EOQ and inflation uncertainty

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Abstract

Inflation uncertainty is introduced into a basic EOQ model and its potential to make mischief is explored. Among other things, it is shown that even when the expected rate of inflation is less than the marginal cost of capital, the appropriate discount rate to use in computing a discounted expected total inventory cost will not necessarily be negative. Therefore, the classic EOQ square-root formula may drastically underestimate the optimal lot size. The results may be generalized to the more complex inventory models.

Keywords: Inventory; Economic order quantity; Inflation; Uncertainty

1. Introduction

Between 1975 and 1985 a series of related papers appeared that in one way or another considered the effects of inflation on inventory [1–5]. In the 1990s, this earlier work, which focused on some fundamentally static inventory models, has been extended to consider more complex and dynamic situations (e.g. [6–10]). In every case of which I am aware, it has been implicitly assumed that the rate of inflation is known with certainty. Yet, inflation enters the inventory picture only because it may have an impact on the present value of the future inventory cost, and the future rate of inflation is inherently uncertain and unstable. In January of 1999, for example, the Producer Price Index for the United States surged 0.5%, although Wall Street economists had expected a 0.1% increase (CBS MarketWatch, February 18, 1999). Then in February 1999, when economists expected the index to drop by 0.1%, it dropped by 0.4% (CNNfn, March 12, 1999). Indeed, there is empirical evidence to suggest both that increases in inflation uncertainty accompany increases in inflation [11], and that increases in inflation uncertainty are preceded by increases in the rate of inflation [12]. There is also evidence to the effect that political instability effects increases in both the rate of inflation and inflation uncertainty [13]. If, therefore, one is going to be introducing the rate of inflation into an inventory model, it would seem, a priori, equally appropriate and at least as important to consider the role and impact of inflation uncertainty in inventory decisions. That is the purpose of this paper.

In particular, inflation uncertainty is introduced into a basic EOQ model à la Bierman and Thomas [2]. Even within this most simple structure both the role of inflation uncertainty and its potential to
make mischief become readily apparent. Specifically, and among other things, it is shown that even when the same uncertain rate of inflation affects all prices and costs, risk-neutral operations managers are a priori worse off as a result of the uncertainty, although the firm as a whole might actually be better off. It is also shown that even when the expected rate of inflation is less than the marginal cost of capital the appropriate discount factor, \( \gamma \), in the expression \( e^\gamma \) will not necessarily be negative, and that therefore the classic EOQ square-root formula may drastically underestimate the optimal lot size. These results may be immediately generalized to the more complex inventory models.

2. The inflation-modified model

The following notation is employed. Let \( p \) denote the cost per unit of an order quantity of \( Q \) units, including handling, insurance, shrinkage and other such costs. Let \( K \) denote the ordering cost per order. Let \( D \) denote the demand rate per year in units, and let \( k \) denote the holding cost per unit per year. Here, \( k \) “is defined as the cost of physical storage only, since the cost of capital tied up is represented by the discounting of future expenditures of \( pQ \)” ([2], p. 152). The total cost per order cycle is therefore

\[
C_0 = K + pQ + kQ^2/2D. \tag{1}
\]

Let \( r \) denote the firm’s discount factor, or its (marginal) cost of capital. Then, the total discounted cost for \( T \) cycles of \( Q/D \) years is given by

\[
TC(Q) = C_0 \sum e^{-r(Q/D)j}, \tag{2}
\]

where the summation is over \( j = 0, \ldots, T - 1 \). With a uniform inflation rate of \( i \) for all costs and the selling price, \( \gamma = i - r \) replaces \( -r \) in Eq. (2). When \( \gamma = 0 \), we obtain the classical EOQ square-root rule for the optimal order quantity; or \( Q^* = (2KD/k)^{0.5} \).

