The manufacturing of heavy weight cardboard

Gerard Sierksma\textsuperscript{a, *}, Henrico L.T. Wanders\textsuperscript{b}

\textsuperscript{a}Department of Econometrics, University of Groningen, P.O. Box 800, 9700 AV Groningen, Netherlands
\textsuperscript{b}Faculty of Management and Organization, University of Groningen, P.O. Box 800, 9700 AV Groningen, Netherlands

Received 26 February 1998; accepted 7 May 1999

Abstract

The cardboard company Kappa Graphic Board produces several kinds of cardboard with various weights and thicknesses. If the thickness is above a certain value (namely 1.7 mm), the cardboard is composed of three layers which are glued together, and is called heavy weight cardboard. The three layers are produced on different machines; the production speed depends on the thickness of the layers. In this paper we restrict our analysis to heavy weight cardboard. The objective is to find a composition of the layers such that the throughput time is minimized. The problem is solved by means of an LP-model in which the objective function is a linearized working hours function; also a number of constraints are linearized. The results of the calculations indicate that it is possible to decrease the total number of working hours in the production process without a large increase of production costs. © 2000 Elsevier Science B.V. All rights reserved.

Keywords: Linear optimization; Cardboard production

1. The cardboard production process

The Dutch cardboard company Kappa Graphic Board produces different kinds of cardboard with various weights and thicknesses, namely Eska Puzzlegreen, Eska Mono/Duo, Eska Print, Eska Screen, Eska Lux and Eska Board. Cardboard with a thickness of over 1.7 mm, the so-called heavy weight cardboard, is manufactured from three layers. It is because of the fact that the drying process causes warping that thick cardboard is manufactured by gluing three layers together. The two outer layers have the same thickness. The thicknesses of the outer and inner layers may vary. All cardboard is produced on three machines in a continuous process. The production speed mainly depends on the thickness of the layers. The following specifications can be chosen by the customer: the thickness of the cardboard, the dimensions of the cardboard, and the direction of the cardboard cutting. The thickness of the cardboard that we consider in this paper is usually between 1.7 and 4.0 mm. The dimensions of the cardboard are restricted by the machine width. The knives of the cardboard machine can cut the cardboard horizontally as well as vertically. The inputs of the production process are: waste paper (mostly old newspapers), water, additives or substances, and if necessary, laminating paper and adhesives.

The production process consists successively of the following main steps. The first step is the...
transformation of input material (waste paper) into pulp by mixing the waste paper with hot water. After filtering, sieving, and pressing, the liquid percentage drops to 50%. After the remaining water is evaporated, the inner layer is ready. This inner layer is now laminated with the two outer layers. These outer layers are taken from stock. The product is now ready, except for its dimension specifications, which are performed by the cutting machine. The cut cardboard is stored on pallets, and after a quality control in the wrapping street, the product is either ready or directed to the final processing department. In the final processing department the cardboard might be cut into small formats.

Fig. 1 shows the production process schematically. In this figure the triangles denote storage points. Note that the outer layers are input in the production process. The several types of cardboard are manufactured on three machines. One of the machines, called the PM-7, is used for the production of the outer layers. The other two machines, which are used for the production of the inner layer, are called KM-6 and KM-8, respectively.

In this paper we derive for the possible demanded thicknesses of heavy weight cardboard a choice of the three layers in such a way that the process speed is maximized.

We assume that the company produces in the current situation eight different cardboard thicknesses, with eight different inner layer thicknesses and five different outer layer thicknesses. In reality all thicknesses between 1.7 and 4.0 mm can be produced. PM-7 is the bottleneck machine, because of its restricted production capacity. KM-6 produces all types of cardboard, while on KM-8 only Eska Board is produced, both laminated and unlaminated. Approximately 33% of KM-6 hours and 80% of the KM-8 hours are reserved for the production of laminated Eska Board. The outer layer, produced on the PM-7, is used both as input for the laminated Eska Board and for the production of Eska Puzzlegreen. About 95% of the available machine hours is used for the production of outer layer to be used for heavy weight Eska Board. Table 1 shows the total available machine hours and the machine hours allocated for the production of laminated Eska Board.

