Plane behaviour at high strain rates of a quasi-unidirectional
E-glass/polyester composite: application to ballistic impacts

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Abstract. – Dealing with the plane behaviour of a composite quasi-unidirectional elementary ply made of E glass/polyester, this paper contributes to the study of impact phenomena on ceramic/composite bi-layered targets. Firstly, a quasistatic characterization of the ply in plane tensile and plane shear is carried out. After these tests, an elastoviscoelastic constitutive model with kinematic hardening is derived using the "bi-material" concept. It allows good description of the residual strains, of stiffness losses and of strain rates sensitivities. The second part presents a new fixing method for testing orthotropic materials with the tensile Hopkinson bars apparatus. Then, the elementary ply is tested at high strain rates and these tests are simulated with the constitutive model. Impact simulations incorporating a proposed failure criterion are performed, and then compared with experimental. © Elsevier, Paris

Keywords. – Composite, Dynamic tensile tests, Impact.

Introduction

The aim of this research, conducted at Centre de Recherches et d’Études d’Arcueil (DGA/DRET/ETCA), is the study of the impact of small and medium caliber projectiles on front-faced ceramic/back-faced composite targets ("bi-layered targets"). The kinds of damage in the composite plate of a bi-layered target being the same as those observed in a composite plate impacted without a front-faced ceramic tile (Kammerer, 1996), the plane constitutive relationship of the elementary ply is studied here.

Impacted composite plates usually exhibit two kinds of damage: transverse cracking and delamination (Hitchen and Kemp, 1995). It is possible to consider the composite plate as a stacking of plies and interfaces (Allix et al., 1994): the elementary plies are responsible for the plane constitutive relationship of the composite plate, and consequently for the transverse cracking; the interfaces between the elementary plies are responsible for the out of plane constitutive relationship of the plate, their failure is delamination.

Several studies deal with the constitutive relationship of elementary plies, using different approaches (e.g. Lesne et al., 1994 or Aussedat et al., 1995). But, for our impact applications here, we are interested in the constitutive relationship under high strain rates. The investigated composite is made of quasi-unidirectional E-glass/polyester elementary plies (rate of woof: 5%; specific rate of fibers: 80%).

The first part of our work characterizes, by means of quasistatic tensile tests, the plane tensile and shear behaviours of the elementary ply. Although the tests were carried out under quasistatic loading, the material exhibits strain rates sensitivities according to experimental results at 2 \times 10^{-5}, 2 \times 10^{-4} and 2 \times 10^{-3} \text{s}^{-1}. Thus, it has been possible, in the second part, to introduce the strain rate dependency in our constitutive relationship, before conducting dynamic tests (≈ 200 \text{s}^{-1}).
In order to perform tensile tests with high strain rates, the third part suggests a new fixing method and a new geometry for specimens to test orthotropic materials with the Hopkinson bars apparatus. Since we have already developed a constitutive relationship which takes into account strain rate effects, it seems interesting to test the efficiency of this constitutive relationship to describe the behavior under high strain rates. The comparisons between experimental and numerical data are in good agreement, then we do not need to modify this constitutive relationship to describe the behaviour of the ply under high strain rates.

In the fourth and last part, numerical impact simulations of a spherical projectile against unidirectional stacking composite plates are compared the experiments. Moreover, a simple failure criterion is suggested in this part.

1. Static characterization of the elementary ply

1.1. Specific stress concept

Here, this concept (Neme and Dahan, 1994) is used because it allows an easier examination of uniaxial tensile tests: in our [45°, −45°]_{hs} tensile tests, although the strain gages became unglued at 0.06 shear strain, we observed large shear strain (≃ 0.15) before the failure of the specimen. If we wanted to evaluate the Cauchy stress tensor, we need to record the out of plane strain (across the thickness of the specimen); using the specific stress concept, we avoid this measure.

In fact, when the tensile specimen is supposed to be an \( \vec{L} \) axis cylinder with any shape section, it is only necessary to know both the time evolutions of the applied load and the longitudinal strain to be able to exactly determine the longitudinal specific stress:

\[
\sigma_{LL} = \frac{F(1+m_L)}{A_0 \rho_0}, \quad \varepsilon_{LL} = \ln(1+m_L)
\]

Where \( \sigma = \sigma/\rho \) are the Cauchy specific stresses (J.kg\(^{-1}\)), \( \sigma \) are the Cauchy stresses and \( \rho \) the density. \( m_L \) is the stretching measurement of the longitudinal strain. \( F = F\vec{L} \) is the applied load. \( \rho_0 \) and \( A_0 \) are the initial density and the initial section of the specimen, respectively.