Suppose, however, that the future rate of inflation is not known with certainty. What is known is that \( i \) follows a normal distribution with an expected rate of inflation of \( E[i | v, \sigma^2] = v \) and a variance of \( \sigma^2 \). The good news is that \( \sigma^2 \) is known; the bad news is that \( v \) is not known. Fortunately, our risk-neutral inventory manager has assessed a normal prior density over \( v \) such that \( E[\gamma | \rho, \sigma^2]=\rho \), where \( \sigma^2 \) is the prior variance.

In this situation, the manager’s goal is to choose \( Q = Q^* \) so as to minimize the expected total cost of \( E[TC(Q)] \). To do this, the manager first computes the conditional expectation \( E[e^\gamma | v] = \exp(v + \sigma^2/2) \), and then computes the unconditional expectation \( E[e^\gamma] = \exp(\rho + \sigma^2/2 + \sigma^2/2) \) (see, e.g., [14, p. iv]). Hence,

\[
E[TC(Q)] = E[C_0 \sum e^{(i-r)(Q/D)j}] = C_0 \sum e^{(i-r)(Q/D)j}. \tag{3}
\]

Now, however, \( \gamma = \rho - R + (\sigma^2 + \sigma^2)/2 \). Whether \( \gamma \) is negative or positive thus depends upon three factors. First, whether the company’s cost of capital, \( r \), is greater than the manager’s expectation as to the rate of inflation, \( \rho \). Second, the extent of the manager’s uncertainty about the expected rate of inflation, as reflected in the prior variance, \( \sigma^2 \). And, third, the extent of the uncertainty inherent in the rate of inflation itself, as reflected in the process variance, \( \sigma^2 \). Therefore, the sign of \( \gamma \) is not at all clear. Indeed, the only thing clear about \( \gamma \) is that in our uncertain world it will be positive, rather than zero, when \( \rho = r! \) Moreover, it can and has been argued that the cost of capital is also uncertain [15]. The latter uncertainty comes about because \( r \) is determined by the firm’s marginal investment, which is the last one included in a set of projects. But the last investment included in the set is unknown until the entire set has been determined. Suppose, however, that we allow the inventory manager the luxury of assessing a normal prior density over \( r \), such that \( r = N(R, \sigma^2) \). Then, proceeding as above, we finally determine

\[
\gamma = \rho - R + (\sigma^2 + \sigma^2 + \sigma^2)/2. \tag{4}
\]

The latter expression is even less likely to be negative than were the previous expressions for \( \gamma \).

In any event, it is readily determined that

\[
\frac{d^2 TC(Q)/d\gamma^2}{C_0(Q/D)^3 \sum j^2 e^{(i-r)(Q/D)j}} > 0.
\]

Thus, total cost is a convex function of \( i \). Therefore, by Jensen’s Inequality [16, p. 29],

\[
E[TC(Q^*)] > TC(Q^*|i \equiv \rho).
\]
That is, expecting over the uncertain inflation rate, the expected discounted total cost at the optimum (or any other) lot size, is greater than the total cost evaluated at the expected rate of inflation. Therefore, even the risk-neutral operations manager prefers a higher but known rate of inflation to an uncertain and lower rate of inflation. Since \( \frac{\partial^2 \text{TC}(Q)}{\partial r^2} > 0 \), too, despite \( \frac{\partial \text{TC}(Q)}{\partial r} < 0 \), the risk-neutral operations manager would similarly prefer a lower but known cost of capital to a higher and uncertain cost of capital. At least where managing inventory is concerned, these sorts of uncertainties would appear to be generally discomfitting.

