Justification of manufacturing technologies using fuzzy benefit/cost ratio analysis

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Abstract

The application of discounted cash flow techniques for justifying manufacturing technologies is studied in many papers. State-price net present value and stochastic net present value are two examples of these applications. These applications are based on the data under certainty or risk. When we have vague data such as interest rate and cash flow to apply discounted cash flow techniques, the fuzzy set theory can be used to handle this vagueness. The fuzzy set theory has the capability of representing vague data and allows mathematical operators and programming to apply to the fuzzy domain. The theory is primarily concerned with quantifying the vagueness in human thoughts and perceptions. In this paper, assuming that we have vague data, the fuzzy benefit–cost (B/C) ratio method is used to justify manufacturing technologies. After calculating the B/C ratio based on fuzzy equivalent uniform annual value, we compare two assembly manufacturing systems having different life cycles. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Many authors give the economic justification approaches of manufacturing systems: Meredith and Suresh [1], Lavelle and Liggett [2], Soni et al. [3], Kolli et al. [4], Boaden and Dale [5], Khouja and Offodile [6], Proctor and Canada [7], etc. Kolli et al. [4] classify the approaches into two main groups: single criterion and multi-criterion approaches. These two main groups are then divided into two subgroups: deterministic and non-deterministic approaches. Simple criterion and deterministic approaches contain discounted cash flow techniques (NPV, JRR, PP, etc.). Single criterion and nondeterministic approaches contain sensitivity analysis, decision tree, Monte Carlo simulation, etc. Multi-criteria deterministic approaches contain scoring, AHP, goal programming, DSS, dynamic programming, and ranking methods (ELECTRE, PROMETHEE, ...). Multi-criteria nondeterministic approaches contain fuzzy linguistics, expert system, utility models, and game theoretic models. Wilhelm and Parsaei [8] use a fuzzy linguistic approach to justify a computer-integrated manufacturing system. Kahraman et al. [9] use a fuzzy approach based on the fuzzy present value analysis for the manufacturing flexibility.

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To deal with vagueness of human thought, Zadeh [10], first introduced the fuzzy set theory, which was oriented to the rationality of uncertainty due to imprecision or vagueness. A major contribution of fuzzy set theory is its capability of representing vague data. The theory also allows mathematical operators and programming to apply to the fuzzy domain. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function, which assigns to each object a grade of membership ranging between zero and one.

A tilde ‘’ will be placed above a symbol if the symbol represents a fuzzy set. Therefore, $\hat{P}$, $\tilde{r}$, $\tilde{n}$ are all fuzzy sets. The membership functions for these fuzzy sets will be denoted by $\mu(x|\hat{P})$, $\mu(x|\tilde{r})$, and $\mu(x|\tilde{n})$ respectively. A triangular fuzzy number (TFN), $\tilde{M}$ is shown in Fig. 1. A TFN is denoted simply as $(m_1/m_2/m_3)$ or $(m_1, m_2, m_3)$. The parameters $m_1$, $m_2$ and $m_3$ respectively denote the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event.

Each TFN has linear representations on its left and right side such that its membership function can be defined as

$$
\mu(x|\tilde{M}) = \begin{cases} 
0, & x < m_1, \\
(x - m_1)/(m_2 - m_1), & m_1 \leq x \leq m_2, \\
(m_3 - x)/(m_3 - m_2), & m_2 \leq x \leq m_3, \\
0, & x > m_3. 
\end{cases}
$$

A fuzzy number can always be given by its corresponding left and right representation of each degree of membership:

$$
\tilde{M} = (M^{l(y)}, M^{r(y)}) = (m_1 + (m_2 - m_1)y, m_3 + (m_2 - m_3)y)
$$

\forall y \in [0, 1],

where $l(y)$ and $r(y)$ denotes the left side representation and the right side representation of a fuzzy number, respectively. Zimmermann [12] gives the algebraic operations with triangular fuzzy numbers. Many ranking methods for fuzzy numbers have been developed in the literature. They do not necessarily give the same rank. Two ranking methods are given in the appendix.

