Optimal reinsurance under mean-variance premium principles

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Received 1 June 2000; received in revised form 1 September 2000; accepted 4 October 2000

Abstract

We derive optimal reinsurance under premium principles based on the mean and variance of the reinsurer’s share of the total claim amount. Both global reinsurance and local reinsurance are studied. Examples considered include standard deviation principle and variance principle. © 2001 Elsevier Science B.V. All rights reserved.

MSC: Primary 90A46; Secondary 62P05

Keywords: Optimal reinsurance; Global reinsurance; Local reinsurance

1. Introduction

Reinsurance is the transfer of risk from a direct insurer, the cedent, to a second insurance carrier, the reinsurer. The best known examples of reinsurance are quota share and stop loss. In quota share reinsurance, the reinsurer’s share of the total claim amount $X$ is given by $R_q(X) = aX$, where $a \in (0, 1)$ is a parameter. An easy computation shows that the minimum of $\mathbb{D}^2(X - R(X))$ subject to the constraint $P = \mathbb{D}^2 R(X)$ is attained at $R_q$, where $P$ is the premium of the reinsurer and $\mathbb{D}^2 U$ means the variance of random variable $U$. We assume throughout this paper that $P$ is a fixed positive number.

In stop loss reinsurance, $R_s(X) = (X - b)_+$, $b$ being a parameter. Here and subsequently, $a_+$ means $\max\{a, 0\}$. It is well known that the retention function $R_s$ is a solution of the following minimization problem:

$$\mathbb{D}^2(X - R(X)) = \min_{R} \quad \text{under the condition } P = (1 + \beta)\mathbb{E}R(X),$$

where $\beta > 0$ is a safety loading coefficient (cf. Daykin et al., 1994).

The aim of the paper is to derive optimal reinsurance under other premium principles based on the mean and variance of the reinsurer’s share of the total claim amount. Two premium calculation principles are often used in practice:

$$P = \mathbb{E}R(X) + \beta \mathbb{D}R(X) \quad \text{and} \quad P = \mathbb{E}R(X) + \beta \mathbb{D}^2 R(X),$$

called the standard deviation and the variance principle, respectively. Other rules are:

1. the modified variance principle $P = \mathbb{E}R(X) + \beta \mathbb{D}^2 R(X)/\mathbb{E}R(X)$,
2. the mixed principle $P = \mathbb{E}R(X) + \alpha \mathbb{D}R(X) + \beta \mathbb{D}^2 R(X)$

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Observe that the above-mentioned principles are of the form
\[ \mathbb{E} R(X) = f(P, \mathbb{D} R(X)), \]
where \( f : (0, \infty) \times [0, t(P)) \to (-\infty, \infty) \) with \( \mathbb{D} X < t(P) \leq +\infty \), and the following conditions hold:

(F1) \( f(P, 0) = P \),

(F2) \( f(P, t) \) is nonincreasing, concave, and differentiable in \( t \).

Condition (F1) is related to the property called no unjustified safety loading. In fact, if \( \Pi(X) \) denotes a solution of the equation \( \mathbb{E} X = f(t, \mathbb{D} X) \) for \( t > 0 \), then (F1) implies that \( \Pi(c) = c \) for all \( c > 0 \).

A retention function \( R \) is said to be optimal if it is a solution of the following problem:

\[
\min_{R(X)} \mathbb{D}(X - R(X)) \quad \text{subject to} \quad \mathbb{E} R(X) = f(P, \mathbb{D} R(X)), \quad 0 \leq R(x) \leq x \text{ for all } x \geq 0.
\]

In Section 2 we prove that optimal retention functions are of the following form:

\[ R^*(x) = a(x - b)_+ \quad (1) \]

with \( 0 < a \leq 1, b \geq 0 \). The rule defined by (1) is called change loss reinsurance. In Section 3 optimal rules for local reinsurance are provided.

The results to be presented generalize work published in Gajek and Zagrodny (2000). Using convex programming methods, they demonstrated that the change loss reinsurance is optimal under the standard deviation principle. All proofs are straightforward, in contrast to those of Gajek and Zagrodny (2000).