1.2. Experimental approach, behaviour characterization

Let \( \mathcal{R} = \{0, \vec{1}, \vec{2}, \vec{3}\} \) be an orthonormal set linked to the elementary quasi-unidirectional ply, where \( \vec{3} \) is perpendicular to the plane and \( \vec{1} \) is the direction of fibers. Several tests on different stackings have been conducted: \([0°]_n, [90°]_n, [0°, 90°]_{hs} \) and \([45°, −45°]_{hs} \), with different strain rates (2 \( \times 10^{-5} \), 2 \( \times 10^{-4} \) and 2 \( \times 10^{-3} \) s\(^{-1}\)).

1.2.1. \([0°]_n \) and \([90°]_n \) tensile tests

The tensile tests on \([0°]_n \) specimens show a linear elastic behaviour, a brittle rupture and no rate dependance in the direction of fibers. The behaviour in direction \( \vec{2} \) is given by the tensile tests on \([90°]_n \) specimens. It is not linear, strongly strain rate sensitive and exhibits residual strains and losses of specific stiffnesses (Fig. 1). There are also some recovery phenomena (between unloading and reloading, the plastic strains decrease) and hysteresis cycles. These characteristics can be observed in tensile tests on \([0°, 90°]_{hs} \) specimens too (Kammerer, 1996).

The losses in stiffness, in both directions, are non-reversible phenomena and correspond with the increasing number of cracks (an acoustic analysis during tests shows it). The losses in stiffness are coupled with an induced anisotropy: during the \([90°]_n \) tensile tests, for a specific stress greater than a threshold value (about 40 kJ.kg\(^{-1}\)), the non-reversible decreasing of specific stiffness and the growth of plastic strain begin in the
direction $\tilde{z}$. In the direction $\tilde{1}$, fibers are in compression and the matrix is in tension. Therefore some matrix cracks may appear in a plane perpendicular to $\tilde{1}$ (Fig. 2), inducing a loss in stiffness, and creating (positive) plastic strains in this direction.

The plastic strains in direction $\tilde{z}$ may be explained by the fact that some cracks (in a plane perpendicular to $\tilde{z}$) do not close completely after their opening during the last loading-unloading (Fig. 3).
1.2.2. $[45^\circ, -45^\circ]_n$ tensile tests

These tensile tests allow to obtain the shear constitutive behaviour of the ply by using the following equations (it is assumed that the response of plies is symmetric and that the strains are homogeneous within the specimen):

\[
\varepsilon_{12} = \frac{\varepsilon_{LL} - \varepsilon_{TT}}{2} \quad \text{and} \quad \sigma_{12} = \frac{\sigma_{LL}}{2}
\]

The shear constitutive behaviour, as it is presented on figure 4, has the same features as the tensile constitutive behaviour at $90^\circ$: sensitivities to strain rates, plastic strains and losses in constitutive properties.

1.2.3. Discussion

We choose the elastoviscoplasticity concept to represent the plastic strains and the strain rate sensitivities (Lesne et al., 1994). In order to accurately describe the phenomena of recovery and hysteresis cycles, a kinematic hardening is retained. If the evolution of stiffnesses is drawn versus the maximum of specific stress (Fig. 5), a strain rate sensitivity appears for damage. Therefore, a new constitutive model is suggested to represent such sensitivities. This model is based on the "bi-material" concept and differs from the damage law depending explicitly on damage rate (Allix et al., 1994).
2. Identification of the quasistatic behaviour

2.1. Elastoviscoplastic “bi-material” (Kammerer and Neme, 1995)

The aim of this identification is to obtain the analytical description of the behaviour of the elementary ply, in order to introduce it in the F.E. code Abaqus/explicit. Since the latter uses the objective corotational frame, we do not need to search for the intrinsic closed-form expression of the constitutive relationship, which could be very difficult to find.

We name this model “bi-material” model because it considers that the ply is made of two elementary fictitious materials. The first one (exponent (1)) has the behaviour of the virgin ply, the second one (exponent (2)) has the behaviour of the damaged material just before its final breaking. The transformation of material (1) in material (2) is non-reversible. The proportions of these two materials are characterized by an internal vector variable \( z \): \( z_i = 0 \) means that the ply is undamaged in direction \( i \) and \( z_i = 1 \) that it is made up entirely of material (2) in direction \( i \).

