Numerical modelling of the powder compaction of a cup

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Abstract – The porosity and stress distributions in an axisymmetric cup have been analysed numerically. The finite element programme is based on rate independent, finite strain plasticity theory, and two different porous material models have been implemented. The first material model is a combination of the Gurson model, relevant to rather low porosities, and the Fleck–Kuhn–McMeeking (FKM) model, developed for highly porous powder compacts. The second material model accounts for the possibility of reduced inter-particle cohesion. Friction between tool and work-piece as well as the compaction method have both a significant influence on the porosity distribution in the compacted cup. The inter-particle cohesive strength plays only a minor role in the porosity distribution, but is more important in both the distribution and the level of the von Mises and the hydrostatic stresses.

1. Introduction

In the powder compaction process, it is possible to make near net final shape components of complex geometries and also components of high strength. The production route of sintered components is roughly described in two steps: the compaction of the powder and the subsequent heat treatment, called sintering, which is necessary to bond the powder particles together. In the present study, focus is on the first step, the compaction process. The compaction of metal powders at absolute temperatures lower than 0.3 times the melting temperature of the powder material is considered. Here, plasticity is the dominant deformation process whereas at higher temperatures creep and diffusion play an important role.

In the beginning of the powder compaction process, the porous aggregate typically consists of loose powder particles. Later on in the process the powder particles have deformed, and porosity now exists in the form of isolated voids. Fleck, Kuhn and McMeeking (1992a) suggested a material model for powder compaction as a combination of two porous material models: a micromechanically based particle model proposed by Fleck et al. (1992a) (the FKM model) at high porosities; and a micromechanically based void shrinkage material model, the Gurson model (1977), at low porosities. In the transition range, a linear combination of the two models is used. This combined material model has earlier been used to study the effects of friction, the compaction method, and choice of material parameters on the compaction of cutting tool powders, Redanz (1998). This combined material model as well as the FKM and Gurson models as individual material models have been used to analyse failure in sintered metals under tensile loading (Redanz et al., 1998; Redanz and Tvergaard, 1999). The combined material model is used here to study the effects of the compaction method and friction between the tool and the workpiece on the development of microstructure in a cup.

Most of the porous material yield functions in the literature do not take variable inter-particle cohesion into account. It is either assumed that no inter-particle cohesion is present, and the porous material has no strength in hydrostatic tension (e.g. the Cam–Clay model (Schofield and Wroth, 1968) and the Drucker–Prager material model (1952)), or fully sticking contacts are assumed and the material has as much strength in hydrostatic tension as in hydrostatic compression (e.g. Gurson, 1977; Shima and Oyane, 1976; Fleck, Kuhn
and McMeeking, 1992a). The first group of material models are often used in the study of granular solids, such as soils, and the models in the latter group are used in the study of ductile fracture or powder compaction. However, Fleck (1995) introduced a yield function with variable inter-particle cohesion and a modified version of his yield surface is used in the present study to analyse the effect of inter-particle cohesion on the compaction of a cup. The geometry of the cup is shown in figure 1.

2. Yield functions for porous materials

For porous materials, both the deviatoric and hydrostatic components of stress cause yielding. Hence, the yield criterion for an isotropic material must depend on both the second invariant of the deviatoric stress tensor and the first invariant of the stress tensor. Several yield functions of that form have been developed in the past. Shima and Oyane (1976) suggested a yield criterion based on compaction studies for sintered copper at various apparent densities. Other yield functions of similar form are given in, e.g. (Drucker and Prager, 1952; Green, 1972) and (Doraivelu et al., 1984). The yield condition for a porous material is dependent on the porosity or the void volume fraction. In the present work, the porosity is assumed to be represented by a scalar, $f$, which gives a yield surface of the form $\Phi(\sigma''', \sigma_M, f) = 0$, where $\sigma_M$ is the current yield stress of the matrix material. A fully dense material is obtained with $f = 0$.