2. Global reinsurance

Let \( X \) be a nonnegative random variable on a probability space \((\Omega, S, \mathbb{P})\), called the risk. Suppose \( 0 < \mathbb{D}^2 X < \infty \). Define \( \sup X = \sup\{b : \mathbb{P}(X > b) > 0\} \). The reinsurer premium risk is given by \( R = R(X) \), where \( R : [0, +\infty) \to [0, +\infty) \) is a retention function. We seek for a rule which minimizes the cedent’s risk, measured by the variance of \( X - R \), subject to the constraint \( R \in \mathcal{R}(f) \), where

\[ \mathcal{R}(f) = \{R : \mathbb{E} R = f(P, \mathbb{D} R), \quad 0 \leq R(x) \leq x \text{ for every } x \geq 0\}. \]

Thus the following minimization problem arises:

\[ \min_{R \in \mathcal{R}(f)} \mathbb{D}^2(X - R). \quad (3) \]

**Theorem 1.**

1. Suppose there exist reals \( a, b \) such that the following conditions hold:
   (i) \( 0 < a \leq 1, 0 < b < \sup X \),
   (ii) \( -a \mathbb{E}(X - b)_+ f'_2(P, a \mathbb{D}(X - b)_+) = (1 - a) \mathbb{D}(X - b)_+ \),
   (iii) \( a \mathbb{E}(X - b)_+ = f(P, a \mathbb{D}(X - b)_+) \),

   where \( f'_2(P, t) = \partial f(P, t)/\partial t \). Then \( R^*(x) = a(x - b)_+ \) is a solution of the problem (3).

2. If \( \mathbb{E} X > f(P, \mathbb{D} X) \), then there exist reals \( a, b \) such that conditions (i)-(iii) are satisfied.

**Proof.** Recall that

\[ \mathbb{D}^2(X - R) = \mathbb{D}^2 X - 2 \operatorname{Cov}(X, R) + \mathbb{D}^2 R. \quad (4) \]
Observe that
\[ \text{Cov}(X, R) = \text{Cov}(X - b, R) \leq \text{Cov}((X - b)_+, R) + [\mathbb{E}(X - b)_+ - \mathbb{E}(X - b)]R. \]
Since \( \mathbb{E}(X - b)_+ - \mathbb{E}(X - b) = \mathbb{E}(b - X)_+ \), we have
\[ \text{Cov}(X, R) \leq \text{Cov}((X - b)_+, R) + \mathbb{E}(b - X)_+ \mathbb{E}R. \tag{5} \]
and the equality in (5) holds if \( R(x) = 0 \) for \( 0 \leq x \leq b \). From (4), (5) and the Cauchy–Schwarz inequality it follows that for every \( R \in \mathcal{R}(f) \)
\[
\mathbb{D}^2(X - R) \geq \mathbb{D}^2(X - 2\mathbb{D}(X - b)_+ + R) + \mathbb{D}^2R - 2\mathbb{E}(b - X)_+ f(P, \mathbb{D}R). 
\] 
Putting \( t = \mathbb{D}R/\mathbb{D}(X - b)_+ \), we get
\[
\mathbb{D}^2(X - R) \geq \mathbb{D}^2(X) + (t^2 - 2t)\mathbb{D}^2(X - b)_+ - 2\mathbb{E}(b - X)_+ f(P, t\mathbb{D}(X - b)_+). \tag{6} 
\]
The equality in (6) holds if \( R(x) = c(x - b)_+ \) with a real \( c \) such that \( R \in \mathcal{R}(f) \). By (F2) and (ii), the right-hand side of (6) is a convex function in \( t \) and attains its minimum at \( t = a \). By (i) and (iii), \( R^* \in \mathcal{R}(f) \), and consequently for every \( R \in \mathcal{R}(f) \)
\[
\mathbb{D}^2(X - R) \geq \mathbb{D}^2(X - R^*), \tag{7} 
\]
completing the proof of the first part of the theorem.