This kind of model has been used by Moumni and N’Guyen, 1993, with two elastic materials to simulate the behaviour of materials for which a phase transformation was possible. Here, an elastoviscoplastic, with kinematic hardening, behaviour (Lemaitre and Chaboche, 1985) is chosen for the two materials. The plane Voigt’s notation is used such that, for example, the strain tensor \( \mathbf{\varepsilon} \) and the specific Hooke tensor \( \mathbf{K} \) is defined by:

\[
\mathbf{\varepsilon} = \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{12} \end{bmatrix}, \quad \dot{\mathbf{\varepsilon}} = \begin{bmatrix} \dot{\varepsilon}_{11} \\ \dot{\varepsilon}_{22} \\ \dot{\varepsilon}_{12} \end{bmatrix}, \quad \mathbf{K} = \begin{bmatrix} K_{11} & 0 \\ 0 & K_{22} \end{bmatrix}, \quad \tilde{\mathbf{K}} = \begin{bmatrix} K_{11} & \tilde{K}_{12} \\ \tilde{K}_{12} & K_{22} \end{bmatrix}
\]

For each material (i), let \( \varepsilon^{(i)}_r \), \( \alpha^{(i)}_j \) and \( \Psi^{(i)} \) be the elastic strains, the \( j \)-th internal kinematic hardening variables \( (j \in \{1, \ldots, N\}) \) and the specific free energy, respectively. The latter is assumed equal to \( \mathbf{K}^{(i)} \) and \( \mathbf{C}^{(i)}_j \) are symmetric and positive defined:

\[
2\Psi^{(i)}(\varepsilon_{r}^{(i)}, \alpha^{(i)}_j) = \varepsilon_{r}^{(i)'} K^{(i)} \varepsilon_{r}^{(i)} + \sum_{j=1}^{N} \alpha_j^{(i)'} C_j^{(i)} \alpha_j^{(i)}
\]

Where \( N \), greater than 1, quadratic terms can easily describe a kinematic hardening, which is not linear (Nouailhas, 1989). If \( N = 1 \) the kinematic hardening is linear, if \( N \geq 2 \) it is not.
In the two-dimensional frame, the assumptions of the “bi-material” (Moumni and N’Guyen, 1993) is written as: let, in the ply, the elastic strains ($\varepsilon_r = \varepsilon - \varepsilon_p$, where $\varepsilon_p$ represents the plastic strains), the $N$ kinematic hardening variables $\alpha_j$ and the specific free energy $\Psi (\varepsilon_r, \alpha_j, z)$ be such that (with $j \in \{1, \ldots, N\}$):

\[
\begin{align*}
\varepsilon_r &= (1 - z_2) \varepsilon^{(1)} + z_2 \varepsilon^{(2)} \\
\gamma_{r_{12}} &= (1 - z_6) \gamma^{(1)}_{r_{12}} + z_6 \gamma^{(2)}_{r_{12}} \\
\alpha_j &= (1 - z_2) \alpha_j^{(1)} + z_2 \alpha_j^{(2)} \\
\alpha_{j_{12}} &= (1 - z_6) \alpha_j^{(1)} + z_6 \alpha_j^{(2)}
\end{align*}
\]

\[
2 \Psi (\varepsilon_r, \alpha_j, z) = (1 - z_2) \varepsilon_r^{(1)\prime} \hat{K}^{(1)}_{\varepsilon_r} \varepsilon_r^{(1)} + z_2 \varepsilon_r^{(2)\prime} \hat{K}^{(2)}_{\varepsilon_r} \varepsilon_r^{(2)}
\]

\[
+ (1 - z_6) K^{(1)}_{66} \gamma_{r_{12}}^{(1)} + z_6 K^{(2)}_{66} \gamma_{r_{12}}^{(2)}
\]

\[
+ \sum_{j=1}^{N} [(1 - z_2) \hat{C}_j^{(1)} \alpha_j^{(1)} + z_2 \hat{C}_j^{(2)} \alpha_j^{(2)}]
\]

By using the Lagrange’s multipliers method, the specific free energy, function of the new internal variables, is obtained:

\[
2 \Psi (\varepsilon_r, \alpha_j, z) = \varepsilon_r' K_z \varepsilon_r + \sum_{j=1}^{N} \alpha_j' C_z \alpha_j
\]

where $K_z$ and $C_z$ are defined by:

\[
\begin{align*}
\hat{K}_z^{-1} &= (1 - z_2) \hat{K}^{(1)}_{\varepsilon_r}^{-1} + z_2 \hat{K}^{(2)}_{\varepsilon_r}^{-1} \\
1/K_{66}^{(1)} &= (1 - z_6)/K_{66}^{(11)} + z_6/K_{66}^{(12)} \\
1/C_{66}^{(1)} &= (1 - z_6)/C_{66}^{(11)} + z_6/C_{66}^{(12)}
\end{align*}
\]