The problem is to derive an optimal composition of the inner layer and the outer layers, such that the demanded thicknesses are satisfied and the total working hours are minimized (i.e. the throughput time is minimized). Note that it is also possible to minimize the working hours time for either the PM-7, or the KM-8, or the KM-6 separable, or the KM-6 and the KM-8 combined. The machine costs per hour are assumed to be equal for each machine. There are two types of machine hours, namely the machine hours of the inner layer (KM-6 and KM-8), and the machine hours of the outer layers (PM-7). We do not consider setup times, because these do not influence the solutions.

### Table 1

<table>
<thead>
<tr>
<th></th>
<th>KM-6</th>
<th>PM-7</th>
<th>KM-8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total available machine hours</td>
<td>8000</td>
<td>6000</td>
<td>8000</td>
</tr>
<tr>
<td>Allocated machine hours</td>
<td>2677</td>
<td>5656</td>
<td>6465</td>
</tr>
</tbody>
</table>
Table 2 shows the relationship between the thickness and the weight of the cardboard. In reality, this relationship is not quite unambiguous, since it slightly depends on the time (seasonality effects) and on the type of machine. We distinguish eight different thicknesses.

### Table 2
Relationship between thickness and total weight

<table>
<thead>
<tr>
<th>Index</th>
<th>Thickness (mm)</th>
<th>Weight (g/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.90</td>
<td>1140</td>
</tr>
<tr>
<td>2</td>
<td>2.00</td>
<td>1190</td>
</tr>
<tr>
<td>3</td>
<td>2.25</td>
<td>1325</td>
</tr>
<tr>
<td>4</td>
<td>2.40</td>
<td>1395</td>
</tr>
<tr>
<td>5</td>
<td>2.50</td>
<td>1465</td>
</tr>
<tr>
<td>6</td>
<td>2.75</td>
<td>1600</td>
</tr>
<tr>
<td>7</td>
<td>3.00</td>
<td>1750</td>
</tr>
<tr>
<td>8</td>
<td>3.50</td>
<td>2035</td>
</tr>
</tbody>
</table>

2. A linear programming formulation

The following assumptions and restrictions have to be taken into account:

(a) The company produces eight different thicknesses of heavy weight Eska Board, varying from 1.9 to 3.5 mm. Each thickness has its own inner and outer layers.

(b) The cardboard consists of one inner layer and two outer layers; the two outer layers have the same weight.

(c) The total amounts of cardboard of the various thicknesses are determined exogenously.

(d) There are no shortages in machine hours. Moreover, the whole demand has to be produced in the available machine hour time.

(e) The production speeds are based on norm speeds, which are mainly determined by the thicknesses of the layers; the production speed is a nonlinear decreasing function of the weight.

(f) During the production of outer layer on the PM-7 a certain percentage of waste must be taken into account, caused by the transportation and the gluing process.

(g) The machine widths are different: 2.80 m for the PM-7 and the KM-8. For the KM-6 the width is 2.53 m.

(h) The inner layer weight varies between 500 and 1040 g/m², whereas the outer layer weight varies between 200 and 600 g/m².

(i) The sum of the weights of the two outer layers is at most the weight of the inner layer.

(j) The weight of the outer layers of the 1.9, 2.0, and 2.25 mm cardboard is at most 250 g/m².

(k) The 2.75 mm cardboard is only produced on the KM-8.

We use the following indices:

- \( i \) = the thickness index for \( i = 1, \ldots, 8 \);
- \( k \) = the type of cardboard machine for \( k = 1, 2 \);
- \( k = 1 \) refers to KM-6 and \( k = 2 \) to KM-8.

