given by He and Hutchinson (1989), which was used to study the crack deflection at an interface between dissimilar elastic half-planes. Similarly, for the matrix crack to extend first, we must have

\begin{equation}
    \frac{G_{dc}}{G_{c}} > \frac{G^{*}_{d}}{G^{*}_{\alpha}}
\end{equation}

3. Results and discussion

A computer code has been developed to solve the set of linear algebraic equations in \( A_{s,m,n}^{l,n}, B_{s,m,n}^{l,n}, C_{s,m}^{l,n} \) and \( \lambda_{m,n} \). The non-uniform element mesh is employed in numerical computation and meshes are automatically generated by computer in two steps. In the first, the uniform element mesh appears over the entire cross section of the composite cylinder, and in the following step a fine non-uniform element mesh is added near the crack front such that the element size is inversely proportional to the distance of that element from the crack front. The number of element is chosen to be about 250. The following material and geometric properties are used in obtaining the numerical results: the Young’s modulus for the fiber, \( E_{f} = 400 \) GPa, the Young’s modulus for the matrix, \( E_{m} = 100 \) GPa, the radius of the fiber \( R_{f} = 75 \) \( \mu \)m, the Poisson’s ratio for the fiber, \( \nu_{f} = 0.25 \), the Poisson’s ratio for the matrix, \( \nu_{m} = 0.35 \), the radius of the composite cylinder \( R_{c} = 136 \) \( \mu \)m, the length of the cylinder \( 2L = 500 \) \( \mu \)m.

We first study the effect of the order of the exponential series used in Eqs. (15-17) on the convergence of the numerical solution. Figure 4 shows the variation of the compliance
increment $C^*$ as a function of the order $T$ of the exponential series for the case of the crack angle $\alpha = 180^\circ$ and the debond length $L_0 = 37.5 \mu$m. When the order of the exponential series increases, compliance increment $C^*$ approaches a constant value indicating the convergence of the numerical solution. In this numerical analysis, $T$ is taken to be 8 to achieve a satisfactory convergence within a reasonable computational time.

We investigate the effect of the thickness of the interlayer between the fiber and the matrix on the compliance of the composite cylinder for selecting a proper thickness for the interlayer. Figure 5 shows the variation of the compliance increment $C^*$ with the normalized interlayer thickness, $100(t/R_f)$, for the case of the crack angle $\alpha = 180^\circ$ and the debond length $L_0 = 37.5 \mu$m. When the thickness is less than 5%, the compliance increment $C^*$ is almost a constant; when the thickness is more than 5%, the compliance increment $C^*$ increases very slowly. It means that the existence of the interlayer has very little effect on the compliance increments, if we select the thickness of the interlayer less than 5%. For all cases in this paper, the interlayer thickness is taken as 5% $R_f$.

Figures 6 and 7 give the plot of the compliance increment $C^*$ as a function of the crack angle $\alpha$ for various debond lengths $L_d$ and as a function of the debond length $L_0$ for various crack angles $\alpha$, respectively. Using the data shown in Figure 6, the results of the compliance increment $C^*$ against the crack angle $\alpha$ for different debond length $L_d$ are fitted with a polynomial expression. Substituting the above fitted polynomial expression into Eq. (20), the relation between the strain energy release rate for matrix cracking $G^*_\alpha$ and the crack angle $\alpha$ is obtained and then shown graphically in Figure 8. Following the same procedure and using Figure 7 and Eq. (21), the relation between the strain energy release rate for interface debonding $G^*_d$ and the debond length $L_d$ for various crack angles.
Fig. 5. – Effect of interlayer thickness $t/R_f$ on compliance increment $C^*$ for $\alpha = 180^\circ$ and $L_0 = 37.5 \, \mu m$.

Fig. 6. – Compliance increment $C^*$ as a function of crack angle $\alpha$ for a fixed value of debond length $L_0$.

$\alpha$ is obtained and plotted as shown in Figure 9. From Figure 8, the variation of the strain energy release rate for matrix cracking $G_{\alpha}$ with the crack angle $\alpha$ falls into three stages: when the crack angle $\alpha$ is less than $30^\circ$, $G_{\alpha}$ sharply increases; when the crack angle $\alpha$ is more than $30^\circ$ and less than $150^\circ$, $G_{\alpha}$ basically remains constant and when the crack
angle $\alpha$ is more than $150^\circ$, $G_\alpha^*$ again increases sharply to infinity as the crack angle $\alpha$ goes to $180^\circ$. It can be inferred from Eq. (23) that the critical load for the matrix crack extension, $P_{cr}$, decreases or remains constant with the matrix crack propagation,
Fig. 9. – Strain energy release rate for interface debonding $G_f^i$ as a function of debond length $L_0$ for a fixed value of crack angle $\alpha$.