The specific stresses, the thermodynamic variable $X_j$ associated to $\alpha_j$ and the thermodynamic variable $Z$ associated to $z$, are:

\[
\sigma = \frac{\nabla (\Psi)}{\varepsilon_r} = K_z \varepsilon_r, \quad X_j = \frac{\nabla (\Psi)}{\alpha_j} = C_z, \quad j \in \{1, \ldots, N\}; \quad Z = \frac{\nabla (\Psi)}{\alpha_j}
\]

**Kinematic hardening evolution**: A dissipation potential as the sum of two power functions is chosen (Lemaitre and Chaboche, 1985):

\[
\Omega_{\alpha} (\sigma, X_j, \alpha_j, z)
\]

\[
\begin{align*}
\varepsilon_r(n_2) &= \frac{k_2^{-n_2}}{n_2 + 1} \left( H (\varepsilon_{p_{22}}) \left| \sigma - \sum_{j=1}^{N} \hat{X}_j \right| - R_{0_2} + \frac{1}{2} \sum_{j=1}^{N} \left( X_{j_{12}} - \alpha_j' \hat{C}_z \hat{X}_j \right)^2 \right) \varepsilon_r^{n_2 + 1} \\
+ \frac{k_6^{-n_6}}{n_6 + 1} \left( H (\sigma_{12}) \left| \sigma - \sum_{j=1}^{N} X_{j_{12}} \right| - R_{0_6} + \frac{1}{2} \sum_{j=1}^{N} G_j \left( X_{j_{12}}^2 - C_j \hat{X}_{j_{12}}^2 \right) \right)^{n_6 + 1}
\end{align*}
\]

$H$ is the Heaviside function: $H(x) = x$ if $x \geq 0$, and $H(x) = 0$ if $x < 0$.

A, symmetric and positive, can be a function of damage $z_2$ to describe a possible induced anisotropy (here $A = (1 - z_2) A^{(1)} + z_2 A^{(2)}$, where $A^{(i)}$ are constant). $k_i$, $n_i$ and $R_0$, are positive reals, if necessary functions
of \( z_j \). \( G_j \) are symmetric and positive. \( \langle x \rangle \) is the “positive part of \( x \)”. \( \|U\|_A = (U' A U)^{1/2} \); this notation can describe the anisotropic evolution of viscoplasticity, as in Nouailhas, 1989.

By differentiation of the dissipative potential versus \( \varepsilon \) and \( X_j \), the evolution laws are obtained:

\[
\dot{\varepsilon}_p = \text{grad} (\Omega_p) = \frac{1}{k_p^2} \left( H(\varepsilon_{p,22}) \|\tilde{\sigma} - \tilde{X}\|_A - R_{0,2} \right)'' \frac{H(\varepsilon_{p,22}) A(\tilde{\sigma} - \tilde{X})}{\|\tilde{\sigma} - \tilde{X}\|_A} = \tilde{p}_2 \frac{H(\varepsilon_{p,22}) A(\tilde{\sigma} - \tilde{X})}{\|\tilde{\sigma} - \tilde{X}\|_A} ; \quad \text{where} \quad \tilde{X} = \sum_{j=1}^{N} \tilde{X}_j
\]

\[
\dot{\alpha}_{p,22} = \text{grad} (\Omega_p) = \frac{1}{k_p^6} \left( \|\tilde{\sigma}_{12} - \tilde{X}_{12}\| - R_{0,6} \right)^{n_6} \text{sign} (\tilde{\sigma}_{12} - \tilde{X}_{12}) = \tilde{p}_6 \text{sign} (\tilde{\sigma}_{12} - \tilde{X}_{12})
\]

\[
\dot{\tilde{X}}_j = -\text{grad} (\Omega_p) = \dot{\varepsilon}_p - \tilde{p}_2 G_j \tilde{X}_j ; \quad \alpha_{j,22} = -\text{grad} (\Omega_p) = \dot{\alpha}_{p,22} - \tilde{p}_6 G_{j,6} \tilde{X}_{12}
\]

These equations show that the plastic strain \( \varepsilon_{p,22} \) cannot be negative. Then, this model is able to take into account the crack closure in compression loading: when it occurs, the cracks close and we suppose that the elementary ply recovers its elastic behaviour (material 1).