The exogenous variables are:

- \( \text{weight}(i) \) = the total weight of the laminated cardboard of thickness \( i \) in g/m²;
- \( \text{amount}(k, i) \) = the total amount of cardboard of thickness \( i \) to be manufactured on machine \( k \) (tonnes);
- \( \text{hours}(k) \) = the total available machine hours on machine \( k \);
- \( \text{width}(k) \) = the machine width of machine \( k \) (meters);
- \( \text{hours7} \) = the total available machine hours on PM-7 for the production of outer layer as input for the KM-6 and the KM-8;
- \( \text{width7} \) = the machine width of PM-7 (meters);
- \( \text{waste}(k) \) = percentage of outer layer waste produced on PM-7 as input for the KM-6 and KM-8.

Although the allocation to the cardboard machines and the total amount of cardboard (distinguished in thicknesses) is exogenously determined, the composition of the inner and outer layers is to be determined.

The endogenous variables are:

- \( \text{innerweight}(k, i) \) = the weight (g/m²) of the inner layer of cardboard \( i \) produced on machine \( k \);
outerweight\((k, i)\) = the weight \((g/m^2)\) of the outer layer of cardboard \(i\) produced on machine \(k\);

innerspeed\((k, i)\) = the production speed of cardboard \(i\) on machine \(k\). This variable includes both the production of the inner layer, and the gluing of the outer layers;

outerspeed\((k, i)\) = the production speed of outer layer \(i\) on machine \(k\);

innerhour\((k, i)\) = the number of hours used on machine \(k\) for the production of cardboard type \(i\);

outerhour\((k, i)\) = the number of hours used on PM-7 for the production of outer layers \(i\) for machine \(k\).

The objective is to minimize the total number of hours used on the machines KM-6, PM-7, and KM-8, i.e.,

\[
\min \left( \sum_{k=1}^{2} \sum_{i=1}^{8} \text{innerhour}(k, i) \right.
\]

\[
+ \sum_{k=1}^{2} \sum_{i=1}^{8} \text{outerhour}(k, i) \right).
\]

(1)

The following constraints are needed. Constraint (2) indicates that laminated cardboard consists of one inner layer and two outer layers, which is the mathematical formulation of assumption (b). The weight of the glue, about 25 g/m², does not influence the results of the model and is therefore not used in the model.

\[
\text{innerweight}(k, i) + 2 \times \text{outerweight}(k, i) = \text{cardboard}(i).
\]

(2)

Formulas (3) and (4) determine the lower and upper bounds of the weights of the outer and inner layer respectively, and correspond to assumption (h).

\[
500 \leq \text{innerweight}(k, i) \leq 1040,
\]

(3)

\[
200 \leq \text{outerweight}(k, i) \leq 600.
\]

(4)

Formula (5) restricts the weight of the outer layers in such a way that the weight of the two outer layers are at most the weight of the inner layer. This constraint corresponds to assumption (i).

\[
2 \times \text{outerweight}(k, i) \leq \text{innerweight}(k, i).
\]

(5)

Constraint (6), which models assumption (k), restricts the weight of the outer layers of the 1.9, the 2.0 and the 2.25 mm cardboard to 250 g.

\[
\text{outerweight}(k, i) \leq 250 \quad \text{for} \quad i = 1, 2, 3.
\]

(6)

Formula (7) requires that the production of the inner layer on machine \(k\) is limited by the total available machine hours on machine \(m\). Eq. (8) gives a similar restriction for the production of the outer layers on PM-7. For both (7) and (8) assumption (d) is used, which indicates that all production must be manufactured within the available time and that no shortages are allowed.

\[
\sum_{i=1}^{8} \text{innerhour}(k, i) \leq \text{hours}(k),
\]

(7)

\[
\sum_{k=1}^{2} \sum_{i=1}^{8} \text{outerhour}(k, i) \leq \text{hours7}.
\]

(8)

We will now derive a formulation to calculate the number of hours that are necessary to produce a certain amount of tons of the inner layer. By \(\text{innerpaper}(k, i)\) we denote the amount of inner layer of thickness \(i\) (t) to be produced on machine \(k\). Since each type of cardboard has one inner layer, we have that

\[
\text{innerpaper}(k, i) = \frac{\text{innerweight}(k, i)}{\text{weight}(i)} \times \text{amount}(k, i).
\]