2.1. The combined material model

Fleck, Kuhn and McMeeking (1992a) suggested a material model as a combination of two material models based on different micromechanical morphologies.

In the beginning of the powder compaction process at high porosities, the porous aggregate consists of particles in point contact. Fleck, Kuhn and McMeeking (1992a) developed a yield function by representing the initial geometry as a dense random packing of spherical particles and assuming that the contact patches which form between the deforming particles do not interact with each other

$$\Phi_{FKM} = \left( \frac{5}{18} \frac{\sigma_c}{p_y} + \frac{2}{3} \right)^2 + \left[ \frac{\sqrt{5}}{3} \frac{\sigma_M}{p_y} \right]^2 - 1 = 0 \quad (1)$$

Figure 1. The geometry of the cup studied.
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with \( \sigma_e = \sqrt{3s_{ij}s^{ij}/2} \) representing the von Mises stress, where \( s^{ij} = \sigma^{ij} - G^{ij}\sigma^k_k/3 \) is the deviatoric stress tensor, and \( p_y \) is the yield strength of the porous material under hydrostatic loading

\[
p_y = 2.97(1 - f)^2 \left( \frac{p_y}{f} - \frac{f}{f} \right) \sigma_M.
\]

(2)

The model is applicable at high porosities only, up to the limit of dense random packing of equi-sized spheres, \( f = \tilde{f} = 0.36 \). At porosities below about 0.25, the contacts start to interact and the particles become less and less spherical in shape. The yield surface in (1) is referred to as the FKM model and is shown in the deviatoric–hydrostatic space in figure 2a. The yield surface for the full micromechanical model contains a vertex on the hydrostatic stress axis in \((\sigma_e, -\sigma^k_k/3)\) space. For numerical computation purposes, the vertex is rounded off by a quadratic approximation to the yield surface near the vertex, as in Fleck, Otoyo and Needleman (1992b).

Later in the compaction process, the porosity exists in form of isolated voids. Here, the micromechanical basis for the Gurson model (1977) is suitable. The appropriate yield condition is then

\[
\Phi_G = \frac{\sigma^2}{\sigma_M^2} + 2q_1f \cosh \left\{ \frac{q_2 \sigma^k_k}{2 \sigma_M^2} \right\} - (1 + (q_1f)^2) = 0.
\]

(3)

The constants \( q_1 \) and \( q_2 \) were introduced in Tvergaard (1981, 1982) to bring the predictions of the model into closer agreement with full numerical analyses for periodic arrays of voids. The Gurson yield surface is compared with the FKM model at various porosities in figure 2a.

The FKM model (1) is used at porosities higher than \( f_1 \), and at porosities lower than \( f_2 \), the Gurson model (3) is used \((f_1 > f_2)\). In the transition range, \( f_1 > f > f_2 \), a linear combination of the two models is used

\[
\Phi_{\text{comb}} = W_{\text{FKM}}\Phi_{\text{FKM}} + W_{\text{G}}\Phi_{\text{G}} = 0
\]

(4)
Figure 3. Yield surfaces at different degrees of inter-particle cohesion. Both the yield surface suggested by Fleck (1995) and the modified yield surface are shown.

with the weight functions $W_{FKM} = (f - f_2)/(f_1 - f_2)$ and $W_G = (f_1 - f)/(f_1 - f_2)$. The choice of $f_1 = 0.25$ and $f_2 = 0.10$ is made as in (Fleck, Kuhn and McMeeking, 1992a).

Because of the different micromechanical bases in the combined material model, this model is suitable across the whole range of porosities during the powder compaction process. The yield functions at various porosities are shown in figure 2b for the combined model.

2.2. Material model with reduced inter-particle cohesion

The micromechanical basis for the FKM model discussed in the previous section is that the particles are bonded by fully sticking contacts. However, this is normally not the case for powders used in powder compacted components. Either there is no cohesive strength between the powder particles or the powder is mixed with a binder to bond the particles together in order to obtain some degree of inter-particle cohesion. Thus, the green compact produced from a powder consisting of initially spherical particles breaks very easily when it is taken out of the mould before the subsequent heat treatment. A binder may be necessary to make the process possible.

