Corporate spin-offs, bankruptcy, investment, and the value of debt

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Abstract

In a risk-neutral stochastic environment where bankruptcy is possible, it is well-established that coinsurance incentives may lead creditors to prefer mergers over spin-offs, while shareholders may prefer spin-offs. This paper shows that there are two distinct reasons for this. One is due to the concavity of the debt payoff function in the face value of the debt, while the other arises from imperfect covariation in ultimate firm values. For the latter reason, conventional measures of covariation are not sufficient to evaluate the impact on ex-ante debt value. Also considered are the effects of mergers and spin-offs on investment decisions. © 2000 Elsevier Science B.V. All rights reserved.

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JEL classification: G34; L20

1. Introduction

Among the many motives for merger and spin-off activities, we focus upon the effects such decisions may have upon the values of claims on firm assets. That there may be some truth to several theories concerning mergers and spin-offs is evidenced by the inconclusive results arising from empirical analyses of the effects of such re-organizations on the values of claims on assets. Berger and Ofek (1995) provide a survey of empirical studies. As explained by Higgins and Schall (1975), in the absence of synergy, tax distortions, or decisions to adjust debt subsequent to a corporate re-organization, the sum of bondholder value and shareholder value is invariant to the re-organization.

The idea that shareholders benefit from spin-offs at the expense of bondholders has been formalized by Higgins and Schall (1975), Galai and Masulis (1976), Scott (1977), and MacMinn and Brockett (1995) among others. We return to their analysis, which we call the coinsurance motive for corporate restructuring, and decompose their overall effect into two distinct effects. We then show that the magnitude of one of these incentives to spin-off depends upon how dispersed the asset/debt ratios are of the spun-off firms. The magnitude of the other incentive to spin-off is shown to depend upon the nature of covariation among the stochastic values of the partitioned business assets. While a change in the covariance statistic is not sufficient to have a determinate impact on the incentive for firms to re-organize, we identify a type of change in covariation that is sufficiently structured to have a determinate impact. The results should help corporate financiers sifting through the many impacts of restructuring on the value
of contingent claims on corporate assets to ascertain whether the coinsurance motive merits weighty or negligible consideration. The final component of our analysis studies the implications of the interactions between stochastic realizations and structural organization for investment decisions made by a firm facing a significant probability of bankruptcy.

2. The problem

At time 0 in a two period model, two independent firms contemplate a merger. Our problem is to analyze the impacts of such a merger on the aggregate value of debt for risk-neutral creditors. Firm A has debt which will have face value $D_A$ at time 1 while the time 1 face value of firm B’s debt is $D_B$. The realization of firm $j$ value gross of debt at time 1 is $V_j, j \in \{A, B\}$, where the $V_j$ are random and are supported on $V_A \times V_B \in [0, T_A] \times [0, T_B]$. $T_A > D_A$. $T_B > D_B$. The time 1 summed net value of debt for the independent firms is $\min[D_A + D_B, V_A + V_B]$. The time 1 value of the merged firm would be $\min[D_A + D_B, V_A + V_B]$. From Scott (1977), among others, it is well know that

$$\min[D_A + D_B, V_A + V_B] \leq \min[D_A, V_A] + \min[D_B, V_B], \tag{1}$$

and so $E[\min[D_A + D_B, V_A + V_B]] \geq E[\min[D_A, V_A]] + E[\min[D_B, V_B]]$, where $E[\cdot]$ is the expectation operator taken over the joint distribution of time 1 gross firm values. Following the reasoning of MacMinn and Brockett (1995), the Modigliani and Miller theorem then implies that, given the pertinent assumptions, mergers decrease the aggregate expected wealth of shareholders. Questions we address include why this inequality arises and what determines the magnitude of the inequality. We also consider how mergers and other organizational alterations that affect stochastic impacts alter the incentive to invest.