By the same token, let \( P \) denote the price for which the inventoried product sells at time \( t = 0 \). Suppose the first unit in any cycle is sold immediately upon receipt, the next unit is sold \( 1/D \) years later, and so forth. Then, the discounted revenue received during the first order cycle will be given by

\[
R_0(Q^*) = P + P[(1 + i)/(1 + r)]^{1/D} + P[(1 + i)/(1 + r)]^{2/D} + \cdots + P[(1 + i)/(1 + r)]^{(Q^* - 1)/D}.
\]

Let \( d = [(1 + i)/(1 + r)]^{1/D} \). We now can write \( R_0(Q^*) = Pd^k \), where the summation runs from \( k = 0 \) to \( k = Q^* - 1 \). When \( i = r \), \( d = 1 \) and \( R_0(Q^*) = PQ^* \), since \( \sum(1)^k \) comprises \( Q^* \) terms. From [16, p. 33], when \( i \neq r \),

\[
R_0(Q^*) = P[(1 - d^Q)/(1 - d)].
\]

Hence, comparable to Eq. (2) for total cost, over \( T \) cycles of \( Q/D \) years the inventoried product will be generating a stream of total discounted revenues of

\[
\text{TR}(Q^*) = R_0(Q^*)\sum e^{(i-r)(Q/D)j}.
\]

And, as is the case with the discounted cost, the discounted revenue stream is also a convex function of both discount factors. Therefore,

\[
E[\text{TR}(Q^*)] > \text{TR}(Q^*)|i = \rho).
\]

Whether the risk-neutral managers of the firm as a whole would prefer the variable and uncertain rate of inflation or cost of capital to their invariant prior expectations thus depends upon the relative convexities of the \( \text{TR}(Q^*) \) and \( \text{TC}(Q^*) \) functions, with respect to the two discount factors; or, upon the convexity of \( H = \text{TR}(Q^*) - \text{TC}(Q^*) \) with respect to the discount factors.

3. Solving the model

To determine the effects of inflation-rate uncertainty on the optimal fixed lot size, \( Q^* \), when \( \gamma \neq 0 \), it useful to write, as above,

\[
\sum e^{(Q/D)j} = \left[1 - e^{(Q/D)T}\right]/\left[1 - e^{(Q/D)}\right].
\]

For reasons that will shortly become evident, suppose that the inventory-planning horizon extends over \( T = \alpha(D/Q) \) cycles. Here, \( \alpha > 0 \) is an arbitrary parameter. This parameter is required to be finite only when \( \gamma > 0 \), and it is otherwise unrestricted. Then,

\[
\sum e^{(Q/D)j} = \left[1 - e^{\gamma\alpha}\right]/\left[1 - e^{(Q/D)}\right].
\]

After making this substitution in Eq. (3), it is also useful to rewrite the resulting equation as

\[
E[\text{TC}(Q)] = \left[C_0(D/Q)[(Q/D)(1 - e^{\gamma\alpha}]/(1 - e^{(Q/D)})]\right].
\]

As shown in the Appendix, the expected total cost is minimized where

\[
- \frac{KD/Q^2}{2} + k/2 = -(C_0D/Q^2)[1 + \gamma(Q/D)e^{\gamma\alpha}/(1 - e^{(Q/D)})].
\]

That is, irrespective of the sign of \( \gamma \), the inventory-planning time horizon, as reflected in \( k \), is irrelevant to the determination of the optimal lot size of \( Q^* \). Thus, the introduction of \( \alpha \) allows us to readily derive Eq. (5). Once having fulfilled this critical end, \( k \) does not reappear in any of the subsequent analyses. As is also shown in the Appendix, the second-order condition for a minimum necessarily holds when \( \gamma \leq 0 \). Finally, it is shown that a “sufficiently large” \( Q^* \) will always exist for which the second-order condition holds when \( \gamma > 0 \), should that condition not hold for a “small” \( Q^* \).
When $\gamma = 0$, $\sum e^{i(Q/D)j} = D/Q$, because the summation of $e^0 = 1$ is over a total of $T = D/Q$ cycles. In this fortuitous happenstance, the optimum lot size is determined where the bracketed term on the left-hand side of Eq. (5) is equal to zero; or, $Q*_{\gamma = 0} = (2KD/k)^{0.5}$. Otherwise, the sign of the bracketed term at the lot-size optimum will be the opposite of the sign of the term in square brackets that is on the right-hand side of the equation.