Fleck (1995) introduced the cohesion factor $\eta$ for which fully sticking contacts are represented when $\eta = 1$ and no cohesive strength is present when $\eta = 0$. He suggested a yield criterion with the inter-particle cohesion factor $\eta$ as a parameter. This yield criterion is shown for the special case $\eta = 0$ in figure 3. Here, the axes are normalized by the yield strength of the porous material in hydrostatic tension, $p_y$, (2). Thus, the curves are independent of porosity and not directly comparable with the yield surfaces shown in figure 2. The determination of the yield surface (Fleck, 1995) is based on an axisymmetric stress situation, where negative values of the true stress are possible. For positive true stresses the yield surface ‘tilts’ to the left as shown in figure 3 and for negative true stresses it tilts to the right.

However, Akisanya et al. (1997) showed experimentally for copper powder that the yield surface does not tilt and they also found that the vertices on the hydrostatic axis are more rounded than those on the yield surface suggested by Fleck (1995). Therefore, in the present work, a modified expression for Fleck’s yield surface is
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suggested as

$$\Phi = \left( \frac{5}{18} \frac{\sigma_x}{p_y} + \frac{2}{3} \right)^2 + \left[ \frac{\sqrt{\frac{2}{3}} \sigma_k^k}{p_y} \right]^2 + \frac{5}{9} (1 - \eta) \left[ 1 + \frac{\frac{1}{3} \sigma_k^k}{p_y} - \frac{1}{3} \frac{\sigma_x}{p_y} \right] - 1 = 0$$

(which reduces to the original FKM model (1) for fully sticking contacts, $\eta = 1$. The modified yield surface is shown in figure 3 for different values of $\eta$. It should be noted here, that the modified yield criterion is only valid at high porosities since the combination with the Gurson model is no longer possible.

In the present study, it is assumed that the cohesive strength does not change during the compaction process, $\eta = 0$. In practice, the areas of contact weld together during the process and true metallurgical bonds are formed (Dowson, 1990), which would suggest some increase of $\eta$ during the process. However, as discussed above, these bonds are not always strong enough for the handling of the green compact, and then a binder is needed. The determination of a possible nonzero $\eta$ is left for further study.

Fleck (1995) also found that the shape of the yield surface is only slightly sensitive to the degree of inter-particle friction.

3. Basic equations

The finite strain formulations are based on a convected coordinate Lagrangian formulation of the field equations, in which $g_{ij}$ and $G_{ij}$ are the metric tensors in the reference configuration and the current configuration, respectively, with determinants $g$ and $G$. The initial state is taken as the reference configuration. The Lagrangian strain tensor is $\eta_{ij} = \frac{1}{2} (G_{ij} - g_{ij})$ or expressed in terms of the displacement components

$$\eta_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i} + u_k^k u_{k,j})$$

with $u^i$ being the displacement components on the reference base vectors and $( )_{;i}$ denoting the covariant derivatives in the reference frame. The contravariant components of the Cauchy stress tensor, $\sigma^{ij}$, and the Kirchhoff stress tensor, $\tau^{ij}$, are related by the expression $\tau^{ij} = \sqrt{G} g^{ij}$ where $\sqrt{G_g}$ denotes the deformed volume per unit undeformed volume.

The elastic contribution to the change in porosity is neglected since plastic deformations are dominant compared to the elastic deformations. The matrix material is plastically incompressible, thus, the change in porosity is

$$\dot{f} = (1 - f) G^{ij} \dot{\eta}_{ij}^p.$$  

Here, $\dot{()}$ denotes differentiation with respect to a loading parameter.

Normality for the porous aggregate is assumed so that the plastic part of the total strain increment is expressed by

$$\dot{\eta}_{ij}^p = \Lambda \frac{\partial \Phi}{\partial \sigma^{ij}},$$

where $\Lambda$ is the plastic multiplier.