Moreover, the second term of the value of each variable \( \alpha_j \) can describe the non-linearities of the evolution of the kinematic hardening (if \( \forall j, G_j = 0 \), the description is linear; it is possible to prove that \( \forall j, G_j = 0 \) is equivalent to \( N = 1 \)).

**Damage evolution**: This evolution is described by an homographic function of power of damage.

Then, let be the two threshold surfaces (for simplifications, even if it does not precisely reflect the reality, the evolutions of \( z_2 \) and \( z_6 \) are uncoupled):

\[
F_k(Z_k, z_k) = f_k((\tilde{Z}_k)) - z_k
\]

where \( f_k : \mathbb{R}^+ \to [0, 1/\tilde{x} \mapsto f_k(\tilde{x}) = \frac{\tilde{x}^{m_k}}{\tilde{x}^{n_k} + Z_k} \), \( k \in \{2, 6\}; (m_k, Z_{1,k}) \in \mathbb{R}^+ \). This expression is chosen because: \( f_k(0) = 0 \), \( f_k'(0) = 0 \) and \( \lim_{x \to \infty} f_k(x) = 1 \). Then the damage variable cannot have negative values or values superior to one.

Like for incremental plasticity: \( \dot{z}_k = f_k'(((\tilde{Z}_k))) (-\tilde{Z}_k) H(F_k) \) with \( k \in \{2, 6\} \).

**Positivity of dissipation**: This can be checked, according to these expressions:

\[
\Phi^p = \Phi_p + \Phi^z \quad \text{where} \quad \Phi^z = -Z^T \dot{Z} = \sum_{k=2,6} -Z_k f_k'((\tilde{Z}_k)) (-\tilde{Z}_k) H(F_k) > 0
\]

\[
\Phi_p = \tilde{p}_2 H(\varepsilon_{p,22}) \|\tilde{\sigma} - \tilde{X}\|_A + \sum_{j=1}^{N} \tilde{p}_2 \tilde{X}_j^T G_j \tilde{X}_j + \tilde{p}_6 \|\tilde{\sigma}_{12} - \tilde{X}_{12}\| + \sum_{j=1}^{N} \tilde{p}_6 G_{j,6} \tilde{X}_{12}^2 \geq 0
\]

### 2.2. Numeric simulation. Identification of ply parameters. Discussion

After the inversion of the constitutive relationship, meaning that we determine the specific stress increment versus the strain increment and the thermodynamic variables increment, its explicit Euler integration is computed. Then, with a minimization algorithm, the constitutive relationship parameters of the elementary ply are identified.
Table 1. Ply parameters: results of minimizations.

<table>
<thead>
<tr>
<th></th>
<th>11</th>
<th>12</th>
<th>22</th>
<th>66</th>
<th>Units</th>
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<td>$K_{11}$</td>
<td>1/21.7</td>
<td>-1/110</td>
<td>1/111.5</td>
<td>1/13.9</td>
<td>1/MI.kg$^{-1}$</td>
</tr>
<tr>
<td>$K_{22}$</td>
<td>1/21.7</td>
<td>-1/170</td>
<td>1/13.98</td>
<td>1/13.9</td>
<td>1/MI.kg$^{-1}$</td>
</tr>
<tr>
<td>$C_{44}^{[1]}$</td>
<td>1250</td>
<td>0</td>
<td>96490</td>
<td>5116</td>
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</tr>
<tr>
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<td>7308</td>
<td>171</td>
<td>kJ.kg$^{-1}$</td>
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<tr>
<td>$G_{44}^{[1]}$</td>
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<td>0</td>
<td>101500</td>
<td>1815</td>
<td>kJ.kg$^{-1}$</td>
</tr>
<tr>
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<td>0</td>
<td>7691</td>
<td>1203</td>
<td>kJ.kg$^{-1}$</td>
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<td>0.2</td>
<td>0.045</td>
<td>0.293</td>
<td>$z_2$</td>
<td>kg.kJ$^{-1}$</td>
</tr>
</tbody>
</table>

$A = 382$ kJ.kg$^{-1}$.s$^{1/2}/z_2$

$A_2 = 3.512; R_{0,2} = 0$

$m_2 = 4.71; Z_{12} = 232.54$.kg$^{-1}$

$u_0 = 3.9; Z_{1,0} = 542.3$.kg$^{-1}$

Fig. 6. Quasistatic tensile tests on $[90]^0_4$ specimens: experiments and simulations.
by means of an optimization of the differences between experiments and simulations (Kammerer, 1996); thanks to the calculation of specific stresses resulting from experimental strains and thanks to the calculation of strains from experimental specific stresses, an error function is evaluated and minimized. The retained experiments are quasistatic tensile tests on both \( [90^\circ]_n \) and \( [45^\circ, -45^\circ]_n S \).