To determine the sign of the latter term, let $y = yQ/D$ and multiply numerator and denominator of the resulting expression by $e^{-y}$ so that

$$[1 + (yQ/D)e^{yQ/D}(1 - e^{yQ/D})]$$

$= 1 + y/(e^{-y} - 1)$

$= (e^{-y} - 1 + y)/(e^{-y} - 1)$.

Substituting $e^y = \sum y^j/j! (t = 0, \ldots, \infty)$ [16, p. 109], and dividing numerator and denominator by $y$, the latter equation may be written as

$$(1 - y + y^2/2 - y^3/6 + \cdots - 1 + y)/(1 - y + y^2/2 - y^3/6 + \cdots - 1)$$

$= (y/2 - y^2/6 + \cdots)/(1 + y/2 - y^2/6 + \cdots).$

When $1 > y > 0$, the latter expression is seen to be negative but greater than $-1$; or, $-1 < [1 + (yQ/D)e^{yQ/D}(1 - e^{yQ/D})] < 0$ for $\gamma > 0$. When $-1 < y < 0$, the latter expression is positive, but less than $+1$; or, $1 > [1 + (yQ/D)e^{yQ/D}/(1 - e^{yQ/D})] > 0$ for $\gamma < 0$. Therefore, $[- KD/Q^2 + k/2]$ and $y$, or equivalently and more critically $\gamma$, will be of the same sign.

By the second-order condition for a minimum, $\partial^2 E[TC(Q)]/\partial Q^2 > 0$. Thus the inference that with $\gamma < 0$ the optimum lot size is determined where $[- KD/Q^2 + k/2] < 0$, further implies a smaller optimum lot size of $Q*_{\gamma < 0} < Q*_{\gamma = 0}$ than will obtain either when $\gamma = 0$, or in the classic EOQ model. Alternatively, $\gamma > 0$ implies a larger optimum lot size of $Q*_{\gamma > 0} > Q*_{\gamma = 0}$ than in the classic model. Whether there is a substantive difference between $Q*_{\gamma > 0}$ and $Q*_{\gamma = 0}$, at least when $\gamma < 0$, is another matter [4].

The introduction of inflation uncertainty into the picture, however, lessens the likelihood of $\gamma < 0$, and Chandra and Bhaner demonstrate “the importance of taking into account inflation and time discounting, especially when inflation rates are high” [5, p. 729]. That statement can now be amended to say “or when there is considerable uncertainty as to either the inflation rate or the marginal cost of capital.”

4. Comparative statics

The effect on $Q^*$ of a change in any particular parameter, $\lambda$, may be determined by totally differentiating $\partial E[TC(Q^*)]/\partial \lambda$ with respect to $\lambda$ and then rearranging terms to obtain $dQ^*/d\lambda = -[\partial^2 E[TC(Q^*)]/\partial Q \partial \lambda]/[\partial^2 E[TC(Q^*)]/\partial Q^2]$. The denominator is positive by the second-order condition. Therefore, the sign of $dQ^*/d\lambda$ will be the opposite of that of the term in the numerator – the cross-partial derivative. As shown in the Appendix, however, we may write $\partial E[TC(Q^*)]/\partial Q$ as

$$\partial E[TC(Q^*)]/\partial Q$$

$= \beta[1 + (\gamma Q/D)(e^{-\gamma Q/D} - 1)]$$

$= [KD/Q + pD + kQ/2]$$

$+ [1 + (Q/D)/(e^{-\gamma Q/D} - 1)].$ (6)

Since $\beta > 0$, the sign of the cross-partial derivative with respect to $\lambda$ will be the same as that of $\partial^2 \cdot /\partial \gamma$. Looking first at $\gamma$, the sign of $\partial (\cdot )/\partial \gamma$ will depend solely upon the sign of $[\partial yQ/D]/\partial \gamma$. After differentiating, that sign is seen to depend solely upon the sign of $e^{-\gamma Q/D} - 1 + (\gamma Q/D)e^{-\gamma Q/D}$. Once again letting $y = \gamma Q/D$ and taking the series expansion of $e^{-\gamma}$, the latter sum is immediately seen to be negative for all $\gamma$. Hence, $dQ^*/d\gamma > 0$. Put otherwise, not only does $\gamma > 0$ effect $Q^*_{\gamma > 0} > Q^*_{\gamma = 0}$, but the larger is that positive value of $\gamma$, the greater will be the extent to which the classic EOQ square-root rule underestimates the optimal lot size under inflation and uncertainty as to the future rate of inflation.