We now show that there exist reals \( a, b \) such that conditions (i)–(iii) are satisfied. Define
\[
\phi(s, t) = t\mathbb{E}(X - s)_+ - f(P, t\mathbb{D}(X - s)_+), \quad 0 \leq s \leq b_1, \quad 0 \leq t \leq 1, 
\]
where \( b_1 \) is a real such that \( \phi(b_1, 1) = 0 \). Since \( \phi(0, 1) > 0, \lim_{b \to \infty} \phi(b, 1) = -f(P, 0) = -P < 0 \) and \( \phi(s, 1) \)
is continuous, \( b_1 \) exists and \( 0 < b_1 < \sup X \). By (F2), for every \( 0 \leq s \leq b_1 \), there is only one point, say \( t(s) \), such that \( \phi(s, t(s)) = 0 \). Since \( \phi \) is continuous the function \( t(s) \) is continuous. Define
\[
\gamma(s, t) = \mathbb{D}^2(X) + (t^2 - 2t)\mathbb{D}^2(X - s)_+ - 2\mathbb{E}(s - X)_+ f(P, t\mathbb{D}(X - s)_+) 
\]
for \( 0 \leq s \leq b_1, \quad 0 \leq t \leq 1 \). Since for every \( 0 \leq s \leq b_1 \) the function \( \gamma(s, t) \) is strictly convex in \( t \), there is only one point, say \( \tau(s) \), such that \( \gamma(s, \tau(s)) = \min_{0 \leq t \leq 1} \gamma(s, t) \). Obviously, \( \tau(0) = 1 \). Moreover, the function \( \tau(s), \quad 0 \leq s \leq b_1 \), is continuous since \( \gamma'(s, t) \) is continuous. We have \( 0 < \tau(0) < 1 = \tau(1) \) and \( b_1 \).

Remark 1. Observe that if conditions (i)–(iii) hold, and if \( \mathbb{P}(X \leq b) > 0 \), then it must hold \( \mathbb{E}X > f(P, \mathbb{D}X) \). In fact,
\[
\mathbb{E}X \geq a\mathbb{E}X > a\mathbb{E}(X - b)_+ = f(P, a\mathbb{D}(X - b)_+) \geq f(P, a\mathbb{D}X) \geq f(P, \mathbb{D}X), 
\]
\[ \text{since } \mathbb{D}^2(X - b)_+ \leq \mathbb{D}^2X \text{ and } \mathbb{E}(X - b)_+ \leq \mathbb{E}X \text{ for every } b > 0. \]

Remark 2. It is clear that the assumptions of Theorem 1 can be weaken but we do not go into details since both (F1) and (F2) are fulfilled in the most common models (see Examples 1–4 of the paper).

Remark 3. Important contribution to solving the problem of optimal purchase of reinsurance was given by Deperez and Gerber (1985). They called a retention function optimal if it maximized the expectation of the cedent’s utility function over the set of all retention rules, i.e. the following problem was studied:
\[
\text{maximize } \mathbb{E}u(R - H(R) - X) \text{ over all } R = R(X), 
\]
where \( H(R) \) is a convex and Gâteaux differentiable premium principle and \( u \) is an increasing and concave function. The work of Deperez and Gerber (1985) was extended by Young (1999) to cover the case of Wang’s premium.
principle which is not Gâteaux differentiable. The difference between our approach and that of Deperez and Gerber (1985) is: (a) the use of different target function and (b) minimization of target function under constraints.

**Example 1** (Expected value principle). Let $P = (1 + \beta)E_R$, where $\beta > 0$ is a safety loading coefficient. Of course, $f(P, t) = P/(1 + \beta)$. From Theorem 1 it follows that $R^*(x) = (x - b)_+$ is an optimal retention function if $(1 + \beta)E(X - b)_+ = P$, which is a well-known fact (cf. Pesonen, 1984).

**Example 2** (Standard deviation principle). Let $P = \mathbb{E}R + \beta \mathbb{D}^2 R$ with $\beta > 0$. Then $f(P, t) = P - \beta t$ and Theorem 1 shows that if $\mathbb{E}X + \beta \mathbb{D}X > P$, then an optimal retention function is given by $R^*(x) = a(x - b)_+$, where $a, b$ are reals such that

$$-\beta \mathbb{E}(b - X)_+ + (1 - a)\mathbb{D}(X - b)_+ = 0, \quad a\mathbb{E}(X - b)_+ + a\beta \mathbb{D}(X - b)_+ = P.$$ 

This result is due to Gajek and Zagrodny (2000). Explicit formulae can be found for $a, b$ in some models of risk. For instance, suppose $X$ is a Bernoulli risk, i.e. $X$ is a random variable that takes the value $M > 0$ with probability $p \in (0, 1)$ and zero with probability $q, q = 1 - p$. Then

$$a = \frac{P[pP - \beta(M(p - q\beta^2) - P)\sqrt{pq}]}{p[P^2 + 2\beta^2 M pq - \beta^2 M^2 q(p - q\beta^2)]}, \quad b = \frac{[M(p - \beta^2 q) - P]\sqrt{pq} + \beta q P}{(p - \beta^2 q)(\sqrt{pq} + \beta q)}.$$ 

For concreteness, let $M = 10^6, p = 10^{-4}, \beta = 2.3$ and $P = 10^4$. Then $a = 0.433987, b = 2455$ and the reinsurer is obliged to pay $432,921 provided $X = 10^6$.