For an easier identification, we choose \( N = 2 \). The optimized parameters (Tab. 1) allow to simulate the plane tensile (Fig. 6) and plane shear behaviours (Fig. 7) of the ply. The following remarks comparing experimental and simulated curves can be made:

- there is a good agreement between experiments and simulations,
- the strain rate sensitivities are as well represented in the specific stress-strains curves as in the losses of specific stiffness curves (Fig. 8),
- the damage induced anisotropy is correctly represented by the constitutive model in the tensile \( [90^\circ]_n \) tests,
- as regards the plastic strains and recovery phenomena, in tensile tests on both \( [90^\circ]_n \) and \( [45^\circ, -45^\circ]_n S \) specimens, numerical and experimental results are of the same order of magnitude.

2.3. INTERMEDIATE CONCLUSION

From quasistatic tensile tests with different strain rates, a constitutive model for the elementary ply has been written to simulate its behaviour. This constitutive model is based on the concept of “bi-material”. It enables to correctly represent the plastic strains, the properties losses, the damage induced anisotropy and to take into account the strain rate sensitivities in plane tension and plane shear. But it does not describe the failure of the elementary ply.
It is also necessary to complete this constitutive relationship to take into account the plane compressive behaviour: then, we assumed that, when the elastic strains become negative, the cracks are closed and the elementary ply recovers its elastic characteristics (material 1).

3. Tensile Hopkinson bars apparatus: tests and simulations

The Hopkinson bars apparatus allows the study of behaviour of materials at high strain rates. This apparatus is particularly used in dynamic compression: it is easy to put a cylindrical specimen between the two parallel plane surfaces of the incident and transmitted bars (Lichtenberger, 1987). Unfortunately, in dynamic tensile loadings, the specimen must be held on the bars. In the case of isotropic materials, the most common configuration is a simple screwing (Harding et al., 1960, 1983) or the bonding of the specimen onto the bars (Lataillade et al., 1996). In the case of orthotropic materials, this screwing configuration cannot be used, and consequently the main difficulties lie in method used to fix the specimen. Moreover, we do not want to use a third material, as a bond: its constitutive behaviour will generally not be well known.

3.1. New design for the fixing method of orthotropic specimens

In order to carry out tensile tests at high strain rates with the Hopkinson bars apparatus, a new fixing method suitable for orthotropic materials has been designed. It (Fig. 9) has several advantages: it requires neither a third material, the behaviour of which might not be fully documented, nor any intermediate part between bars and sample, and thus the numerical simulation is quite easy. The specimen has a hourglass shape (Fig. 10) and it is put between the ends of incident and transmitted bars (Fig. 11). The plane contact surfaces, between the specimen and the bars, must be, as far as possible, the largest and the most perpendicular to the axis of both bars.

This geometry of fixing has the advantage of being mere (with the minimum of parts, and consequently with the minimum of contacts) and therefore, it enables numerical simulations, with the finite elements method, of the set [incident bar, specimen, transmitted bar].

The incident bar has an anvil at its opposite end. The incident tensile wave is created by launching a cylindrical hollow projectile against this anvil.

![Fig. 9. – Ends of Hopkinson bars.](image)

![Fig. 10. – Dynamic tensile specimens: two different geometries (mm).](image)
3.2. Dynamic tensile tests on E-glass/polyester: experiments and simulations

In this configuration, and because the measured strains are low (compared with those measured in metals), every slack between the ends of bars and the specimen, and every defect of alignment between the plane of specimen and the axis of the bars lead to errors because the simulation is carried out with the assumption of perfect contacts. A pre-tension system (Fig. 11) and wedges have been used to improve the measures. Dynamic tensile tests on $[0^\circ]_n$, $[90^\circ]_n$, $[0^\circ]$, $[90^\circ]_{ns}$ and $[45^\circ]$, $[-45^\circ]_{ns}$ specimens have been conducted in this way.