It can be seen that

\[
0.001 \times \text{innerweight}(k, i) \times \text{length}(k, i) \times \text{width}(k)
\]

\[
= 1000 \times \text{innerpaper}(k, i)
\]

with \(\text{length}(k, i)\) the amount of cardboard (in meters), of thickness \(i\) produced on machine \(k\). The entities 0.001 and 1000 are correction factors for balancing the dimensions for the weight variables (kg). It also follows that

\[
\text{length}(k, i) = 60 \times \text{innerspeed}(k, i) \times \text{innerhour}(k, i)
\]

with \(\text{innerspeed}(k, i)\) being the inner layer production speed in m/min. Since \(\text{innerspeed}(k, i)\) and \(\text{innerhour}(k, i)\) do not have the same time dimension, it is necessary to apply a correction by multiplying \(\text{innerspeed}(k, i)\) by 60. It is now easy to derive that
The formulation to express the number of hours necessary to produce the outer layers can be derived in a similar way. The difference is that a cardboard consists of two outer layers, so the nominator has to be multiplied by 2. Another difference is the amount of waste.

\[
\text{outerhour}(k, i) = \frac{1000 \times \text{innerpaper}(k, i)}{60 \times 0.001 \times \text{innerweight}(k, i) \times \text{width}(k) \times \text{innerspeed}(k, i)}
\]

\[
= \frac{1000}{60 \times 0.001 \times \text{innerweight}(k, i) \times \text{width}(k) \times \text{innerspeed}(k, i)} \times \frac{\text{innerweight}(k, i)}{\text{weight}(i)} \times \text{amount}(k, i)
\]

\[
= \frac{1000 \times \text{amount}(k, i)}{60 \times 0.001 \times \text{weight}(i) \times \text{width}(k) \times \text{innerspeed}(k, i)}
\]

Hence \((1 - \text{waste}) \times \text{production} = \text{demanded}\), so that \(\text{production} = \text{demanded}/(1 - \text{waste})\). Throughout, we assume a constant waste percentage for all thicknesses \(i\). Furthermore, we assume a waste percentage of outer layer of 25 % and 30 % as input on the KM-6 and the KM-8, respectively.

The relationship between the inner layer weight and the production speed is nonlinear (assumption (e)). In order to remove the nonlinearities, the relationship will be approximated by a piece-wise linear function. Fig. 2 shows the relation between the speed and the inner layer weight for inner layers produced on the KM-6. In order to linearize the nonlinearities, we may use the following constraints:

\[
\text{innerweight}(k, i) = \sum_{j=1}^{5} \text{smplinweight}(k, j) \times \mu(k, j, i), \quad (11)
\]

\[
\text{innerspeed}(k, i) = \sum_{j=1}^{5} \text{smplinspeed}(k, j) \times \mu(k, j, i), \quad (12)
\]

\[
\sum_{j=1}^{5} \mu(k, j, i) = 1, \quad (13)
\]

\[
\mu(k, j, i) \geq 0, \quad (14)
\]

with \(\text{smplinweight}(k, j)\) and \(\text{smplinspeed}(k, j)\) the chosen break points. In our formulation we used five break points including the two endpoints. For a treatment of linearizing nonlinear constraints, we refer to e.g. [1] and [2].

The constraints for the outer layers can be derived in a similar way. The constraints read

\[
\text{outerhour}(k, i) = \frac{2 \times 1000 \times \text{amount}(k, i)}{60 \times 0.001 \times \text{weight}(i) \times \text{width}(k) \times \text{outerspeed}(k, i) \times [1 - \text{waste}(k)]}, \quad (10)
\]

\[
\text{outerweight}(k, i) = \sum_{j=1}^{7} \text{smplouterweight}(k, j) \times \lambda(k, j, i), \quad (15)
\]

\[
\text{outerspeedapp}(k, i) = \sum_{j=1}^{7} \text{smplouterspeed}(k, j) \times \lambda(k, j, i), \quad (16)
\]

\[
\sum_{j=1}^{7} \lambda(k, j, i) = 1, \quad (17)
\]

\[
\lambda(k, j, i) \geq 0. \quad (18)
\]

To linearize these nonlinearities we used seven break points, which we have not specified here. Note that the LP-model has 313 constraints, whereas 80 constraints are necessary to remove the nonlinearities. The model has 560 variables, of which 432 variables are used to remove the nonlinearities.