**Example 3** (Variance principle). Let $P = \mathbb{E}R + \beta \mathbb{D}^2 X$ with $\beta > 0$. Suppose $\mathbb{E}X + \beta \mathbb{D}^2 X > P$. Then an optimal retention function is given by $R^*(x) = a(x - b)_+$, where $a, b$ satisfy the following equations:

$$-2a\beta \mathbb{E}(b - X)_+ + 1 - a = 0, \quad a\mathbb{E}(X - b)_+ + a^2\beta \mathbb{D}^2 (X - b)_+ = P.$$ 

If $X$ is a Bernoulli risk, then

$$a = \frac{[p(4q\beta P + p)]^{1/2}}{p(2q\beta M + 1)}, \quad b = \frac{p + 2pq\beta M - [p(4q\beta P + p)]^{1/2}}{2q\beta [p(4q\beta P + p)]^{1/2}}.$$ 

Let $M = 10^6, p = 10^{-4}, \beta = 1/10^3$ and $P = 10^4$. In this case, $a = 0.316, b = 1082$ and the reinsurer covers $315,743$ if $X = 10^6$.

**Example 4** (Modified variance principle). Suppose $P = \mathbb{E}R + \beta \mathbb{D}^2 X/\mathbb{E}R$, with $\beta > 0$. Then $f(P, t) = (P + (P^2 - 4\beta t^2)^{1/2})/2$ for $0 \leq t \leq P/(2\beta^{1/2})$. If $2\beta^{1/2} \mathbb{D}X < P < \mathbb{E}X + \beta \mathbb{D}^2 X/\mathbb{E}X$, then $R^*(x) = a(x - b)_+$ is an optimal retention function with $a, b$ defined by

$$-2a\beta \mathbb{E}(b - X)_+ + (1 - a)(P^2 - 4\beta a^2 D^2 (X - b)_+)^{1/2} = 0, \quad a\mathbb{E}(X - b)_+ + \frac{a\beta \mathbb{D}^2 (X - b)_+}{\mathbb{E}(X - b)_+} = P.$$ 

Suppose $X$ is a Bernoulli risk. Then

$$a = \frac{2q\beta P + u(p + q\beta)P}{(p + q\beta)(2q\beta M + uP)}, \quad b = M - \frac{2q\beta M + uP}{2q\beta + u(p + q\beta)}$$ 

with $u = (1 - 4pq\beta)/(p + q\beta)^2)^{1/2}$. For concreteness, let $M = 10^6, p = 10^{-4}, \beta = 0.05$ and $P = 10^4$. In this case, $a = 0.272, b = 266,437$ and the reinsurer has to pay $199,620 provided $X = 10^6$. 

3. Local reinsurance

Let \( X, X_1, X_2, \ldots \) be a sequence of identically distributed random variables defined on a common probability space \( (\Omega, \mathcal{S}, \mathbb{P}) \). Let \( N \) be an integer-valued random variable with \( 0 < \mathbb{P}N < \infty \). Throughout this section we assume that the random variables \( N, X, X_1, X_2, \ldots \) are independent. We use \( X_1, X_2, \ldots \) as the sequence of successive claims occurring in a time interval. \( N \) models the number of claims over that period. Consider local reinsurance with a common retention function \( R \), i.e. each claim is divided between the cedent and the reinsurer as follows: for the \( i \)th claim of size \( X_i \) the part \( R(X_i) \) is carried by the reinsurer. The cedent wants to have a reinsurance arrangement which minimizes the variance of his payoff subject to the following constraints:

\[
0 \leq R(X) \leq X \quad \text{and} \quad \mathbb{E}R(X) = f(P, \mathbb{D}R(X)).
\]