Several geometries of the center part of specimens (Fig. 10) have been tested in order to localize the fracture within the center part of $[90^\circ]_n$ specimens: for example a constant section (Type I) or a reduced section (Type II). The second geometry ensures that the rupture is localized in the center of the sample, where the gauge has been stuck, whereas in the first case, the rupture occurs near the transmitted shoulder (Fig. 12).

Six strain gauges have been stuck on the apparatus to simulate these tests, and then to compare the experiments with the simulations: the first on the incident bar, four on the specimen ($J_{nil}$ and $J_{nil}^R$ are located in the center of the specimen, one on each face, $J_{nil}$ and $J_{il}$, symmetrically to $J_{nil}$, as presented on figures 10 and 11), the last one is located on the transmitted bar.

The finite element code ABAQUS 5.4 code (Hibbit, Karlsson and Sorensen), with implicit integration has been used to realize dynamic simulation of these tests. The mesh, on figure 13, represents the set [incident bar, specimen, transmitted bar]. We use semi-infinite elements which can absorb every reflection wave and avoid the meshing of the whole bars. The specimen is meshed with plane stress elements.

The constitutive relationship has been written in a “UMAT” subroutine. It means that the user, with the ABAQUS code, can define his own material. In this subroutine, the user must compute the stresses, the internal variables and the Jacobian matrix at the end of each time increment. This subroutine evaluates the stress increment versus the strain increment and the thermodynamic variables increment: an explicit Euler integration is computed.
The measure of the load is provided by the incident strains recorded by the incident gauge; the experiment/simulation comparison is achieved by looking at the differences between experimental strains measured on the specimen gauges and computed strains obtained on the meshed specimen, as presented in Figures 14, 15 and 16 (the specimens have not broken during these recordings).

A few remarks can be made:

- In direction $\vec{2}$, there is a good agreement between experiment and simulation. Moreover, several computations show that, at high strain rates, the viscoplastic characteristic of the elementary ply vanishes out, and only the stiffness losses are important (Kammerer, 1996).

- In direction $\vec{1}$, the behaviour is essentially elastic. At high strain rates, it has not been possible to test it as far as the rupture does not occur in the center of specimens (Fig. 12): the shoulders break by shearing. In fact, in this configuration, we study the plane shear behaviour of the specimen in the shoulders.

- The dynamic tensile tests on $[45^\circ, -45^\circ]_N$ specimens have allowed to test the elementary ply in plane shear and bi-tension. Although we have simulated the slack between the specimen and the ends of bars, there is still a difference between the simulation and the experiment (Fig. 16). In our opinion, the quasistatic tests

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**Fig. 13.** – Mesh of the set \{incident bar, specimen, transmitted bar\}.

**Fig. 14.** – Dynamic tensile test on $[90^\circ]_N$ specimen: experiment and simulation.

**Fig. 15.** – Dynamic tensile test on $[0^\circ]_N$ specimen: experiment and simulation.
should be performed on a larger range of strain and strain rates, so that the constitutive relationship can be identified more accurately.

- Figure 17 presents three dynamic simulations of an $[90^\circ]_n$ tensile specimen: a simulation with the elastoviscoplastic "bi-material" constitutive model (see first part), a simulation with an elastic "bi-material" constitutive model and a simulation with the elastic constitutive model. Comments on these results can be made:

The description of the stiffness losses is important and must be done if we want a good agreement between experiments and simulations. Moreover, these results show that the viscoplastic parts of the constitutive relationship vanishes at high strain rates in direction $\overrightarrow{\gamma}$.

4. Impact tests and simulations

4.1. Failure criterion

In each direction, the plane tensile breaking is brittle. Because of the quasi-unidirectionality of the elementary ply (rate of woof: 5%), it is possible to assume that the ultimate tensile load is controlled by the fibers breaking. This assumption is reinforced by some values of maximum strains: at low strain rates, they are equivalent (Tab. II). But there is a sensitivity to strain rate as it is shown in the two last rows of Table II.

Figure 18 shows the maximum strains versus the logarithm of strain rate.