One of the parameters that contributes to a larger $\gamma$ is the variance of the prior density, $\sigma^2$. That prior variance can always be expressed in terms of the process variance as $\sigma^2 = \sigma^2/n'$, where $n'$ reflects the extent of the manager’s prior information as to
the process mean, or in this case the expected rate of inflation. When in typical Bayesian fashion the prior density is revised as new (sample) information is received, the manager's posterior density will also be normal. Regardless of what that new information happens to be, the variance of the posterior density will always be given by \( \sigma^2 = \frac{\sigma^2}{n} + \frac{n'}{n} \), where \( n \) is the number of sample observations [17, p. 128]. Therefore, the term \( \sigma^2 + \sigma^2 \) in Eq. (4) can be replaced by \( \frac{n + n'}{n} + \frac{n'}{n} \rightarrow \sigma^2 \)

as \( n \rightarrow \infty \). As a consequence, when the manager's judgments as to the expected rate of inflation are systematically revised as new data about the rate of inflation are received, ceteris paribus \( \gamma \) will decline. The implication of this decline is that, ceteris paribus, we can anticipate that the optimum lot size will also decline over time.

The cross-partials with respect to \( K, k, \) and \( p \) are also readily computed and their signs readily determined:

\[
\frac{\partial \gamma}{\partial K} = \frac{\gamma}{(e^{-\gamma Q}D - 1)} < 0, \quad (7a)
\]

\[
\frac{\partial \gamma}{\partial k} = \frac{Q}{2} - \frac{(Q/D)(e^{-\gamma Q}D - 1)}{D} > 0, \quad (7b)
\]

\[
\frac{\partial \gamma}{\partial p} = \frac{D[1 - (Q/D)(e^{-\gamma Q}D - 1)]}{D[1 + (Q/D)(e^{-\gamma Q}D - 1)]} = \pm. \quad (7c)
\]

The sign of \( \gamma \) is the opposite of that of \( (e^{-\gamma Q}D - 1) \). Therefore, \( \frac{\partial \gamma}{\partial K} < 0 \) and hence \( \frac{\partial Q^*}{\partial K} > 0 \). As in the classic EOQ model, the optimum lot size increases when the set-up cost increases. Also as in the classic EOQ model, the optimum lot size decreases when the carrying cost increases.

This is so, since as previously determined, \( |1 + (\gamma Q/D)(e^{-\gamma Q}D - 1)| < 1 \), which means that \( \frac{\partial \gamma}{\partial \gamma} > 0 \), and therefore that \( \frac{dQ^*}{dk} < 0 \). The sign of \( \frac{\partial \gamma}{\partial p} \), however, will be the same as the sign of \( 1 + (\gamma Q/D)(e^{-\gamma Q}D - 1) \), and therefore it will be the opposite of the sign of \( \gamma \). Hence, \( \frac{dQ^*}{dp} \) and \( \gamma \) have the same sign.

The latter result asserts the following. Suppose either the expected rate of inflation exceeds the expected marginal cost of capital, or that there is sufficient uncertainty inherent in either the inflation rate or the cost of capital so as to effect \( \gamma > 0 \). Then, the greater is the cost of acquiring the product to be inventoried, the greater will be the optimal lot size. When the expected rate of inflation is sufficiently lower than the expected marginal cost of capital, so that even allowing for the uncertainty factor we get \( \gamma < 0 \), then higher product-acquisition costs result in lower lot sizes. In essence, fears of inflation or uncertainty about inflation encourage greater purchases at “current” prices. With a high marginal cost of capital and lower inflation rates, however, uncertainty notwithstanding it will be preferable to tie up less capital than otherwise in acquiring inventory.