The corresponding minimization problem is then given by

\[
\min_{R \in \mathcal{R}(f)} \mathbb{D}^2 \left[ \sum_{i=1}^{N} (X_i - R(X_i)) \right],
\]

where \( \mathcal{R}(f) \) is defined by (2). The problem (8) can be treated in a similar way as that of (3), since

\[
\mathbb{D}^2 \left[ \sum_{i=1}^{N} (X_i - R(X_i)) \right] = \mathbb{E}N\mathbb{D}^2(X - R) + (\mathbb{E}X - \mathbb{E}R)^2\mathbb{D}^2N
\]

with \( R = R(X) \) (see e.g. Rolski et al. (1998, Corollary 4.2.1)). Put

\[
\gamma(s, t) = \mathbb{E}N[\mathbb{D}^2X + (t^2 - 2t)t\mathbb{D}^2(X - s)_+ - 2\mathbb{E}(s - X)_+ f(P, t\mathbb{D}(X - s)_+)]
\]

\[
+ (\mathbb{E}X - f(P, t\mathbb{D}(X - s)_+))^2\mathbb{D}^2N
\]

for \( 0 \leq t \leq 1, s \geq 0 \), and define \( \gamma'_2(s, t) = \partial \gamma(s, t)/\partial t \).

**Theorem 2.**

1. Suppose there exist \( a, b \) such that
   (i) \( 0 < a \leq 1, 0 < b \leq \sup X \),
   (ii) \( \gamma'_2(b, a) = 0 \),
   (iii) \( a\mathbb{E}(X - b)_+ = f(P, a\mathbb{D}(X - b)_+) \).
   Then \( R^*(x) = a(x - b)_+ \) is a solution of the problem (8).

2. Suppose \( \mathbb{E}X > P \) and suppose

\[
-f'_2(P, t(0)\mathbb{D}X) \leq \frac{\mathbb{E}N\mathbb{D}X}{\mathbb{D}^2N\mathbb{E}X},
\]

where \( t(0) \) is a real such that \( t(0)\mathbb{E}X = f(P, t(0)\mathbb{D}X) \). Then there exist reals \( a, b \) such that conditions (i)–(iii) are satisfied.

**Proof.** Taking into account (9) and using the same line of argument as in the proof of Theorem 1, we get

\[
\mathbb{D}^2 \left[ \sum_{i=1}^{N} (X_i - R(X_i)) \right] \geq \gamma(b, t)
\]

for all \( R \in \mathcal{R}(f) \) and \( 0 \leq t \leq 1 \). This becomes equality for \( R(x) = c(x - b)_+, c \) being a real such that \( R \in \mathcal{R}(f) \). Observe that for all \( b, t \geq 0 \),

\[
\mathbb{E}X - f(P, t\mathbb{D}(X - b)_+) \geq \mathbb{E}X - f(P, 0) = \mathbb{E}X - P > 0,
\]

for all \( R \in \mathcal{R}(f) \) and \( 0 \leq t \leq 1 \). This becomes equality for \( R(x) = c(x - b)_+, c \) being a real such that \( R \in \mathcal{R}(f) \).
and, in consequence, the function $\gamma(b, t)$ is convex in $t \geq 0$. By (12) and (ii), for every $R \in \mathcal{R}(f)$,
\begin{equation}
\mathbb{D}^2 \left[ \sum_{i=1}^{N} (X_i - R(X_i)) \right] \geq \gamma(b, a) = \mathbb{D}^2 \left[ \sum_{i=1}^{N} (X_i - R^*(X_i)) \right].
\end{equation}

Since $R^* \in \mathcal{R}(f)$, $R^*$ is a solution of (8) which completes the proof of the first part of the theorem. For the proof of the second part, we use the same arguments as in the proof of Theorem 1 but with one subtle difference. Now $\tau(0)$ may be not equal to 1 and we assume that $t(0) \leq \tau(0)$, i.e.
\begin{equation}
\gamma_2'(0, t(0)) \leq 0,
\end{equation}
which is equivalent to
\begin{equation}
-\mathbb{D}^2 N[\mathbb{E}X - f(P, t(0)\mathbb{D}X)]f_2'(P, t(0)\mathbb{D}X) \leq \mathbb{E}N(1 - t(0))\mathbb{D}X.
\end{equation}

Since $t(0)\mathbb{E}X = f(P, t(0)\mathbb{D}X)$ with $t(0) < 1$ (see (13)), the inequality (11) implies (16), completing the proof.