By setting the macroscopic plastic work rate equal to the plastic work rate in the matrix material

$$\sigma^{ij} \dot{\eta}_{ij}^p = F(f) \sigma_M \dot{\varepsilon}_M^p$$
and using the incremental relation between the effective plastic strain in the matrix, \( \varepsilon_p^p \), and the equivalent tensile yield strength of the matrix material, \( \sigma_M \), with \( \dot{\varepsilon}_p^p = (1/E_t - 1/E)\dot{\sigma}_M \), an expression for the increment of the tensile equivalent yield stress in the matrix material is obtained

\[
\dot{\sigma}_M = \frac{E E_t}{E - E_t F(f)\sigma_M} \sigma^{ij} \dot{\varepsilon}_p^{ij}.
\]

where \( E \) is the Young's modulus and \( E_t \) is the slope of the uniaxial true stress-logarithmic strain curve for the matrix material. In (9) and (10), \( F(f) \) represents the volume fraction of deforming material, which depends on the yield criterion used. It is given by \( F = 1 - f \) when the Gurson material model is used, since all the matrix material is taken to yield. An expression for \( F(f) \) in the FKM model and the model with reduced inter-particle cohesion is chosen as in Fleck, Otoyo and Needleman (1992) as

\[
F = \frac{45}{\sqrt{3}}(1 - f)^2 \left( \frac{\dot{f}}{f} \right)^{3/2}.
\]

The expression for \( F(f) \) in the transition range in the combined material model is of the same, weighted form as the yield function in (4).

Initiation of plastic yielding occurs when \( \Phi = 0 \) and \( \dot{\Phi} > 0 \). During plastic yielding, the consistency condition, \( \Phi = 0 \), must be fulfilled. The latter is used to determine the value of the plastic multiplier, \( \Lambda \). Then, (8) can be written in the form

\[
\dot{\varepsilon}_p^{ij} = \frac{1}{H} \frac{\partial \Phi}{\partial \varepsilon^{ij}} \frac{\partial \Phi}{\partial \sigma_{kl}} \sigma^{kl},
\]

with

\[
H = - \left( \frac{\partial \Phi}{\partial f} (1 - f) G^{ij} + \frac{\partial \Phi}{\partial \sigma_M} \frac{E E_t}{E - E_t F(f)\sigma_M} \sigma^{ij} \right) \frac{\partial \Phi}{\partial \sigma^{ij}}.
\]

The incremental constitutive relations for time independent plasticity are of the form

\[
\dot{\varepsilon}_p^{ij} = L^{ijkl} \dot{\varepsilon}_p^{kl},
\]

where \( L^{ijkl} \) are the instantaneous moduli.

The elastic stress–strain relationship is taken to be of the form

\[
\varepsilon^{ij} = \mathbf{G}^{ijkl} \dot{\varepsilon}_p^{kl},
\]

where \( \varepsilon^{ij} \) is the Jaumann derivative of the Cauchy stress tensor and

\[
\mathbf{G}^{ijkl} = \frac{E}{1 + \nu} \left\{ \left( \frac{1}{2} (G^{ik} G^{jl} + G^{ij} G^{kl}) + \frac{\nu}{1 - 2\nu} G^{ij} G^{kl} \right) \right\}
\]

is the finite strain generalization due to Budiansky, see Hutchinson (1973), with \( \nu \) as Poisson’s ratio.

The total strain is taken to be the sum of the elastic and plastic parts, so that the elastic part of the strain increment is \( \dot{\varepsilon}_e^{ij} = \dot{\varepsilon}_p^{ij} \). Substituting (12) into this expression and that again into (15), the instantaneous
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moduli from (14) are determined. This procedure is described in detail for the Gurson material model in Tvergaard (1990b).

