Using hierarchical pseudo bills of material for customer order acceptance and optimal material replenishment in assemble to order manufacturing of non-modular products

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Abstract

In assemble-to-order manufacturing, often product families with many options and features are offered to the market. Demand for an individual option or feature can be quite low and uncertain. The efficient checking of materials availability during customer order acceptance, and the materials requirement planning are crucial for effective control in this type of industry. Traditionally, modular bills of materials are used for this purpose. If interdependencies exist between features and options in the product family, modular bills are inadequate; instead generic bills of material can be used to model the production options available in a product family. In this paper we develop a variant of the modular bill of material, called the hierarchical pseudoitem Bill of Material, which is the mirror image of the choice tree in the generic bill of materials, and which can be used for checking materials availability, for allocating materials to customer orders, and for materials replenishment. Furthermore, we propose a model to optimize the master production schedule levels for options and features that drive the material replenishments process. The optimization aims at balancing the stock keeping costs of specific parts for options, with the revenues that result from selling products variants with those options. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

During the past decades the discrete assembly industry has been faced with an increased product variety, shorter product life cycles and a fragmentation of markets. Tighter competition and the emergence of buyer markets have forced companies to offer a continuously growing and changing product variety to prevent losing market share \cite{1}. As a result more than a few million product variants are already offered in the automobile, aviation and medical equipment industries \cite{2}. What’s more, in non-assembly types of industries like insurance, banking, beverage and personal care, this proliferation of product variety can nowadays also be discerned \cite{3}. Unfortunately, these volatile market conditions in the discrete assembly industry have induced
a number of problems. Wemmerlöv [4] indicates four problem areas with regard to material coordination in the assembly industry:

(a) **material structuring.** A proper data storage of product structures is important to control production and supply of materials. In this context a large product variety will increase the number of end product variant structures that have to be stored and maintained and consequently the complexity of the data storing process.

(b) **forecasting.** Forecasting demand for each end product variant, becomes an almost impossible task when many variants are involved because the average demand per variant will be low.

(c) **buffer allocation.** A high product variety thus leads to a high demand uncertainty. This uncertainty has to be buffered against with safety stocks. The chances that these stocks are unbalanced and thus will lose their effectiveness, will increase in case of high product variety.

(d) **order acceptance.** Each final product variant is often a combination of subsystem variants. Upon arrival of an order all the required subassembly variants have to be checked on their stock availability. Fast and integral stocks checks are essential to enable short feedback cycles concerning planned delivery times to customers.

Wemmerlöv further states that assemble-to-order (ATO) manufacturing is relatively the best manufacturing strategy to deal with a high product variety because it “defers the commitment of material and capacity as long as possible”. An ATO strategy implies the final assembly of end product variants upon arrival of a customer order from subsystem variants. Important logistic functions that should be addressed while adopting this manufacturing strategy are therefore product structuring, buffering against uncertainty, material supply planning and customer order processing.

The response to these problems have been threefold.

Firstly, the production and final assembly processes are designed such that the end product variety can be realized in the final assembly phase (design for assembly, modular product design, postponement of product specificity).

Secondly, the final assembly processes, and sometimes also the subassembly processes, are transformed from batch processes to flow processes. This allows for the successive assembly of all end product variants with batch size one (from a multi-model assembly line to a mixed model assembly line). The material supply to these mixed model assembly lines requires a very tight coordination of the different components and subassemblies needed for each end product, to the different positions in the assembly line.

Thirdly, special bills of material are developed to support the customer order acceptance process and the materials supply process. Generally, a customer does not express his requirements in the technical characteristics of the product, such as motor type, brake type, transmission type, etc., but in terms of functional features such as horse power, color, accuracy, speed, etc. A manufacturer may offer a wide range of products, some of which may fit with the functional specifications of the customer. Searching through the product family to look for the best fitting product can be a time consuming process, unless the search process is efficiently organized.