Buckley [11], Ward [13], Chiu and Park [14], Wang and Liang [15], Kahraman and Tolga [16] are among the authors who deal with the fuzzy present worth analysis, the fuzzy benefit/cost ratio analysis, the fuzzy future value analysis, the fuzzy payback period analysis, and the fuzzy capitalized value analysis.

2. Fuzzy benefit/cost ratio analysis

The benefit–cost ratio can be defined as the ratio of the equivalent value of benefits to the equivalent value of costs. The equivalent values can be present values, annual values, or future values. The benefit–cost ratio (BCR) is formulated as

$$
BCR = B/C,
$$

where $B$ represents the equivalent value of the benefits associated with the project and $C$ represents the project’s net cost [17]. A $B/C$ ratio greater than or equal to 1.0 indicates that the project evaluated is economically advantageous.
In B/C analyses, costs are not preceded by a minus sign. The objective to be maximized behind the B/C ratio is to select the alternative with the largest net present value or with the largest net equivalent uniform annual value, because B/C ratios are obtained from the equations necessary to conduct an analysis on the incremental benefits and costs. Suppose that there are two mutually exclusive alternatives. In this case, for the incremental BCR analysis ignoring disbenefits the following ratios must be used:

$$\frac{\Delta B_{2-1}}{\Delta C_{2-1}} = \frac{\Delta PV_{B_{2-1}}}{\Delta PV_{C_{2-1}}}$$  \hspace{1cm} (4)$$

or$$
\frac{\Delta B_{2-1}}{\Delta C_{2-1}} = \frac{\Delta EU_{AB_{2-1}}}{\Delta EU_{AC_{2-1}}},  \hspace{1cm} (5)$$

where $\Delta B_{2-1}$ is the incremental benefit of Alternative 2 relative to Alternative 1, $\Delta C_{2-1}$ is the incremental cost of Alternative 2 relative to Alternative 1, $\Delta PV_{B_{2-1}}$ is the incremental present value of benefits of Alternative 2 relative to Alternative 1, $\Delta PV_{C_{2-1}}$ is the incremental present value of costs of Alternative 2 relative to Alternative 1, $\Delta EU_{AB_{2-1}}$ is the incremental equivalent uniform annual benefits of Alternative 2 relative to Alternative 1 and $\Delta EU_{AC_{2-1}}$ is the incremental equivalent uniform annual costs of Alternative 2 relative to Alternative 2.

Thus, the concept of B/C ratio includes the advantages of both NPV and NEUAV analyses. Because it does not require to use a common multiple of the alternative lives (then B/C ratio based on equivalent uniform annual cash flow is used) and it is a more understandable technique relative to rate of return analysis for many financial managers, B/C analysis can be preferred to the other techniques such as present value analysis, future value analysis, rate of return analysis.

In the case of fuzziness, the steps of the fuzzy B/C analysis are given in the following:

**Step 1:** Calculate the overall fuzzy measure of benefit-to-cost ratio and eliminate the alternatives that have

$$\tilde{B}/\tilde{C} = \frac{\sum_{t=0}^{n} B_{2}(t) (1 + r(t))^{-t}}{\sum_{t=0}^{n} C_{2}(t) (1 + r(t))^{-t}} < 1,$$

where $\tilde{r}$ is the fuzzy interest rate and $r(t)$ and $l(t)$ are the right and left side representations of the fuzzy interest rates and $\tilde{T}$ is $(1, 1, 1)$, and $n$ is the crisp life cycle.

**Step 2:** Assign the alternative that has the lowest initial investment cost as the defender and the next-lowest acceptable alternative as the challenger.

**Step 3:** Determine the incremental benefits and the incremental costs between the challenger and the defender.

**Step 4:** Calculate the $\Delta \tilde{B}/\Delta \tilde{C}$ ratio, assuming that the largest possible value for the cash in year $t$ of the alternative with the lowest initial investment cost is less than the least possible value for the cash in year $t$ of the alternative with the next-lowest initial investment cost.