The uniaxial true stress-logarithmic strain curve for the matrix material is represented by the piecewise power law

\[
\varepsilon = \begin{cases} 
\frac{\sigma}{E}, & \sigma \leq \sigma_y, \\
\left(\frac{\sigma}{E\sigma_y}\right)^n, & \sigma > \sigma_y,
\end{cases}
\]

(17)

where \(\sigma_y\) is the uniaxial yield stress and \(n\) is the strain hardening exponent.

Friction between the tool and the workpiece is modelled by the Coulomb friction model in which \(\mu\) is the coefficient of friction. For \(\mu = 0\), contact with frictionless sliding is present. At high normal pressures, the Coulomb friction model predicts a friction stress higher than the shear yield stress of the material, which is not possible in practice. Hence, an upper limit for the friction stress is included. The chosen upper limit for the friction stress, \(\sigma_y/\sqrt{3}\), is an approximation which does not include any dependence on porosity or hardening of the material. The numerical procedure of the implementation of contact and friction can be found in Tvergaard (1990a) and will not be repeated here.

4. Numerical method and results

4.1. Numerical method

An incrementally linear, forward extrapolation method is used to obtain the numerical solutions. The equations of equilibrium are expressed in terms of the principle of virtual work. Equilibrium is fulfilled for the current values of stresses, \(\sigma^{ij}\), strains, \(\eta^{ij}\), etc. The equations for the incremental values of stresses, \(\dot{\sigma}^{ij}\), strains, \(\dot{\eta}^{ij}\), etc. are obtained when the principle of virtual work, with the contact and friction terms included, is expanded about this current state. With body forces neglected, the expansion of the principle of virtual work takes the form

\[
\int_V \left( \dot{\sigma}^{ij} \delta \eta_{ij} + \tau^{ij} \delta u^k \delta u_{k,j} \right) dV + \int_{S_T} \left( \dot{T}_n \delta u_n + \dot{T}_t \delta u_t \right) dS
\]

\[
= \int_S \dot{T}^i \delta u_i dS - \left[ \int_V \tau^{ij} \delta \eta_{ij} dV + \int_{S_T} \left( T_n \delta u_n + T_t \delta u_t \right) dS - \int_S T^i \delta u_i dS \right].
\]

(18)

Here, \(S_T\) is the surface where the porous material is in contact with the mould and \(T_n\) and \(T_t\) are the tractions in the normal and tangential directions, respectively. The term in square brackets on the right hand side of (18) is included to prevent the solution from drifting away from the true equilibrium state due to the accumulation of incremental errors.

The compaction process for the axisymmetric cup is shown in figure 4. In the numerical analysis, it is possible to move the three punches, \(P_1\), \(P_2\) and \(P_3\), independently. In practice, the powder is poured into the mould and, due to gravity, the initial dimensions become those of a cup as seen in figure 4. If \(P_2\) and \(P_3\) had been upper punches (figure 4 turned upside-down), the initial geometry of the porous body would have been that of a straight cylinder. A remeshing scheme (see, for example (Pedersen, 1998)) would be necessary to model this situation since the elements otherwise get very destorted during the numerical compaction process.

It has been chosen here to study two compaction routes: single action pressing, where \(P_1\) moves while \(P_2\) and \(P_3\) are fixed; and double action pressing where \(P_1\) and \(P_3\) move at the same speed and \(P_2\) is held still.
The finite element meshes used in the present study consist of 396 isoparametric, eight noded axisymmetric elements with 1279 nodes. Each element contains 4 Gauss points. Due to symmetry around the center line, only half of the body is analysed.

When friction is present, the friction coefficient is set to $\mu = 0.17$, as has been found experimentally for the compaction of a simple cylinder by Kim et al. (1997). The matrix material to be analysed here is specified by $\sigma_y/E = 0.003$, $n = 10$, and $\nu = 1/3$. The initial porosity is $f = 0.35$ in all the examples presented here.

4.2. Double and single action pressing

The comparison of double and single action pressing is shown in figure 5. Friction between the tool and the workpiece is represented by the friction coefficient, $\mu = 0.17$, and the combined material model is used. The initial volumes of the two bodies are identical, but the initial dimensions of the two cups are different since the dimensions of the compacted or green cup must be identical for comparison purposes.