3. Perfect rank correlation

Following Scott (1977), the state space may be decomposed into six regions, these being regions I–VI as defined in Table 1. The unmerged and merged values are the same in regions I (i.e., $\{(V_A, V_B) : V_A \leq D_A, V_B \leq D_B\}$) and IV (i.e., $\{(V_A, V_B) : V_A > D_A, V_B > D_B\}$), but unmerged value is less than the merged value otherwise. If the state space can be restricted so that only regions I and IV have strictly positive measure, then ex-ante expected value of debt is invariant to the merger. Thus it is required that $\text{prob}[V_A \leq D_A, V_B > D_B] = \text{prob}[V_A > D_A, V_B \leq D_B] = 0$. This is unlikely when the state space is diffused over two dimensions rather than concentrated on a curve because regions I and IV are diagonally opposed. It is impossible when $V_A \times V_B$ has strictly positive measure on all measurable subsets of $[0, T_A] \times [0, T_B]$. It is more likely if the variables are dependent. Consider the case where there is perfect rank dependence:

<table>
<thead>
<tr>
<th>Region</th>
<th>Conditions</th>
<th>Unmerged value</th>
<th>Merged value</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$V_A \leq D_A, V_B \leq D_B$</td>
<td>$V_A + V_B$</td>
<td>$V_A + V_B$</td>
</tr>
<tr>
<td>II</td>
<td>$V_A \leq D_A, V_B &gt; D_B, V_A + V_B \leq D_A + D_B$</td>
<td>$V_A + D_B$</td>
<td>$V_A + V_B$</td>
</tr>
<tr>
<td>III</td>
<td>$V_A \leq D_A, V_B &gt; D_B, V_A + V_B &gt; D_A + D_B$</td>
<td>$V_A + D_B$</td>
<td>$D_A + D_B$</td>
</tr>
<tr>
<td>IV</td>
<td>$V_A &gt; D_A, V_B &gt; D_B$</td>
<td>$D_A + D_B$</td>
<td>$D_A + D_B$</td>
</tr>
<tr>
<td>V</td>
<td>$V_A &gt; D_A, V_B \leq D_B, V_A + V_B \leq D_A + D_B$</td>
<td>$D_A + V_B$</td>
<td>$V_A + V_B$</td>
</tr>
<tr>
<td>VI</td>
<td>$V_A &gt; D_A, V_B \leq D_B, V_A + V_B &gt; D_A + D_B$</td>
<td>$D_A + V_B$</td>
<td>$D_A + D_B$</td>
</tr>
</tbody>
</table>
Definition 1. Random variables $V_A$ and $V_B$ are said to have perfect rank dependence if $V_B = \psi(V_A)$ except on sets of measure zero and $\psi(\cdot)$ is an uniformly continuous and strictly monotone function.

Because of systemic risk in the economy, we will assume positive monotonically, i.e., $\psi'(\cdot) > 0$, in which case the random variables are said to express perfect positive rank dependence. The difference between merged and unmerged ex-ante debt values is

$$\Gamma = E[\min[D_A + D_B, V_A + \psi(V_A)]] - E[\min[D_A, V_A]] - E[\min[D_B, \psi(V_A)]],$$

and we can assert the following.

Proposition 1. Given perfect positive rank dependence between $V_A$ and $V_B$, and that the distribution of $V_A$ has strictly positive support on all measurable intervals in $[0, T_A]$, then $\Gamma > 0$ if $D_B \neq \psi(D_A)$.

Proof. By uniform continuity of $\psi(\cdot)$, for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $|\psi(x_1) - \psi(x_2)| < \varepsilon$ for all $x_1$ and $x_2$ for which $|x_1 - x_2| < \delta$. Writing $\delta = \delta(\varepsilon)$, it follows that if $|\psi(x_1) - \psi(x_2)| \geq \varepsilon$ for some $x_1$ and some $x_2 \neq x_1$, then $|x_1 - x_2| \geq \delta(\varepsilon)$. Specifically for points $D_A$ and $\psi^{-1}(D_B)$, we have $|\psi(D_A) - D_B| \geq \varepsilon$ for $\varepsilon$ sufficiently small, since $D_B \neq \psi(D_A)$, and so $|D_A - \psi^{-1}(D_B)| \geq \delta(\varepsilon)$. Here strict monotonicity of $\psi(\cdot)$ ensures that the inverse function uniquely defines a point. $D_A$ is the point in the domain $V_A \in [0, T_A]$, where $V_B = \psi(V_A)$ intersects $V_A = D_A$, while $\psi^{-1}(D_B)$ is the point in the domain $V_A \in [0, T_A]$, where $V_B = \psi(V_A)$ intersects $V_B = D_B$. Thus there is a measurable interval in $V_A \in [0, T_A]$ with measure not smaller than $\delta(\varepsilon)$ along which $(V_A, \psi(V_A)) \notin \{\text{Region I} \cup \text{Region IV}\}$. It follows from Table 1 and Eq. (2) that $\Gamma > 0$. 