The fuzzy incremental BCR is

$$\frac{\Delta \tilde{B}}{\Delta \tilde{C}} = \frac{\sum_{t=0}^{n} B_{2}(t) (1 + r(t))^{-t} - \sum_{t=0}^{n} B_{1}(t) (1 + r(t))^{-t}}{\sum_{t=0}^{n} C_{2}(t) (1 + r(t))^{-t} - \sum_{t=0}^{n} C_{1}(t) (1 + r(t))^{-t}},$$

If $\Delta \tilde{B}/\Delta \tilde{C}$ is equal or greater than $(1, 1, 1)$, Alternative 2 is preferred.

In the case of a regular annuity, the fuzzy $\tilde{B}/\tilde{C}$ ratio of a single investment alternative is

$$\tilde{B}/\tilde{C} = \frac{A_{2}(n, r(t))}{C_{2}(n, r(t))},$$

where $\tilde{C}$ is the first cost and $\tilde{A}$ is the net annual benefit, and $\gamma(n, r) = ((1 + r)^n - 1)/(1 + r)^n r$.

The $\Delta \tilde{B}/\Delta \tilde{C}$ ratio in the case of a regular annuity is

$$\Delta \tilde{B}/\Delta \tilde{C} = \frac{(A_{2}(n, r(t)) - A_{1}(n, r(t)))}{C_{2}(n, r(t)) - C_{1}(n, r(t))},$$

where $\tilde{A}$ and $\tilde{C}$ are the fuzzy annual benefits and costs.
Step 5: Repeat steps 3 and 4 until only one alternative is left, thus the optimal alternative is obtained.

The cash-flow set \(\{A_t = A: t = 1, 2, \ldots, n\}\), consisting of \(n\) cash flows, each of the same amount \(A\), at times 1, 2, ..., \(n\), with no cash flow at time zero, is called the equal-payment series. An older name for it is the uniform series, and it has been called an annuity, since one of the meanings of “annuity” is a set of fixed payments for a specified number of years. To find the fuzzy present value of a regular annuity \(\{A_t = A: t = n\}\), we will use Eq. (10). The membership function \(\mu(x|\tilde{P}_n)\) for \(\tilde{P}_n\) is determined by

\[
f_n(y|\tilde{P}_n) = f_i(y|\tilde{A}) \cdot f_{\tilde{r}}(y|\tilde{r})
\]

for \(i = 1, 2\) and \(\gamma(n, r) = (1 - (1 + r)^{-n})/r\). Both \(\tilde{A}\) and \(\tilde{r}\) are positive fuzzy numbers. \(f_i(.)\) and \(f_{\tilde{r}}(.)\) shows the left and right representations of the fuzzy numbers, respectively.

In the case of a regular annuity, the fuzzy \(\tilde{B}/\tilde{C}\) ratio may be calculated as in the following:

The fuzzy \(\tilde{B}/\tilde{C}\) ratio of a single investment alternative is

\[
\tilde{B}/\tilde{C} = \left(\frac{A_1(n, r^{(y)})}{FC_1(n, r^{(y)})}, \frac{A_2(n, r^{(y)})}{FC_2(n, r^{(y)})}\right),
\]

where \(FC\tilde{C}\) is the first cost and \(\tilde{A}\) is the net annual benefit.

The \(\Delta\tilde{B}/\Delta\tilde{C}\) ratio in the case of a regular annuity is

\[
\Delta\tilde{B}/\Delta\tilde{C} = \left(\frac{(A_2^{(y)} - A_1^{(y)}) \gamma(n, r^{(y)})}{FC_2^{(y)} - FC_1^{(y)}}, \frac{(A_2^{(y)} - A_1^{(y)}) \gamma(n, r^{(y)})}{FC_2^{(y)} - FC_1^{(y)}}\right).
\]