The normal solution to this problem is to make use of features and options to identify a specific variant within a product family. A feature can be viewed as a product family characteristic for finished products. Each feature, e.g. the color of a product, is associated with a set of mutually exclusive options. An option is, therefore, a value assigned to a characteristic that represents a property of a product that is relevant for a customer. In this case, the master production schedule items are typically found at the level of major components or important subsystems [5] that represent mutually exclusive (combinations of) options and features. Forecasting the demand for finished products is often based upon the forecasted demand for the various options. Translating the option forecast to MPS item forecasts is traditionally accomplished by using so called modular bill of material. The MPS items are grouped in planning modules in a modular bill of material. A planning module, therefore, becomes a set of parts needed to manufacture a finished product with a certain...
option or combination of options within a product family or, otherwise, a set of parts that is used in every finished product in the family. A forecast for a planning module is based upon the forecast for the whole product family, multiplied by the planning percentage associated with the associated option.

The use of modular bills of material is based on a number of assumptions [6]. These are:

- There is a one-to-one relationship between an option and a planning module. The bill-of-material associated with a planning module contains all of the components needed to manufacture the product with that option.
- There are no dependencies between the options associated with different features. This implies that any option can be chosen for a given feature without affecting the choices of options for other features. In other words, orthogonality exists between the features.
- No common items are found in the lower levels of the bills-of-material for the options belonging to a given feature.

Van Veen [6] has shown that these assumptions often do not hold true when there is a large number of variants within a product family. As the number of variants increases, the number of options will normally increase and the likelihood of interdependencies will increase. To solve that problem Hegge and Wortmann [7], Van Veen [6] and Hegge [8] have developed a product configurator based on the so-called generic bills of material (see also [2]). For a detailed description of the generic bill of material concept we refer to [7]. Here we give a short summary of its use for selecting a product variant.

A generic bill of material captures a generalized product structure which is the technical twin of the commercial choice sheet, i.e., each commercial variant specification mirrors a technical variant specification in the generic bill of material system. The commercial choice sheet shows the commercial features of the product family, for each feature parameter. The value ranges of the parameters for each feature define the combinations of options that are available. For complex products, not all combinations of options may be available. For instance, a motor with a certain horsepower may only be available in combination with a subset of the range of transmission systems available. The search process through the combination of functional product features can be modeled as a choice three. At each decision point the choice made results in selected parameter values. The choice menu at a certain point in the choice tree can be determined by the parameter values selected at earlier decision points in the choice tree. In this way the interdependencies between options are modelled. After all choices have been made, the parameters selected uniquely determine a valid product variant.

As an example we consider a manufacturer of desk chairs which offers a range of products to its customers. The generic bill of material of the desk chair product family consists of an underframe, a seat and back. The underframe consists of a stand and wheels. The seat consists of a seat frame and upholstery. The back consists of a back frame and upholstery. The bold boxes and bold lines in Fig. 1 show the generic bill of material of this product family (adapted from [8]).

Each generic item in the product structure has a code which is a four digit number, followed by a single “G” if the item is a generic item, that is, if it represents a group of product variants.

The choices that can be made within the desk chair product family are characterized by the following possibilities:

- the upholstery of a chair can have any of three colours, white, red or blue, with parameter values (WH), (RD) and (BL).
- the underframe can have a swiveling or a non-swiveling stand, with parameter values (SW) and (NO),
- the underframe can have wheels or no wheels with parameter values (W) and (N).

However, there are restrictions imposed on the combinations that can be chosen. These are:

(a) all of the product variants with wheels have a non-swiveling stand,
(b) all of the blue variants have a swiveling stand.

These choice possibilities and restrictions are represented in Fig. 1 in the decision tables that are
associated with the generic items in the BOM. The selections made at each level in the BOM result in parameter values which can be input to the decision tables at lower levels in the BOM. In the decision tables, dashed lines indicate the options to choose from and solid lines indicate a forced choice. If no choice needs to be made, the parameter values determined at higher levels are just carried through to lower levels.

The generic BOM contains a choice tree that is used to select a product variant. However, the selected product variant can only be assembled and delivered if all components and subassemblies needed for the product variant are available. As has been pointed out by Vollmann et al. [5] checking availability of materials and allocating materials to customer orders should be done concurrently with the selection of the options that require these materials. A fine example of how this can be done for product families that can be adequately modeled by modular bills of material, can be found in [9]. However, in this paper we deal with product families which cannot be modeled with modular bills of materials, in particular because complex interdependencies exist between options. For this situation the relationship between features and options available in the product family can be modeled by generic bills of materials [7]. Now the question arises how to model the material requirements for these features and options, for checking materials availability, for materials allocation and for materials replenishment. In this paper we develop a variant of the modular bill of material which is the mirror image of the choice tree in the generic bill of material and which can be used for these tasks. Like in the modular bill of material, the bill relates
pseudoitems to options in the choice process. Unlike the modular bill of material, the relationship between two pseudoitems is hierarchical. Therefore, we refer to this bill as a hierarchical pseudoitem bill of material.