If $V_A$ is held fixed at say $V_A = V_A^0$, then the reason for $\Gamma > 0$ becomes clearer. The function $\min[2\alpha D_A + 2(1-\alpha)D_B, 2\alpha V_A^0 + 2(1-\alpha)\psi(V_A^0)]$ is concave in $\alpha$, and so $\min[D_A + D_B, V_A^0 + \psi(V_A^0)] \geq \min[D_A, V_A^0] + \min[D_B, \psi(V_A^0)]$. Now suppose that $V_A = \psi(V_A) \forall V_A \in [0, T_A]$ so that $2\min[\frac{1}{2}(D_A + D_B), V_A^0] \geq \min[D_A, V_A^0] + \min[D_B, V_A^0]$. The weak inequality is an equality when $D_A = D_B$. Setting $D_A = \tilde{D} + \beta$ and $D_B = \tilde{D} - \beta$, where $\tilde{D} = \frac{1}{2}(D_A + D_B)$ and $\beta \geq 0$, an increase in the value of $\beta$ constitutes a majorization so that the magnitude of $\Gamma$ increases with $\beta$ (see Marshall and Olkin, 1979, p. 11). An increase in $\beta$ causes $V_A/D_B = V_A/(\tilde{D} + \beta)$ to diverge away from $V_A/D_B = V_A/(\tilde{D} - \beta)$. Thus one would expect creditors to become increasingly concerned about spin-off plans as the proposed debt/asset ratios for the resulting firms become increasingly divergent. Conversely, ex-ante aggregate shareholder value would increase with such a spin-off.

4. Imperfect correlation

Proposition 1 has provided a reason why, having controlled for the degree of correlation, the ex-ante risk-neutral value of debt in the case of a merger may exceed the summed ex-ante risk-neutral values if the merger does not take place. It will now be shown that alteration in the degree of correlation provides another reason why creditors would prefer a merged firm. Specifically, a sufficiently structured correlation decreasing shift in the distribution of random variables $(V_A, V_B)$ will be identified that preserves the marginal distributions of $V_A$ and $V_B$, and so does not affect either $E[\min[D_A, V_A]]$ or $E[\min[D_B, V_B]]$, yet increases the value of $E[\min[D_A + D_B, V_A + V_B]]$.

The correlation shift to be considered arises from a rearrangement, or permutation, of the interactions between marginal distributions. Following Hennessy and Lapan (1997) or Epstein and Tanny (1980), we construct a counting measure for two vectors, $V_A$ and $V_B$, of realizations of random variables $V_A$ and $V_B$. Coordinate $j \in \{1, 2, \ldots, n\}$ in vector $V$, $i \in \{A, B\}$ is denoted by $V^i_j$, and $(V_A^j, V_B^j)$ is one of $n$ realizations in sample space $\Theta = \{V_A, V_B\} = \{(V_A^1, V_B^1), (V_A^2, V_B^2), \ldots, (V_A^n, V_B^n)\}$. Multiple replications of realizations can, of course, occur. Realization $(V_A^k, V_B^k)$ may not be very well aligned in the sense that $V_A^k$ may be among the smaller ordinates in $V_A$,
while $V^k_A$ may be among the larger ordinates in $\tilde{V}_B$. Consider now another realization in state space $(V^+_A, V^+_B)$. When $V^k_A < V^k_B$ and $V^l_B > V^l_A$, then the vectors would be in some sense better aligned if the state space were rearranged by interchanging $V^l_B$ with $V^l_A$. Using the arrangement increasing (AI) partial ordering of Hollander et al. (1977) and also Boland et al. (1988), symbolized by $\geq^A$, this concept of interchange can be formalized.

**Definition 2.** For a given vector, $\tilde{x} = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$, let $\tilde{x} \uparrow=(x_{\pi(n)}, x_{\pi(1)}, \ldots, x_{\pi(n-1)}, x_{\pi(n-2)}, \ldots, x_{\pi(1)})$ be the vector that has the components of $\tilde{x}$ arranged in increasing order. For any permutation $\pi$ of $(1, 2, \ldots, n)$, let $\tilde{x}_\pi = (x_{\pi(1)}, x_{\pi(2)}, \ldots, x_{\pi(n)})$. For vectors $\tilde{x}, \tilde{y}, \tilde{u}$, and $\tilde{v}$, assign $\{\tilde{x}, \tilde{y}\} = \tilde{a}\{\tilde{u}, \tilde{v}\}$ if there exists a permutation $\pi$ of $(1, 2, \ldots, n)$ such that $\tilde{x}_\pi = \tilde{u}$ and $\tilde{y}_\pi = \tilde{v}$.