3. Solutions

Table 3 shows the current situation of the composition of the inner layer and the outer layers. There is no difference between the KM-6 and the KM-8 composition of the inner layer and the outer layers. On both machines the company does not
produce cardboard with a thickness of 2.75 mm. Table 5 shows the total number of machine hours necessary to produce the production budget with this current composition.

Our model proposes a slight change in the currently used composition schedule. KM-6 and KM-8 could have different compositions of the inner layer and the outer layers for the same thickness of cardboard. Table 4 shows the composition as a result of applying the LP-model.

Table 4 shows no values of the weight for the 2.75 mm cardboard on KM-6, because it was not allowed to produce this thickness on the KM-6. This follows from assumption (k). Table 5 shows the machine hours that are necessary to produce this composition schedule. The main difference with the current situation can be found in the small thicknesses, where our model proposes a heavier inner layer.
Table 5
Number of machine hours (in 1 yr)

<table>
<thead>
<tr>
<th>Situation</th>
<th>KM-6</th>
<th>PM-7</th>
<th>KM-8</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old composition</td>
<td>2677</td>
<td>5656</td>
<td>6465</td>
<td>14798</td>
</tr>
<tr>
<td>Standard LP-model</td>
<td>2536</td>
<td>5656</td>
<td>6465</td>
<td>14657</td>
</tr>
<tr>
<td>Reduction waste paper loss</td>
<td>2331</td>
<td>5334</td>
<td>5527</td>
<td>13192</td>
</tr>
<tr>
<td>Extension machine hours PM-7</td>
<td>2331</td>
<td>6360</td>
<td>5527</td>
<td>14218</td>
</tr>
<tr>
<td>Production speed KM-6</td>
<td>2353</td>
<td>5656</td>
<td>6415</td>
<td>14424</td>
</tr>
<tr>
<td>Production speed KM-8</td>
<td>2484</td>
<td>5656</td>
<td>5922</td>
<td>14062</td>
</tr>
<tr>
<td>Production speed KM-6 &amp; KM-8</td>
<td>2258</td>
<td>5656</td>
<td>5922</td>
<td>13836</td>
</tr>
</tbody>
</table>

From Table 5 we can see that the total number of machine hours decreases with $(14798 - 14657) = 141$ hours per year after applying the LP model; this is a reduction of about 1% compared to the current situation.

4. Scenario analysis

In the previous section we analyzed the effects of using the model for the total number of machine hours necessary to produce the specified amount of cardboard in the several thicknesses. In this section we will analyze the following scenarios.

The scenarios are:

i. reducing the outer layer waste;
ii. increasing the total number of machine hours on the PM-7;
iii. increasing the production speed on the KM-6, iv. on the KM-8, and
v. on both the KM-6 and the KM-8, respectively.

The first question to be answered is: What are the effects on the composition of the layers in case the outer layer waste percentage decreases to 15% for both machines? As can be seen from Table 6, the smallest thicknesses are obtained when the weight of the outer layer, and the weight of the inner layer are smaller compared to the current situation. The logic behind this phenomenon is that a reduction of the waste of the outer layer results in a higher production of it, so that for the production of the same thicknesses smaller sizes of the inner layer can be used, resulting in higher production speeds. Therefore, a reduction of the waste of outer layer results in a reduction of the amount of machine hours. If the waste paper loss can be reduced to 15%, the performance in necessary number of machine hours will increase considerably. As can be seen from Table 5, the waste paper loss reduction results in a decrease of necessary machine hours of $14798 - 13192 = 1606$ hours, which is equal to a reduction of the necessary number of machine hours of 11% compared to the current situation.