The deformed finite element meshes for double and single action pressing are shown in figures 5a and c, respectively, for the compaction stage, $\Delta V/V_0 = 0.22$, where $\Delta V$ is the volume reduction and $V_0$ is the initial volume of the cup. The initial meshes are shown as well. It is seen in the double action case, that the mesh deforms very homogeneously, whereas the elements around the inner, rounded corner in the single action case are quite distorted. Here, remeshing would be required for further compaction. It it seen from figure 5c that the deformation in the cup wall is very small but that the bottom of the cup (top part of figure 5c) is very compacted. This difference causes material to flow ‘around the corner’ from the bottom to the cup wall. Due to the geometry of the cup and due to friction, the elements in the area around the corner become very distorted.

**Figure 4.** The compaction process of the axisymmetric cup. Three independent punches are present, $P_1$, $P_2$ and $P_3$. The finite element meshes used in the present study consist of 396 isoparametric, eight noded axisymmetric elements with 1279 nodes. Each element contains 4 Gauss points. Due to symmetry around the center line, only half of the body is analysed.
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Figure 5. Deformed finite element meshes and porosity contours using the combined material model at $\Delta V/V_0 = 0.22$. The friction between tool and work-piece is specified by $\mu = 0.17$. (a)–(b) Double action pressing. (c)–(d) Single action pressing.
Curves of constant porosity are shown for double action pressing in figure 5b. The porosity jump between each curve if $\Delta f = 0.02$. At the corners between the punches and the die walls, the material is closely packed due to friction. As a result, the porosity in the cup wall midway between the upper and lower punches is quite high. The effect of the rounded corner inside the cup wall is a higher compaction of the material at the bottom of the cup next to the corner, which prevents the material from sliding around the corner, and a less compacted region on the vertical face adjacent to the corner.

Figure 5d shows the porosity contours in the green specimen as a result of single action pressing. Again, friction results in a denser compaction in the corner between the die and punch $P_1$. The porosity increases along the outer wall of the cup, since the friction does not allow the material to slide down into the corners. The effect of the rounded, inner corner is much stronger in the single action pressing case as seen from the deformed mesh, figure 5c. At the bottom of the cup, most of the material is closely packed with the smallest porosities at the inner corner. As seen for double action pressing, this results in an obstacle for the material flow from the bottom to the inner wall of the cup. Therefore, the material compacts more easily on the outer wall, and this effect is so strong that material at the inner wall near the rounded corner experiences tensile loading. The material here does not compact at all. On the contrary, with the combined material model, tensile stresses give rise to a porosity increase and at porosities approaching the upper limit for the material model, $f = 0.36$, the calculations break down. Therefore, the change in porosity is set to zero when the total porosity has attained a maximum value, here set to $f_{\text{max}} = 0.351$. This numerical approximation for representing uncompacted powder has only been used in the computation of figures 5c and d. In practice, this corresponds to a region in the specimen where nothing really happens; neither tensile stresses of significance nor compaction of the powder. This seems realistic in the powder compaction process.

4.3. The effect of friction

The effect of friction is studied in more detail for the combined material model. Figures 6a and b show the porosity contours as a result of double action pressing with and without friction, respectively, at the compaction stage $\Delta V/V_0 = 0.31$. The porosity distance between each curve if $\Delta f = 0.01$. Figure 6a, where $\mu = 0.17$, is a later compaction stage for the cup shown in figure 5a, and b, but is otherwise identical. The average porosity is lower here but the porosity distribution is quite similar to that of figure 5b, only the gradients are higher. The two ‘bubbles’ of high porosity on the inner side of the cup wall are very clear here. The one closest to the rounded corner is present due to the material flow discussed above, and the other one together with the porous area at the outer side of the cup wall are present due to a combination of friction and double action pressing.