Assign $\{\tilde{x}, \tilde{y}\} \leq^A \{\tilde{u}, \tilde{v}\}$ if there exists a finite number of transposition vectors $\{\tilde{z}^1, \tilde{z}^2, \ldots, \tilde{z}^k\}$ such that:

1. $\{\tilde{x}, \tilde{y}\} \leq^A \{\tilde{z}^1\}$ and $\{\tilde{x} \uparrow, \tilde{z}^1\} = \tilde{a}\{\tilde{u}, \tilde{v}\}$, and
2. $\tilde{z}^{i-1} \leq^A \{\tilde{z}^i\}$ can be obtained from $\tilde{z}^i$ by an interchange of two components of $\tilde{z}^i$, the first of which is less than the second.

To illustrate, rearrange $\{\tilde{V}_A, \tilde{V}_B\}$ in the following manner: $\{\tilde{V}_A, \tilde{V}_B\} = \{\tilde{0}, 1, 2, 6\}, (3, 0, 4, 5)\}$ is the minimal arrangement. Some arrangements are incomparable, as can be seen by consideration of arrangements $\{(0, 1, 2, 6), (0, 5, 4, 3)\}$ and $\{(0, 1, 2, 6), (3, 0, 4, 5)\}$. The maximal arrangement is of particular interest to us because it imposes a relationship between $\tilde{V}_A$ and $\tilde{V}_B$ that is consistent with perfect positive rank dependence, i.e., $\psi^*(\cdot) > 0$ and $\tilde{V}_B = \tilde{\psi}(\tilde{V}_A)$ except on sets of measure zero.

Since our interest is in how an AI rearrangement of the sample space affects the value of functions of forms $E\{\min(D_A, V_A)\}$ and $E\{\min(D_B, V_A + V_B)\}$, we must relate the $\geq^A$ ordered mapping of state space into an order relationship for functions. Hollander et al. (1977) identified the concept of AI function (also called a decreasing in transposition function):

**Definition 3.** Function $G(\tilde{V}_A, \tilde{V}_B)$ is said to be AI if $G(\tilde{V}^+_A, \tilde{V}^+_B) \geq G(\tilde{V}_A, \tilde{V}_B)$ whenever $\{\tilde{V}^+_A, \tilde{V}^+_B\} \geq^A \{\tilde{V}_A, \tilde{V}_B\}$, and is said to be arrangement decreasing (AD) if $G(\tilde{V}^+_A, \tilde{V}^+_B) \leq G(\tilde{V}_A, \tilde{V}_B)$ whenever $\{\tilde{V}^+_A, \tilde{V}^+_B\} \geq^A \{\tilde{V}_A, \tilde{V}_B\}$.

We also need to formally define the order induced by AI and AD rearrangements of the sample space.

**Definition 4.** An increase (decrease) in the covariation order is said to arise from an AI (AD) rearrangement of the sample space.

This order is a special case of one studied by Athey (1999). Boland et al. (1988) show that the function $E\{V_A, V_B\} = (1/n)\sum_{j=1}^n V_A^j V_B^j$ is AI, and so $\text{cov}(V_A, V_B) = (1/n)\sum_{j=1}^n V_A^j V_B^j - (1/n)^2\sum_{j=1}^n V_A^j \sum_{j=1}^n V_B^j$ is AI. That is, an increase in the covariation order between two random variables does increase covariance. However, the two concepts are not synonymous, since covariance is a total ordering (a chain) while the covariation order is partial. The rearrangement of $\{(0, 1, 2, 6), (0, 5, 4, 3)\}$ to $\{(0, 1, 2, 6), (3, 0, 4, 5)\}$ increases covariance between the two vectors, but it is neither increasing nor decreasing in the covariation order.

From Hollander et al. (1977), functions of the form $E\{\min(D_A + D_B, V_A + V_B)\}$ are Schur concave and so are AD in rearrangements of $\{\tilde{V}_A, \tilde{V}_B\}$. Also, since rearrangements do not affect marginal distributions, $E\{\min(D_A, V_A)\}$ and $E\{\min(D_B, V_B)\}$ are arrangement invariant. Therefore, function

$$A = E\{\min(D_A + D_B, V_A + V_B)\} - E\{\min(D_A, V_A)\} - E\{\min(D_B, V_B)\}$$

is AD, and so we have the following proposition.