The second scenario assumes an extension of the total number of machine hours on the PM-7, i.e. a reduction of idle time, with 30%. What are the effects of this extension? The result is a reduction of $14798 - 14218 = 580$ machine hours (see Table 5), about 4% compared to the current situation. The composition of the inner and outer layer is the same as in the scenario of reducing outer layer loss, but the differences are found in the necessary machine hours. In the first scenario an improvement in waste paper loss, i.e. a change in the parameters of the production process, is assumed, whereas in the second scenario only the idle time on the paper machine is reduced. So, waste paper loss has more effects on the total production capacity than a reduction of the idle time.

In the third scenario it is assumed that technical improvements on the cardboard machines increase the production speed. First, we assume a production speed increase on the KM-6 with 10%. The results are shown in Tables 5 and 6. The scenario leads to a total machine hour reduction of $14798 - 14218 = 580$ hours, or a reduction of 2.52% compared to the current situation.

The next scenario assumes an increase of the production speed on the KM-8 of 10%. The effects are as follows. Obviously, since the KM-8 has a higher production speed than the KM-6 and the machine width of the KM-8 is larger, it follows that increasing the production speed on the KM-8 results in more production (or, equivalently, in a lower number of machine hours to produce all products), than increasing the production speed on the KM-6. The reduction in machine hour time is
Table 6
Composition of the layers

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Machine</th>
<th>Layer</th>
<th>Thicknesses (mm)</th>
<th>1.90</th>
<th>2.00</th>
<th>2.25</th>
<th>2.40</th>
<th>2.50</th>
<th>2.75</th>
<th>3.00</th>
<th>3.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current composition</td>
<td>KM-6 &amp;</td>
<td>Inner</td>
<td></td>
<td>690</td>
<td>740</td>
<td>825</td>
<td>895</td>
<td>965</td>
<td></td>
<td>950</td>
<td>1035</td>
</tr>
<tr>
<td></td>
<td>KM-8</td>
<td>Outer</td>
<td></td>
<td>225</td>
<td>225</td>
<td>250</td>
<td>250</td>
<td>250</td>
<td></td>
<td>400</td>
<td>500</td>
</tr>
<tr>
<td>Standard LP-model</td>
<td>KM-6</td>
<td>Inner</td>
<td></td>
<td>740</td>
<td>714.5</td>
<td>825</td>
<td>825</td>
<td>825</td>
<td></td>
<td>1040</td>
<td>1040</td>
</tr>
<tr>
<td></td>
<td>KM-8</td>
<td>Outer</td>
<td></td>
<td>200</td>
<td>237.8</td>
<td>250</td>
<td>285</td>
<td>320</td>
<td></td>
<td>355</td>
<td>497.5</td>
</tr>
<tr>
<td>Reduction waste paper loss</td>
<td>KM-6</td>
<td>Inner</td>
<td></td>
<td>640</td>
<td>690</td>
<td>825</td>
<td>745</td>
<td>815</td>
<td></td>
<td>875</td>
<td>1040</td>
</tr>
<tr>
<td></td>
<td>KM-8</td>
<td>Outer</td>
<td></td>
<td>250</td>
<td>250</td>
<td>250</td>
<td>325</td>
<td>325</td>
<td></td>
<td>437.5</td>
<td>497.5</td>
</tr>
<tr>
<td>Extension machine hours</td>
<td>KM-6</td>
<td>Inner</td>
<td></td>
<td>640</td>
<td>690</td>
<td>825</td>
<td>745</td>
<td>815</td>
<td></td>
<td>875</td>
<td>1040</td>
</tr>
<tr>
<td></td>
<td>KM-8</td>
<td>Outer</td>
<td></td>
<td>250</td>
<td>250</td>
<td>250</td>
<td>325</td>
<td>325</td>
<td></td>
<td>437.5</td>
<td>497.5</td>
</tr>
<tr>
<td>Production speed KM-6s</td>
<td>KM-6</td>
<td>Inner</td>
<td></td>
<td>740</td>
<td>785</td>
<td>825</td>
<td>825</td>
<td>825</td>
<td></td>
<td>1040</td>
<td>1040</td>
</tr>
<tr>
<td></td>
<td>KM-8</td>
<td>Outer</td>
<td></td>
<td>200</td>
<td>202.5</td>
<td>250</td>
<td>285</td>
<td>320</td>
<td></td>
<td>355</td>
<td>497.5</td>
</tr>
<tr>
<td>Production speed KM-8</td>
<td>KM-6</td>
<td>Inner</td>
<td></td>
<td>740</td>
<td>790</td>
<td>825</td>
<td>825</td>
<td>825</td>
<td>950</td>
<td>1040</td>
<td>1040</td>
</tr>
<tr>
<td></td>
<td>KM-8</td>
<td>Outer</td>
<td></td>
<td>200</td>
<td>200</td>
<td>250</td>
<td>285</td>
<td>320</td>
<td>325</td>
<td>355</td>
<td>497.5</td>
</tr>
<tr>
<td>Production speed on both the KM-6 and KM-8</td>
<td>KM-6</td>
<td>Inner</td>
<td></td>
<td>690</td>
<td>690</td>
<td>825</td>
<td>825</td>
<td>825</td>
<td></td>
<td>1040</td>
<td>1040</td>
</tr>
<tr>
<td></td>
<td>KM-8</td>
<td>Outer</td>
<td></td>
<td>225</td>
<td>250</td>
<td>250</td>
<td>285</td>
<td>320</td>
<td></td>
<td>355</td>
<td>497.5</td>
</tr>
</tbody>
</table>