In figure 6b, no friction is present, $\mu = 0$. The compaction of a straight cylinder with no friction would result in a constant porosity throughout the specimen. This is not at all the case with the compaction of the cup. The concentration of the compaction in the corners between the punches and the die wall are not present without friction. As was also seen in the earlier examples with friction, the porosity gradients are high around the inner, rounded corner. Almost full densification has happened at the bottom of the cup and porosities up to $f = 0.15$ are present at the inner wall close to the corner.

To prevent the formation of this highly porous area, a test was run with a different type of single action pressing to that presented in figure 5c and d. Here, only $P_3$ of figure 4 was moved, which resulted in a concentration of high porosity at the bottom of the cup close to the inner, rounded corner. Hence, a double action pressing scheme, where the speed of $P_3$ is higher than that of $P_1$ while $P_2$ is fixed, could be used to avoid the highly porous regions in the vicinity of the inner corner. It is concluded, that the porosity distribution is extremely sensitive to the compaction route, which is therefore a very strong tool to control the porosity distribution, especially when the punches may be moved independently as in the present study.
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Figure 6. Curves of constant porosity as a result of double action pressing using the combined material model at $\Delta V/V_0 = 0.31$. (a) No friction is present, $\mu = 0$. (b) Friction between the tool and the work-piece is present, $\mu = 0.17$.

Figure 7. The degree of compaction vs. the applied load for double and single action pressing with and without friction.

The degree of compaction, $\Delta V/V_0$, versus the applied pressure from $P_1$ (the average normal stress), $\dot{\sigma}/\sigma_y$, is shown in figure 7. As would be expected, the load necessary to compact the cup with double action pressing when friction is present is higher than without friction, e.g. at $\Delta V/V_0 = 0.31$ the applied pressure is 12% higher.
Figure 8. Curves of constant porosity as a result of double action pressing using the variable cohesion material model at $\Delta V/V_0 = 0.15$. Friction between the tool and the work-piece is $\mu = 0.17$. (a) Fully sticking contacts, $\eta = 1$. (b) Almost no cohesion is present, $\eta = 0.1$.

when friction is present. It is also seen from figure 7, that single action pressing demands about 15% higher pressures than double action pressing. Analogous results were found in Redanz (1998) for the compaction of a cutting tool.

4.4. The effect of inter-particle cohesion

In this section, the material model from (5) discussed in Section 2.2, which has a variable cohesion parameter included, is used. The model is only valid at higher porosities, $f \gtrsim 0.15$, thus the cups in this section are only deformed slightly. However, it is expected that the porosity and stress distributions shown here give a good indication of what would happen later on in the process, as seen from figure 5b and 6a. Double action pressing with the friction coefficient, $\mu = 0.17$, is analysed.

Curves of constant porosity in the cup, with $\Delta f = 0.01$, are shown in figure 8 at the compaction stage, $\Delta V/V_0 = 0.15$, for two different values of the cohesion parameter. A material with fully sticking contacts, $\eta = 1$, is presented in figure 8a and a material with very little inter-particle cohesion, $\eta = 0.1$, is shown in figure 8b. On the yield surface of the cohesion model, a vertex exists on the hydrostatic axis. This vertex is rounded off for computational purposes to avoid a corner on the yield surface which would lead to a non-unique determination of the direction of the plastic strain increment (8). At $\eta = 0$, the round off at the right side of the yield surface causes a problem, since an initially stress free specimen, $(\sigma_1^p/3, \sigma_2^p) = (0, 0)$, would then start outside the yield surface. Hence, the small nonzero value $\eta = 0.1$ is used here.

It is seen, that the porosity levels in the two compacts are almost the same and that the distributions are quite similar as well. In the case of fully sticking contacts between particles, figure 8a, the porosity gradients are a little smaller in general compared to figure 8b, except at the bottom of the cup around the inner, rounded
corner, where the gradients are higher. The porosity distributions in general are quite similar to the double action pressing results presented in the previous sections.