5. A numerical example

To illustrate the results, consider a central situation in which \( p = 1, k = 2, K = 100, \) and \( D = 100 \). The classic EOQ square-root rule results in an optimal order quantity of \( Q^* = 100 \). Table 1 shows how this optimal order quantity changes

| Table 1 |
| A numerical example, \( p = 1, k = 2 \) |

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( K = 75 )</th>
<th>( k = 1.5 )</th>
<th>( p = 0.75 )</th>
<th>( K = 100, D = 100 )</th>
<th>( p = 1.25 )</th>
<th>( k = 2.5 )</th>
<th>( K = 125 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.3</td>
<td>73</td>
<td>93</td>
<td>85</td>
<td>84</td>
<td>82</td>
<td>76</td>
<td>92</td>
</tr>
<tr>
<td>-0.2</td>
<td>77</td>
<td>99</td>
<td>89</td>
<td>88</td>
<td>87</td>
<td>80</td>
<td>98</td>
</tr>
<tr>
<td>-0.1</td>
<td>81</td>
<td>106</td>
<td>94</td>
<td>93</td>
<td>93</td>
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<td>104</td>
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<td>87</td>
<td>115</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>89</td>
<td>112</td>
</tr>
<tr>
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<td>93</td>
<td>127</td>
<td>108</td>
<td>109</td>
<td>110</td>
<td>96</td>
<td>122</td>
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<td>102</td>
<td>148</td>
<td>118</td>
<td>121</td>
<td>123</td>
<td>102</td>
<td>138</td>
</tr>
<tr>
<td>0.3</td>
<td>116</td>
<td>197</td>
<td>136</td>
<td>142</td>
<td>148</td>
<td>118</td>
<td>167</td>
</tr>
</tbody>
</table>

*Note: All numbers have been rounded off to the nearest integer.*
with varying values of \( \gamma \) in the \([ -0.3, 0.3] \) range, and for different values of \( p, k, \) and \( K \), considered individually.

For ease of interpretation, all the optimal order quantities have been rounded off to the nearest integer. The figures may be verified by substitution into Eq. (5). The numbers in the table are interpreted as follows. In the central case, when \( \gamma = -0.3 \) the optimal order quantity decreases from its classic \( Q^*_y = 100 \) level by 15% to 85. By contrast, when \( \gamma = 0.3 \), the optimal order quantity increases by 42% to 142. When \( p \), say, increases to 1.25, the \( Q^*_y = 100 \) value is unaltered, but the other (rounded off) optimal inventory levels decline or remain unchanged for \( \gamma < 0 \), and increase for \( \gamma > 0 \). Precisely the reverse is true when \( p \) declines to 0.75. That is, when \( \gamma = -0.2 \), for example, the optimal inventory increases from 88 to 89 when \( p \) declines, but it decreases from 121 to 118 when \( \gamma = 0.2 \). These results are what were foretold by the comparative statics implications of Eq. (7c). That is, when \( \gamma < 0 \), increases in the selling price result in decreased optimal lot sizes; and, when \( \gamma > 0 \), increases in the selling price result in increased optimal lot sizes.

Similarly, the results in the \( K \) and \( k \) columns were foretold by the comparative statics implications of Eqs. (7a) and (7b). Regardless of the sign of \( \gamma \), increases in the ordering cost increase the optimal lot size, and increases in the holding cost reduce the optimal lot size.