Up to this point, we assumed that the alternatives had equal lives. When the alternatives have life cycles different from the analysis period, a common multiple of the alternative lives (CMALs) is calculated for the analysis period. Many times, a CMALs for the analysis period hardly seems realistic (CMALs (7, 13) = 91 years). Instead of an analysis based on present value method, it is appropriate to compare the annual cash flows computed for alternatives based on their own service lives. In the case of unequal lives, the following fuzzy \(\tilde{B}/\tilde{C}\) and \(\Delta\tilde{B}/\Delta\tilde{C}\) ratios will be used:

\[
\tilde{B}/\tilde{C} = \left(\frac{PVB^{(y)}\beta(n, r^{(y)})}{PVC^{(y)}\beta(n, r^{(y)})}, \frac{PVB^{(y)}\beta(n, r^{(y)})}{PVC^{(y)}\beta(n, r^{(y)})}\right),
\]

\[
\Delta\tilde{B}/\Delta\tilde{C} = \left(\frac{PVB_2^{(y)}\beta(n, r^{(y)}) - PVC_1^{(y)}\beta(n, r^{(y)})}{PVC_2^{(y)}\beta(n, r^{(y)}) - PVC_1^{(y)}\beta(n, r^{(y)})}, \frac{PVB_2^{(y)}\beta(n, r^{(y)}) - PVC_1^{(y)}\beta(n, r^{(y)})}{PVC_2^{(y)}\beta(n, r^{(y)}) - PVC_1^{(y)}\beta(n, r^{(y)})}\right).
\]

where \(PVB\) is the present value of benefits, \(PVC\) the present value of costs and

\(\beta(n, r) = ((1 + n)^i/((1 + r)^n - 1))\).

3. Application

In this hypothetical application, two assembly manufacturing systems will be considered, namely a transfer machine with robot workhead (RW) and a robot assembly cell (RAC). Figs. 2 and 3 show these two manufacturing systems.

Fig. 2. RW — transfer machine with robot workhead.
The robot assembly cell uses one robot. This sophisticated robot has six degrees of freedom. Tables 1 and 2 give the data for RAC and RW, respectively. The tax rate (TR) is 40% and is not fuzzy. The crisp life cycles are 7 years for RAC and 5 years for RW. The fuzzy discount rate is (15%, 20%, 25%) per year.

To fuzzify the data for RAC and RW, it is assumed that the model parameters are triangular fuzzy numbers. The parameters like the average time for the robot to assemble a part, the part quality, the average downtime at a station due to a defective part, the number of production hours per shift, the plant efficiency, the cost of a robot, the cost of a part feeder, the cost of a robot gripper, the annual cost of a supervisor have been accepted as triangular fuzzy numbers. For example, the average time of the RAC to complete an assembly i, is used while calculating the number of shifts and the annual labour cost. It is formulated by

\[
A T F_i = N P_i \otimes [A P_i \oplus P Q_i \oplus A D_i],
\]

where \( N P_i \) is the fuzzy number of parts in each assembly i, \( A P_i \) the fuzzy average time (in seconds) for the robot to assemble one part, \( P Q_i \) the fuzzy part quality (the fraction of defective parts to good parts), and \( A D_i \) the fuzzy average downtime at a station due to a defective part.

The other details are not given in the paper but the calculated results are.

In year 6, a new robot assembly cell is required. So, \( ATCF_6 < 0 \). While calculating after-tax cash flow, the following formula has been used:

\[
ATCF_i = (R E V_i - L A B_i)(1 - TR) \oplus DEP_i \times TR - I_i.
\]
Now, using Eq. (13), we will calculate $\tilde{B}/\tilde{C}$ ratios for each alternative. First, let us calculate $PVB_{RAC}$, $PVB_{RW}$, $PVC_{RAC}$, and $PVC_{RW}$.