**Proposition 2.** A decrease in the covariation order between $\tilde{V}_A$ and $\tilde{V}_B$ increases the value of $A$. 


Noteworthy AI functions are the upper orthant functions, UO(\(\tilde{V}_A, \tilde{V}_B, \tilde{V}_A, \tilde{V}_B\)) = \(\sum_{j=1}^{\eta} I\{V_A^j \geq \tilde{V}_A, V_B^j \geq \tilde{V}_B\}\), and the lower orthant functions, LO(\(\tilde{V}_A, \tilde{V}_B, \tilde{V}_A, \tilde{V}_B\)) = \(\sum_{j=1}^{\eta} I\{V_A^j \leq \tilde{V}_A, V_B^j \leq \tilde{V}_B\}\), where \(I\) is the indicator function which is equal to one when conditions \(\chi\) are all true and zero otherwise, and where \(\tilde{V}_i, i \in \{A, B\}\) are scalars. Generalizing to the continuous case, an AI rearrangement of state space increases \(\text{prob}[V_A^j \geq \tilde{V}_A, V_B^j \geq \tilde{V}_B]\) and \(\text{prob}[V_A^j \leq \tilde{V}_A, V_B^j \leq \tilde{V}_B]\). A distribution, \(\{\tilde{V}_{A1}, \tilde{V}_{B1}\}\), which dominates distribution \(\{\tilde{V}_{A2}, \tilde{V}_{B2}\}\) in the sense that \(\text{prob}[V_A^{j_1} \geq \tilde{V}_A, V_B^{j_1} \geq \tilde{V}_B] \geq \text{prob}[V_A^{j_2} \geq \tilde{V}_A, V_B^{j_2} \geq \tilde{V}_B] \forall \{\tilde{V}_{A1}, \tilde{V}_{B1}\}\) is said to dominate \(\{\tilde{V}_{A2}, \tilde{V}_{B2}\}\) in the upper orthant order. Symbolically, \(\{\tilde{V}_{A1}, \tilde{V}_{B1}\} \geq_{\text{uo}} \{\tilde{V}_{A2}, \tilde{V}_{B2}\}\). For given marginals, the distribution that dominates all others (except those differing by sets of measure zero) in the \(\geq_{\text{uo}}\) sense is that in which there is perfect positive rank dependence. Among all distributions with given marginals, this will give the least value of \(A\), i.e., the least gain in ex-ante debt value from a merger, and so will give the least shareholder equity boost from a spin-off.

Combining Propositions 1 and 2, it is clear that inequality (1) is driven in part by the concavity of the min\([\ldots]\) function and in part by imperfect covariation in the \(\geq_{\text{uo}}\) sense. It is interesting to note that Slepian (1962) has shown that an increase in any covariance coefficient for the normal distribution, holding means and other cross moments constant, increases the distribution in the \(\geq_{\text{uo}}\) sense. This is also true of the lognormal distribution.

5. Corporate reorganization and investment

Assume that the time 1 value of a firm is partly determined by the level of investment \(I\) at time 0. This investment may arise from internal sources or it may be borrowed. Let internal sources be fixed at \(w\), while external sources comprise the remainder. There are also other assets in the business, and these amount to value \(K\). The cost of external sources is assumed to be an increasing and convex function of the amount required, \(C(I - w)\). Suppose that the time 0 investment converts to time 1 product \(F(I)\) which sells for random price \(\hat{P}\). Here, \(F(I)\) is increasing and concave. The time 0 problem facing a firm with limited liability is then to choose \(I\) so as to maximize function

\[\Omega = E[\max(\hat{P} F(I) - C(I - w) + K, 0)]\]  

(4)

The first-order condition may be written as

\[E(\hat{P} | \hat{P} \geq P^*) F_I(I) = C_I(I - w)\]  

(5)

where \(P^* = [C(I - w) - K] / F(I)\) and \(|\chi|\) means conditional upon criterion \(\chi\). We assume that the second-order condition for a maximum holds, although the concavity of \(F(I)\) and the convexity of \(C(I - w)\) are not sufficient to ensure a global maximum. Given concavity of (4) in \(I\), a shift in the distribution of \(\hat{P}\) that increases the value of \(E(\hat{P} | \hat{P} \geq P^*)\) for all \(P^*\) is sufficient to increase optimal \(I\). Such stochastic shifts are said to be ordered according to the mean residual life ordering (Hollander and Proschan, 1975; Alzaid, 1988), and we denote them by the order relation \(\geq_{\text{mr}}\). Alzaid (1988, p. 38) has shown that if \(G(\hat{P}) \geq_{\text{mr}} H(\hat{P})\) and the distribution means are equal, then \(H(\hat{P})\) is a Rothschild and Stiglitz (1970) mean-preserving contraction with respect to distribution \(G(\hat{P})\).