In all instances it is apparent that \( Q^* \) is an increasing and strictly concave function of \( \gamma \). In this regard, the example affirms that although negative values for \( \gamma \) will not cause drastic deviations from \( Q^*_y = 0 \) unless \( | - \gamma | \) is fairly substantial, even quite modest positive values of \( \gamma \) can wreak havoc with \( Q^*_y = 0 \). Thus, looking at Eq. (4), suppose that the expected marginal cost of capital, \( R \), is equal to the expected rate of inflation, \( \rho \). It is not hard to imagine circumstances under which the sum of the three variances in that equation would equal 0.4, say. Under those circumstances, \( \gamma = 0.2 \). Ignoring the uncertainty about the cost of capital and the rate of inflation would result in setting \( Q^*_y = 0 \), in the central case, whereas taking those uncertainties into consideration results in \( Q^*_y = 121 \). In this example, then, ignoring the implications of uncertainty results in a 17.4% understatement of the optimal lot size.

### 6. Conclusions

One of the few things upon which virtually all economists agree is that prices next year – any prices – are likely to be higher than are prices this year – any year. The only issue separating these economists is “how much” higher, or what the future rate of inflation will be. This is the case even though, for example, in recent years the United States as a case in point has enjoyed its lowest rates of inflation in over a decade.

The pervasiveness and reality of inflation is also recognized in operations management. For the past two decades the recognition has prompted attempts to incorporate inflation into the analysis of inventory systems and to evaluate its impact upon optimal inventory policy in a dynamic and evolving world. The present paper continues that line of inquiry. The focus of that inquiry, however, is on how and whether uncertainty as to the rate of inflation will impact upon optimal inventory policy in that world.

The present exploration of the uncertainty issue takes advantage of all the simplifying assumptions of the classic EOQ model. That model continues to attract interest and to prompt extension [18,19]. In the present extension, it has been further assumed that the same rate of inflation would obtain for all of the model’s individual costs and prices. Insofar as the rate of inflation is determined through a stochastic process, the process was assumed to be normal with a known variance. Insofar as the mean of the process is unknown, it was assumed that the operations manager would assess a normal density over that unknown mean. And the manager was assumed to be risk neutral, to boot. Yet, even within this setting it was shown that inflation and inflation-rate uncertainty, in particular, can indeed affect the optimal lot size, and that even risk-neutral managers would prefer to avoid this form of uncertainty. It was also shown that considerations of the inflation rate and the marginal cost of capital, as well as uncertainty as to those two factors affect how changes in the acquisition cost of the
Appendix A

To establish the first-order condition for a minimum, from Eq. (3)
\[
E[TC(Q)] = [KD/Q + pD + kQ/2][Q/D] \\
\times [(1 - e^{\gamma y})(1 - e^{\gamma Q/D})].
\]
Partially differentiating with respect to Q, we obtain
\[
\partial E[TC(Q)]/\partial Q = [(1 - e^{\gamma y})(1 - e^{\gamma Q/D})][1/D] \\
\times [\{ - KD/Q^2 + k/2\}Q \\
+ [KD/Q + pD + kQ/2] \\
+ [KD/Q + pD + kQ/2][\gamma Q/D] \\
\times e^{\gamma Q/D}/(1 - e^{\gamma Q/D})] \\
= \beta[\{ - KD/Q + kQ/2\} \\
+ [KD/Q + pD + kQ/2] \\
\times [1 + (\gamma Q/D)/(1 - e^{\gamma Q/D})]].
\]
Here, \( \beta > 0 \), since \((1 - e^{\gamma y})\) and \((1 - e^{\gamma Q/D})\) will be of the same sign for any value of \( \gamma \). Setting \( \partial E[TC(Q)]/\partial Q = 0 \) only requires that we set \( \{ \cdot \} = 0 \); or,
\[
[ - KD/Q^2 + k/2]Q + C_0 [D/Q] \\
\times [1 + (Q/D)e^{\gamma Q/D}/(1 - e^{\gamma Q/D})] = 0.
\]
Eq. (5) immediately follows.