**Example 5** (Standard deviation principle). Letting $P = \mathbb{E}R + \beta \mathbb{D}R$, suppose $\mathbb{E}X > P$ and suppose $0 < \beta \leq (\mathbb{E}N\mathbb{D}X)/(\mathbb{D}^2 N\mathbb{E}X)$. An optimal retention function is given by $R^*(x) = a(x - b)_+$, where $a$, $b$ satisfy the following equations:
\begin{align*}
-\mathbb{E}(b - X)_+ + \beta &+ \frac{[\mathbb{E}X - P + a\mathbb{D}(X - b)_+]\mathbb{D}^2 N}{\mathbb{E}N} + (1 - a)\mathbb{D}(X - b)_+ = 0, \\
a\mathbb{E}(X - b)_+ + a\beta \mathbb{D}(X - b)_+ &= P.
\end{align*}

We now study the problem (8) when the constraint (2) is relaxed, namely, we assume that $0 \leq \mathbb{E}R(X) \leq \mathbb{E}X$. This leads to the following optimization problem:
\begin{equation}
\min_{R \in \mathcal{R}_\mathbb{E}(f)} \mathbb{D}^2 \left[ \sum_{i=1}^{N} (X_i - R(X_i)) \right],
\end{equation}
where
\begin{equation}
\mathcal{R}_\mathbb{E}(f) = \{ R | 0 \leq \mathbb{E}R(X) \leq \mathbb{E}X, \mathbb{E}R(X) = f(P, \mathbb{D}R(X)) \}.
\end{equation}

Of course, $\mathcal{R}(f) \subset \mathcal{R}_\mathbb{E}(f)$ so the optimal rule, say $R^{**}$, produces a smaller risk than the rule $R^*$ defined in Theorem 2. From now on, we assume $0 \leq \mathbb{D}N < \infty$.

**Theorem 3.** Suppose that the minimum of the function $t \to (1 - t)^2 \mathbb{E}N \mathbb{D}^2 X + (\mathbb{E}X - f(P, t \mathbb{D}X))^2 \mathbb{D}^2 N$ under constraint $0 \leq f(P, t \mathbb{D}X) \leq \mathbb{E}X$ is attained at $t = a$. Then the solution of the problem (17) is given by
\[ R^{**}(x) = a(x - \mathbb{E}X) + f(P, a \mathbb{D}X). \]

**Proof.** By the Cauchy–Schwarz inequality
\begin{equation}
\mathbb{D}^2 (X - R) = \mathbb{D}^2 X - 2 \text{Cov}(X, R) + \mathbb{D}^2 R \geq \mathbb{D}^2 X - 2 \mathbb{D}X \mathbb{D}R + \mathbb{D}^2 R = (\mathbb{D}X - \mathbb{D}R)^2.
\end{equation}

Here the equality holds if $R(x) = tx + s$, with some reals $s, t$. From (9) and (19) it follows that for every
\[ R \in \mathcal{R}_E(f). \]

\[
\mathbb{D}^2 \left[ \sum_{i=1}^{N} (X_i - R(X_i)) \right] \geq \mathbb{E}N(\mathbb{D}X - \mathbb{D}R)^2 + (\mathbb{E}X - f(P,\mathbb{D}R))^2\mathbb{D}^2N
\]

\[
\geq (1 - a)^2\mathbb{E}N\mathbb{D}^2X + (\mathbb{E}X - f(P,a\mathbb{D}X))^2\mathbb{D}^2N = \mathbb{D}^2 \left[ \sum_{i=1}^{N} (X_i - R^{**}(X_i)) \right]. \tag{20}
\]

Since \( R^{**} \in \mathcal{R}_E(f) \), the proof is completed. \( \square \)

**Example 6.** Let \( P = \mathbb{E}R + \beta \mathbb{D}R \). Theorem 3 implies that the optimal rule is given by

\[ R^{**}(x) = a(x - \mathbb{E}X - \beta \mathbb{D}X) + P, \]

where \( a \) minimizes \((1 - t)^2\mathbb{E}N\mathbb{D}^2X + (\mathbb{E}X - P + t\beta \mathbb{D}X)^2\mathbb{D}^2N\) over \( t \in [(P - \mathbb{E}X)/(\beta \mathbb{D}X), P/(\beta \mathbb{D}X)] \).

**Remark 4.** An interesting problem seems to be the analysis of optimal reinsurance under alternative premium calculation rules like the exponential principle, the Esscher principle or the risk-adjusted principle (see Embrechts, 1996; Embrechts et al., 1997; Gerber, 1980; Rolski et al., 1998; Young, 1999; Wang et al., 1997; Wang, 1998).

**Acknowledgements**

I would like to thank the anonymous referee for the helpful suggestions. I also thank Prof. L. Gajek and Prof. D. Zagrody for arousing my interest in this field.

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