\[
PVB_{RAC} = \left( \frac{15y + 78}{(1.25 - 0.05y)^3} + \frac{15y + 83}{(1.25 - 0.05y)^3} 
+ \frac{24y + 91}{(1.25 - 0.05y)^3} + \frac{16y + 127}{(1.25 - 0.05y)^3} 
+ \frac{16y + 157}{(1.25 - 0.05y)^3} \cdot \frac{108 - 15y}{(1.15 + 0.05y)^3} 
+ \frac{113 - 15y}{(1.15 + 0.05y)^3} + \frac{134 - 19y}{(1.15 + 0.05y)^3} 
+ \frac{161 - 18y}{(1.15 + 0.05y)^3} + \frac{189 - 16y}{(1.15 + 0.05y)^3} \right)
\]

\[
PVC_{RAC} = \left( \frac{32y - 210}{(1.15 + 0.05y)^3} - \frac{21y - 117}{(1.15 + 0.05y)^3} 
- \frac{32y - 146}{(1.25 - 0.05y)^3} - \frac{21y - 96}{(1.25 - 0.05y)^3} \right)
\]

\[
PVC_{RW} = \left( \frac{21y + 109}{(1.25 - 0.05y)^3} + \frac{21y + 116}{(1.25 - 0.05y)^3} 
+ \frac{34y + 127}{(1.25 - 0.05y)^3} + \frac{22y + 178}{(1.25 - 0.05y)^3} 
\right) \cdot \left( \frac{151 - 21y}{(1.15 + 0.05y)^3} + \frac{158 - 21y}{(1.15 + 0.05y)^3} 
+ \frac{187 - 26y}{(1.15 + 0.05y)^3} + \frac{225 - 25y}{(1.15 + 0.05y)^3} \right)
\]

\[
PVC_{RW} = \left( \frac{62y - 190}{1.15 + 0.05y^3} - \frac{34y - 94}{1.15 + 0.05y^3} \right)
\]

The approximate forms of $PVB_{RW}$, $PVB_{RAC}$, $PVC_{RW}$, and $PVC_{RAC}$ are found as in the following, taking $y = 0, 1$ and 0 respectively:

\[
PVB_{RAC} \approx (204.230, 282.505, 383.675),
\]

\[
PVB_{RW} \approx (239.498, 327.578, 436.848),
\]

\[
|PVC_{RAC}| \approx (153.817, 194.549, 233.191),
\]

\[
|PVC_{RW}| \approx (81.739, 106.667, 165.217),
\]

\[
PVB_{RAC} = (88.08y + 239.498, 436.848 - 109.27y),
\]

\[
|PVC_{RAC}| = (40.732y + 153.817, 233.191 - 38.642y)
\]

\[
|PVC_{RW}| = (24.928y + 81.739, 165.217 - 58.55y),
\]

\[
V = \left( \frac{(1.15 + 0.05y)^3(0.15 + 0.05y)^2}{(1.15 + 0.05y)^3 - 1} \right)
\]

\[
L = \left( \frac{(1.15 + 0.05y)^3(0.15 + 0.05y)^2}{(1.15 + 0.05y)^3 - 1} \right)
\]

\[
Z = \left( \frac{(1.25 - 0.05y)^3(0.25 - 0.05y)^2}{(1.25 - 0.05y)^3 - 1} \right)
\]

\[
S = \left( \frac{(1.25 - 0.05y)^3(0.25 - 0.05y)^2}{(1.25 - 0.05y)^3 - 1} \right)
\]

\[
\tilde{B}/\tilde{C}_{RAC} = \left( \frac{(204.230 + 78.275y)L}{(233.191 - 38.642y)S} \right) \left( \frac{(383.675 - 101.17y)S}{(153.817 + 40.732y)L} \right)
\]

\[
\tilde{B}/\tilde{C}_{RW} = \left( \frac{(239.498 + 88.08y)V}{(165.217 - 58.55y)Z} \right) \left( \frac{(468.848 - 109.27y)Z}{(81.739 + 24.928y)V} \right)
\]

\[
\Delta \tilde{B}/\Delta \tilde{C} = \left( \frac{PVB_{RAC}^{(i)} - PVB_{RW}^{(i)}}{|PVC_{RAC}^{(i)}| - |PVC_{RW}^{(i)}|} \right)
\]

\[
\tilde{B}/\tilde{C}_{RAC} = (0.665, 1.452, 3.283),
\]

\[
\tilde{B}/\tilde{C}_{RW} = (1.163, 3.071, 6.662),
\]

\[
\Delta \tilde{B} = (-113.352, -31.162, 49.926),
\]

\[
\Delta \tilde{C} = (-24, 464, 18.305, 49.384),
\]

\[
\Delta \tilde{B}/\Delta \tilde{C} = (a, b, c).
\]