Now suppose that two firms A and B that are identical in all other ways face different drawings from the same distribution of \(\hat{P}\), say drawings \(\hat{P}_a\) and \(\hat{P}_b\). If they merge, then the objective function may be written as choosing \(I\) to maximize

\[\Omega_{AB} = E[\max(\frac{1}{2}(\hat{P}_a + \hat{P}_b) F(I) - C(I - w) + K, 0)]\]  

(6)

yielding first-order condition

\[E(\frac{1}{2}(\hat{P}_a + \hat{P}_b) | \frac{1}{2}(\hat{P}_a + \hat{P}_b) \geq P^*) F_I(I) = C_I(I - w)\]  

(7)

where \(P^*\) is as previously defined. This condition reduces to (5) when \(\hat{P}_a = \hat{P}_b\), i.e., when both firms would have realized identical drawings in all states of nature. The departure of \(E(\frac{1}{2}(\hat{P}_a + \hat{P}_b) | \frac{1}{2}(\hat{P}_a + \hat{P}_b) \geq P^*)\) from
\( E\{\bar{P}_a | \bar{P}_a \geq P^*\} \) represents a shock to the first-order condition, and can be studied in the usual way to ascertain the impact on optimal \( I \). It can be shown (see, e.g., Hennessy and Lapan, 1997) that the averaging in (7) represents a mean-preserving contraction, i.e., \( \frac{1}{2}(\bar{P}_a + \bar{P}_b) \) is a mean-preserving contraction of \( \bar{P}_a \). Were it to represent a mean-preserving increase or decrease in the \( \geq mr \) sense, then one could determine the effect of the merger on investment. However, the coinsurance due to the merger does not imply either \( \geq mr \) dominance or domination as the following example illustrates. Let \( \bar{P}_a \) and \( \bar{P}_b \) be equiprobably distributed on the atoms 1, 2, 3, and 4, where the state space is described by \( \{(1, 2, 3, 4), (1, 2, 3, 4)\} \). This gives the mixed distribution as the realizations \( (1, 2.5, 2.5, 4) \), each arising with probability \( \frac{1}{4} \). We have \( E\{\bar{P}_a | \bar{P}_a \geq 2.1\} = 3.5 \) and \( E\{\frac{1}{2}(\bar{P}_a + \bar{P}_b) | \frac{1}{2}(\bar{P}_a + \bar{P}_b) \geq 2.1\} = 3 \), but \( E\{\bar{P}_a | \bar{P}_a \geq 2.6\} = 3.5 \) and \( E\{\frac{1}{2}(\bar{P}_a + \bar{P}_b) | \frac{1}{2}(\bar{P}_a + \bar{P}_b) \geq 2.6\} = 4 \). Thus there is a reversal, and the order holds in neither direction. Therefore the impact of a merger or of a spin-off on investment incentives is indeterminate.

6. Conclusion

This paper has decomposed the source of value alterations for risky debt as debt is bundled and unbundled. A corporate spin-off reduces the value of debt in two distinct ways. First, it gives rise to a dispersion of debt for a function that is concave in debt and so the reduction in the ex-ante value of debt is partly due to the Jensen inequality. This effect arises even if there is perfect monotone rank dependence among asset values in the spun-off companies. Second, rank dependence between the values of disaggregated assets may be imperfect, and so creditors would benefit from the stability arising from the pooling of assets. Symmetrically, equity holders gain in these two ways from a corporate spin-off. The effects of restructuring on investment incentives are shown to be indeterminate.

By following the multivariate AI methodology of Boland and Proschan (1988), it would not be difficult to extend this analysis to a multiple random variate problem. It might also be possible to provide a similar decomposition of the gains to insurers arising from risk pooling. As outlined in Skees et al. (1997), this is an issue that is presently of particular interest to North American agricultural policy makers as a possible means of mitigating moral hazard and adverse selection problems. Majd and Myers (1987) have noted that an inequality similar to (1) arises for the effects of mergers on taxation when there is limited or no loss offset. The effect of a merger (or marriage) on expected tax incidence can also be decomposed in the manner of this paper.

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References