equal to $14798 - 14062 = 736$, or 5% compared to the current situation. (See Table 5.)

In the last scenario we analyze the effects of an increase of 10% in the production speed of both the KM-6 and the KM-8. As shown in Table 6, the composition schedule of an increasing production speed on both machines results in a lower weight of the inner layer in the smaller thicknesses on the KM-6 and a larger weight of the inner layer in the 2.40mm thickness on the KM-8, compared to an increase of the production speed at the KM-6 or KM-8 separately. This can be explained by an exchange of orders between both machines. If the KM-8 produces heavier weight inner layers for the 2.40mm thickness, the production capacity loss must be compensated by the KM-6 in producing lower weight inner layers in the smaller thicknesses. The increasing production speeds on both machines result in a reduction of $14798 - 13836 = 962$ hours, or a reduction of 6.5% compared to the current situation.

Table 5 shows the necessary machine hours. As can be seen, the PM-7 is in most cases the bottleneck machine, because of the fact that the PM-7
uses all available machine hours (5656 hours). In order to evaluate the feasibility of the solutions, the necessary machine hours can be compared with the available machine hours, which are shown in Table 1. Note that the restrictions are never violated, i.e. the necessary machine hours are lower than the available machine hours. So, the solutions are in fact feasible.

Table 6 shows the composition schedules for all scenarios, the current situation, and the standard composition. Take, for example, the KM-6 in the standard LP-model. In a thickness of 2.00 mm, the inner layer has a weight of 714.5 g/m$^2$, whereas the outer layer has a weight of 237.8 g/m$^2$. Heavy weight cardboard consists of one inner layer and two outer layers, so that the total weight of the cardboard of 2.00 mm is equal to 1190 g/m$^2$.

These scenario analyses suggest that improvements in the production process may lead to new compositions of the inner layer and the outer layers, such that the total production time reduces. Actually, the calculations indicate that one of the best ways to decrease the total number of machine hours is reducing the outer layer loss.

5. Conclusions

In this paper we answer the question of the planning department of Kappa Graphic Board to design a scheduling tool with which better, faster, flexible, and more reliable production schedules for the three cardboard machines can be calculated. More precisely, the planning management asked for investigating the possibilities of finding compositions of layers for thick cardboard resulting in less production hours. We use a linear programming formulation with which optimal compositions of the layers can be derived. The model also enables us to calculate different production scenarios, such as increasing production speeds, reducing the laminating paper losses, and decreasing the machine idle times. It turns out that in all cases better compositions with less machine hours are possible.

Acknowledgement

We acknowledge the contributions of C.S. Smidtman of Kappa Graphic Board, and of our colleague W.K. Klein Haneveld.

References