Kaufmann and Gupta’s ranking method:

\[
(a + 2b + c)/4 = -0.915,
\]

\[
\tilde{I} = (1, 1, 1),
\]

\[
(a + 2b + c)/4 = 1,
\]
\[ \Delta \tilde{B} / \Delta \tilde{C} < \tilde{I}, \text{ then the preferred alternative is RW.} \]

Chiu and Park’s weighting method \((w = 0.3)\):

\[
\frac{(a + b + c)/3 + wb}{3} = 1.16, \quad \tilde{I} = (1, 1, 1), \quad \frac{(a + b + c)/3 + wb}{3} = 1.3, \quad \Delta \tilde{B} / \Delta \tilde{C} < \tilde{I}, \text{ then the preferred alternative is RW.}
\]

4. Conclusions

This paper develops a fuzzy \(B/C\) ratio analysis to justify a manufacturing technology. The analysis presents an alternative to having to use exact amounts for the parameters used in the justification process. Fuzzy \(B/C\) ratio analysis is equivalent to fuzzy present value analysis. However, fuzzy \(B/C\) ratio based on equivalent uniform annual benefits and costs has the advantage of comparing alternatives having life cycles different from the analysis period, without calculating a common multiple of the alternative lives. The details of fuzzy \(B/C\) ratio are presented in this paper.

The developed fuzzy \(B/C\) ratio analysis only takes into account of a quantitative criterion that is the profitability, while justifying a manufacturing technology. To justify a manufacturing technology, generally, a number of quantitative and qualitative criteria have to be considered. In this case, deterministic or nondeterministic multiple criteria methods such as scoring or fuzzy linguistics should be taken into account.

Appendix

There are a number of methods that are devised to rank mutually exclusive projects such as Chang’s method [18], Jain’s method [19], Dubois and Prade’s method [20], Yager’s method [21], Baas and Kwakernaak’s method [22]. However, certain shortcomings of some of the methods have been reported in [23–25]. Because the ranking methods might give different ranking results, they must be used together to obtain the true rank. Chiu and Park [14] compare some ranking methods by using a numerical example and determine which methods give the same or very close results to one another.

In Chiu and Park’s [14] paper, several dominance methods are selected and discussed. Most methods are tedious in graphic manipulation requiring complex mathematical calculation. Chiu and Park’s weighting method and Kaufmann and Gupta’s [26] three criteria method are the methods giving the same rank for the considered alternatives and these methods are easy to calculate and require no graphical representation. Therefore, these two methods will be explained in the following and will be used in the application section.

Chiu and Park’s [14] weighted method for ranking TFNs with parameters \((a, b, c)\) is formulated as

\[
\frac{(a + b + c)/3 + wb}{3},
\]

where \(w\) is a value determined by the nature and the magnitude of the most promising value.

Kaufmann and Gupta [26] suggest three criteria for ranking TFNs with parameters \((a, b, c)\). The dominance sequence is determined according to priority of:

1. comparing the ordinary number \((a + 2b + c)/4,\)
2. comparing the mode (the corresponding most promising value), \(b, \) of each TFN,
3. comparing the range, \(c − a, \) of each TFN.

The preference of projects is determined by the amount of their ordinary numbers. The project with the larger ordinary number is preferred. If the ordinary numbers are equal, the project with the larger corresponding most promising value is preferred. If projects have the same ordinary number and most promising value, the project with the larger range is preferred.

References