The second-order condition for a minimum is that
\[
[\partial^2 \{ \cdot \}/\partial Q^2] = \partial^2 \{ \cdot \}/\partial Q^2 [1/D] = 0.
\]
But, \( \{ \cdot \} = 0 \) from the first-order condition, and
\[
[(1 - e^{\gamma y})(1 - e^{\gamma Q/D})][1/D] > 0.
\]
Therefore the second-order condition boils down to \( \partial^2 \{ \cdot \}/\partial Q^2 > 0 \); or,
\[
\partial^2 \{ \cdot \}/\partial Q^2 = [KD/Q^2 + k/2] + [ - KD/Q^2 + k/2] \\
\times [1 + (\gamma Q/D)e^{\gamma Q/D}/(1 - e^{\gamma Q/D})] \\
+ [KD/Q + pD + kQ/2] \\
\times \partial \{ \cdot \}/\partial Q [1/D] > 0.
\]
Now,
\[
\partial^2 \{ \cdot \}/\partial Q^2 = (\gamma Q/D)e^{\gamma Q/D}/(1 - e^{\gamma Q/D})\partial^2 \{ \cdot \}/\partial Q^2 + (\gamma Q/D)e^{\gamma Q/D}/(1 - e^{\gamma Q/D})\partial^2 \{ \cdot \}/\partial Q.
\]
Gathering terms and for notational convenience, again letting \( y = \gamma Q/D \), the expression to the left of the inequality may be written as
\[
k + [ - KD/Q^2 + k/2]y e^{y/(1 - e^{\gamma})} \\
+ [KD/Q + pD + kQ/2]y Q/(e^{y/(1 - e^{\gamma})}) \\
\times [1 + y/(1 - e^{\gamma})].
\]
From the first-order condition, \([ - KD/Q^2 + k/2] = - (C_0 D/Q^2)[1 + ye^{y/(1 - e^{\gamma})}] \). Making this substitution into the previous equation and noting that \([KD/Q + pD + kQ/2] = C_0 (D/Q) \), the equation may now be rewritten as
\[
k - (C_0 D/Q^2)[1 + ye^{y/(1 - e^{\gamma})}]\{ye^{y/(1 - e^{\gamma})}\} \\
+ (C_0 D/Q^2)[ye^{y/(1 - e^{\gamma})}]\{1 + y/(1 - e^{\gamma})\} \\
= k + (C_0 D/Q^2)[ye^{y/(1 - e^{\gamma})} - 1].
\]
The latter expression is necessarily positive when \( \gamma < 0 \), since in that case \( e^{-\gamma Q/D} > 1 \). It is also easily verified that the second-order condition holds for \( \gamma = 0 \). The case of \( \gamma > 0 \), however, is more complicated, since under those circumstances \( e^{-\gamma Q/D} - 1 < 0 \).

For \( \gamma > 0 \), and once again taking advantage of the series expansion of \( e^{-\gamma Q/D} \), we may write
\[
y^{\gamma + 1} = [ - \gamma Q/D][1 - \gamma Q/2D + \gamma^2 Q^2/6D^2 - \ldots].
\]
Therefore,
\[
k + (C_0 D/Q^2)[y^{\gamma + 1}/(e^{\gamma + 1}) - 1] \\
= k - \gamma [KD^2/Q^3 + pD^2/Q^2 + kQ/Q]/ \\
[1 - \gamma Q/2D + \gamma^2 Q^2/6D^2 - \ldots].
\]
As \( Q \to \infty \), \( KD^2/Q^3 + pD^2/Q^2 + kQ/Q \to 0 \), whereupon the entire expression approaches \( k > 0 \). Therefore, even when it fails to hold for a relatively small value of \( Q^* \), the second-order condition will necessarily hold for a sufficiently large value of \( Q^* \).
When others have solved this and comparable problems for the case of $\gamma > 0$, they have done so through some search procedure (see, e.g., [7]). As the present theoretical analysis would have pre-saged, the resulting solutions have indeed been large lot sizes, with the size increasing dramatically as the value of $\gamma > 0$ increases.

References