Fig. 10. – Ratio of matrix cracking and interface debonding initiation strain energy release rates as a function of crack angle, $L_0 = 0$. 

EUROPEAN JOURNAL OF MECHANICS. A/SOLIDS. VOL. 17, N° 2, 1998
making the matrix crack propagation unstable or neutrally stable. From Figure 9, it is found that the strain energy release rate for interface debonding $G_d^*$ varies approximately linearly with the denond length $L_0$. For all values of the crack angle less than $180^\circ$, $G_d^*$ decreases as the debond length $L_0$ increases, while $G_d^*$ increasing with the debond length $L_0$ for the crack angle $\alpha = 180^\circ$. The slope of the $G_d^*$ versus $L_0$ plots increases from negative value to positive as the crack angle $\alpha$ increases. According to Eq. (22), it can be inferred that the critical load for the interfacial debond, $P_{cd}$, slightly decreases with the debond length $L_0$. As a result, the interface debond once initiated grows unstably until the interface is debonded entirely.

In order to apply Eqs. (24) and (25) to predict which of the two competing modes occurs first, crack propagation or debond growth for pre-existing damage configurations, the ratio $G_d^*/G_m^*$ has to be calculated. First, we investigate the ratio $G_d^*/G_m^*$ as a function of the crack angle $\alpha$ for the debonding length $L_0 = 0$ as shown in Figure 10. In this case, the ratio $G_d^*/G_m^*$ increases with the crack angle $\alpha$. For a given value of $G_{dc}/G_r$, we can determine the corresponding critical crack angle $\alpha_{cr}$ as illustrated in Figure 10. When the crack angle $\alpha$ is less than the critical crack angle $\alpha_{cr}$, the matrix cracking governs the fracture mode, Eq. (25). This implies that after a pre-existing surface matrix crack extends up to the fiber/matrix interface in the radial direction, the crack continues to extend within the matrix region circumferentially around the fiber. When the crack angle $\alpha$ is greater than the critical crack angle $\alpha_{cr}$, by using Eq. (24), it is inferred that the matrix crack deflects into the fiber/matrix interface causing interface debonding. Figure 11 shows the variation of the ratio $G_d^*/G_m^*$ as a function of the debonding length.

![Graph showing ratio of matrix cracking to interface debonding strain energy release rates for various crack angles and debond lengths.](image-url)

*Fig. 11. – Ratio of matrix cracking and interface debonding strain energy release rates for various crack angles and debond lengths.*
It is found that the ratio $G^*_d/G^*_n$ decreases with the debond length $L_0$ for a fixed crack angle $\alpha$ while increases with the crack angle $\alpha$ less than or equal to $135^\circ$ for a fixed debond length $L_0$. By analyzing the computed results, some interesting observations can be made, particularly with reference to Figure 11. For all crack angles considered, $G^*_d/G^*_n < 0.45$. Therefore, according to Eq. (25), interface debonding would not occur after the crack reaches the fiber, if the interface toughness is such that $G_{dr}/G_r > 0.45$. The matrix crack, therefore, extends around the fiber without initiating interface debond if $G_{dr}/G_r > 0.45$. At $L_0 = 0$, $G^*_d/G^*_n > 0.15$ for all crack angles $\alpha$ considered. Therefore, according to Eq. (24), interface debond always occurs once the matrix crack reaches the fiber/matrix interface, if the interface toughness is such that $G_{dr}/G_r < 0.15$. For intermediate toughness, $0.15 < G^*_d/G^*_n < 0.45$, the fiber/matrix interface debonding would depend on the value of the crack angle $\alpha$ after the matrix crack reaches the interface. The magnitude of $G_{dr}/G_r$ determines a critical crack angle. Based on Eqs. (24) and (25), if the crack angle $\alpha$ is greater than the critical crack angle, interface debond occurs after the matrix crack reaches the fiber/matrix interface; and vice versa. Since for a given crack angle $G^*_d/G^*_n$ decreases as the debond length $L_0$ increases, the interface debonding may cease after growing to certain length, that is, upon satisfying the condition $G^*_d/G^*_n > G_{dr}/G_r$. At this stage, the matrix crack begins to propagate. The value of $G^*_d/G^*_n$, on the other hand, increases with the crack angle $\alpha$ for a fixed debond length $L_0$. For a given crack configuration, if $G^*_d/G^*_n < G_{dr}/G_r$, the matrix


(Manuscript received August 5, 1996; revised May 9, 1997.)