A linear statement linking the maximum strain with the logarithm of strain rate can be written (dashed line on Figure 18). Then, a tensile rupture criterion follows as:

"If $2 \times 10^{-3} \leq \dot{\varepsilon}_i \leq 200 \, s^{-1}$ ($i = 1$ or $2$) and if $\frac{\dot{\varepsilon}_i}{\varepsilon_1 + \chi \ln(\dot{\varepsilon}_i)} \geq 1$, a tensile rupture occurs in direction $\overrightarrow{\gamma}$ ($\varepsilon_1 = 0.233; \chi = 5.144 \times 10^{-3}$)."
4.2. Impact tests

A gas gun has been used to launch steel spherical 11 mm diameter projectiles against unidirectional stacking E-glass/polyester composite plates \((200 \times 200 \times 3 \text{ mm})\). This stacking has been chosen to have the least important delamination (Beaumont, 1990, and Chaillou, 1991). The incident and residual velocities of the projectile are measured, they are shown on Figure 19. The ballistic limit of this plate is about 180 m.s\(^{-1}\).

4.3. Impact simulations

The finite element code Abaqus, with an explicit integration, has been used to simulate these tests. A subroutine “Vumat” has been written in order to describe the constitutive relationship of the elementary ply and
its tensile rupture. Both the mesh of the projectile and the plate is shown in Figure 20. The projectile is a rigid body and the plate is meshed of shell elements with five integration points in their thickness. When the rupture criterion is reached, the Abaqus code prescribes out the values of stresses equal to zero.

Then, some elements can be "destroyed" under the projectile. The velocity of projectile is recorded during its interaction with the composite plate. The residual velocity, after the interaction, is given by the stabilized velocity. The comparison between experiments and simulations is done considering the difference between experimental and simulated residual velocities.

![Fig. 20. - Impact simulations: projectile and composite plate (quarter).](image)

Figure 21 shows four simulations (with the same tensile rupture criterion) using:

- (A): the elastoviscoplastic "bi-material" constitutive model;
- (B): an elastoviscoplastic "bi-material" tension and elastic shearing \( R_{0_z} \to +\infty \);
- (C): an elastic "bi-material" tension and elastic shearing \( R_{0_z} = R_6 \to +\infty \);
- (D): an elastic constitutive model \( K^{(1)} = K^{(2)} \), \( R_{0_z} = R_0 \), \( R_{0_z} \to +\infty \).

![Fig. 21. - Impact tests and simulations: incident and residual velocities.](image)

Several observations can be made about these results. At high incident velocities, the constitutive model discussed in the first part of the paper and the tensile rupture criterion provide a good prediction of the experimental residual velocities. But, for incident velocities near the ballistic limit, there are still some differences. According to the conclusions of Beaumont (1990), they may be due to the coming out of important delamination in the composite plate, even if unidirectional stackings reduce it. When the incident velocity increases, delamination decreases and the simulation becomes more accurate.

There is also a great difference between the ballistic limits obtained by simulations (A) and (B): it shows that the description of shear behaviour is important. Lastly, the simulations (D) justify the choice of a quite elaborate constitutive relationship of the elementary ply.
It seems that for the highest striking velocity value (311 m/s), the simulated value of $\dot{\varepsilon}_{22}$ is decreasing from nearly 2000 s$^{-1}$ at the center of the plate to nearly zero at the boundaries. As a matter of fact, the strain rate is locally greater than 200 s$^{-1}$ which is the value obtained in the Hopkinson tensile tests. Fortunately, the results Figure 21 are good. It could be explained by the fact that when the strain rate is large enough, the behaviour of the material can simply be described with the elastic “bi-material” constitutive model (see the end of part 3).

Nevertheless, it should be pointed out that the true goal of a model is to predict the behaviour of a material under the solicitations which have not been experimentally tested. In fact, if we want the model to be perfectly correct whatever the components of strain tensor, strain rate tensor and the internal variables may be, the model as such ceases to exist and is replaced by a huge experimental data set.

5. Conclusion. Prospects

A plane constitutive relationship for quasi-unidirectional composite has been written to take into account the sensitivities to strain rate of both the stiffnesses losses and the plastic strains. This constitutive relationship, based on the “bi-material” concept, has been identified at low strain rates (quasistatic loadings) and has been tested at high strain rates. To carry out the tensile Hopkinson tests, a new method to fix the specimen at the ends of the bars has been proposed. Then, after retaining a tensile rupture criterion based on maximum strains, impact tests has been performed on a unidirectional stacking composite plate. The simulation of these tests has provided good results.

It is now possible to identify the behaviour of another quasi-unidirectional organic matrix composite, different from $E$-glass/polyester, with the same elastoviscoplastic “bi-material” constitutive relationship.

The ongoing research bears on the study of the interface between two elementary plies, in a non-unidirectional stacking composite plate in order to describe the impact response of such structures thoroughly.

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