The von Mises stress contours normalized by the initial yield strength of the matrix material, \( \sigma_e/\sigma_y \), are shown in figure 9 for the same specimens at the same compaction level, \( \Delta V/V_0 = 0.15 \), as those shown in figure 8. Again, figure 9a contains the result of a material with full cohesion and figure 9b shows that of a material with small cohesive strength between the particles, \( \eta = 0.1 \). The distributions of the von Mises stress are reasonably alike in the two cases. In both cases, the stress is concentrated in the corners between the moving punches and the die, and the double action effect with a decrease in stress on each side half way down the cup wall is seen as well. Also, the von Mises stress is concentrated at the bottom of the rounded corner, whereas an elongated region with lower stresses is present on the vertical face adjacent to the rounded corner. But the important difference is that the average stress in figure 9a is 40–50\% higher than the average in figure 9b. Furthermore, the gradients are higher in the case of fully sticking contacts. The opposite effect is found for the hydrostatic stresses. Here, both the stress level and the gradients are highest when the cohesive strength is small. The distributions of the hydrostatic stresses are much like those of the porosity, figure 8, and will not be shown here.

It can be concluded, that neither the porosity distribution nor the porosity level in the green compact are strongly influenced by the level of inter-particle cohesion. But the von Mises and the hydrostatic stresses are very different in the two cases, which probably has a noticeable effect on the residual stresses.

The degree of compaction, \( \Delta V/V_0 \), versus the applied pressure, \( \dot{\sigma}/\sigma_y \), is shown for \( \eta = 0.1 \) and \( \eta = 1 \) in figure 10. It is seen, that a higher load is needed to compact the material with fully sticking contacts between particles compared to the material with little inter-particle cohesive strength.
5. Conclusions

The powder compaction of a cup has been studied by application of two different porous material models, the combined material model (Fleck, Kuhn and McMeeking, 1992) and a modified version of a model suggested by Fleck (1995), which depends on the inter-particle cohesive strength. The combined material model is valid in the whole range of porosities during the powder compaction, provided that there is full cohesion. By contrast, the variable cohesion model can only be used at higher porosities.

The combined material model has been used to determine the influences of friction and of compaction method on the compacted cup. When a straight cylinder is compacted with no friction between the tool and the work-piece, the porosities throughout the body are constant. This is not the case for the cup in the absence of friction, due to the complex geometry. However, when friction is present, the porosity distribution is much less homogeneous. Hence, the chances for material weaknesses in the form of regions with higher porosity, that may lead to failure in the cup, are increased with increased friction between the tool and workpiece.

The compaction route plays a major role for the final porosity distribution in the compact. Here, single and double action pressing, where the punches have the same speed, have been presented. The predicted porosity distributions in the two cases differ significantly. The speed of each punch can be varied independently, and this way the compaction route which leads to the most homogeneous distribution can be found. To avoid the highly porous regions in the vicinity of the inner corner of the cup, a double action pressing scheme could be used, in which the speed of $P_3$ (from figure 4) is higher than that of $P_1$ while $P_2$ is fixed.

As was also found in Redanz (1998) for a simple compact geometry, a higher applied pressure is necessary when single action pressing is used compared to double action pressing. Also, the body must be compacted with a higher load when friction is present than without.

The variable cohesion material model is used to study the effect of cohesive strength between the powder particles. In the case of the cup, the cohesive strength has hardly any effect on the porosity. However, the von Mises stresses are much higher in the cup where the material has fully sticking contacts than for the material
Numerical modelling of the powder compaction of a cup with little cohesive strength between the particles. The opposite tendency is seen for the hydrostatic pressure. As both the von Mises stress and the hydrostatic pressure lead to plastic yielding in a porous material, it is not surprising that similar porosity distributions can develop in compacts of materials with different inter-particle cohesive strengths even though the stress distributions are different. The differences in the stress fields dependent on the cohesive strength are expected to have much influence on the residual stresses in the compact and this will be studied in a forthcoming paper.

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