The price of a product variant in the market may depend on the options in the variant. Different options will require different materials. Thus, different product variants may have different contributions to the profit of the firm. Therefore, an important issue in materials control for assemble-to-order manufacturing is the control of the availability of parts such that the costs of materials availability is balanced with possible revenues of selling products variants that require these parts. In this paper we also propose a model that can be used to optimize the availability of critical materials taking into account the commercial value of the options, the stock keeping costs of the critical parts and uncertainty in demand per option.

The rest of this paper is organized as follows. In Section 2 we analyze the assemble-to-order process for non-modular products under a large end-product variety, with respect to material supply and customer order acceptance. In Section 3 we derive requirements for the bill of material structure, and we propose the hierarchical pseudobill of material as a technique which satisfies these requirements. In Section 4 we show how the hierarchical pseudobill of material can be used for customer order acceptance against the current inventory position. In Section 5 we propose an optimization model which can be used to determine planning levels for the pseudoitems related to the options or features in the product family. In the optimization the possible revenues of option and features sold are traded-off against the costs incurred by keeping materials in stock, assuming probabilistic knowledge of the levels of demand for features and options. Conclusions are given in Section 6.

2. Analysis of the assemble-to-order process

In this paper we consider an assemble-to-order process which offers a large number of different product variants in a non-modular product structure. These product variants each require a number of parts. Some parts are needed for each product variant, other parts are only needed for a subset of the product variants and some parts are specific for only one product variant.

We assume that the assemble-to-order process is a flow process with a relatively short flow time relative to the replenishment lead times of the parts. For the sake of simplicity we also assume that:

- the assembly flow time for an order is the same for all product variants,
- all parts for a product have to be available at the start of the assembly process, and
- the assembly capacity is highly flexible; that is, all customer orders can be assembled in the given flow time, if all parts are available.

The need for parts is directly related to the need for products, options and features. In modular bills of material, parts are replenished in multiples of the sets needed for options (matched sets of parts). The Master Production Schedule therefore is expressed in these modular sets of parts. For protection against uncertainty in demand option overplanning of the MPS can be used, resulting in coordinated availability of the parts in the modular sets per option.

For non-modular product families showing interdependencies between options and features modular bills of materials are not adequate and a generic bill of material can be used in the product selection process. However, in order to check materials availability, to allocate materials and to replenish materials, the generic bill, which describes the product family in generic terms, must be coupled to a bill of physical items. This can be achieved by applying the same principles on which the modular bill of material concept is based. All parts that are uniquely needed for a specific option or feature are grouped into a set which is related to that option or feature in the product variant selection process. Such a group of parts is called a pseudo item [10].

In the next sections we will elaborate this solution to the problem.
3. Bill of Material structuring for non-modular product families

In the previous section we have introduced the idea that materials availability in non-modular assemble-to-order manufacturing should be controlled by sets of parts, or pseudos. This is not a new idea. Mather [10] already introduced the concept “pseudoitem” to denote a set of physical parts which in the assembly process are always needed as a set. For such a set, a code number is introduced so that material picking of parts and material replenishment can be done for the set as a whole. Thus the code number of a pseudoitem does not refer to a physical part or a physical assembly, but to a set of parts which need to be controlled as an entity. The concept of “pseudo” therefore seems to be perfectly suited for our purpose. The question remaining is how to relate these pseudoitems to the generic bill of material structure introduced by Hegge and Wortmann [7].

The generic bill of material efficiently models all the possible end-products that are offered in a product family. The structure of the generic BOM as developed by Hegge and Wortmann has been designed first of all to support the sales people in selecting a product variant that meets as close as possible the requirements of a customer. The selection process is guided by parameter values, representing product options and features that have a functional meaning for the customer (for instance, for a firm that produces office furniture, the color of a chair, the length of a table, the width of a chair). This selection process is organized as a choice tree. In this way customer requirements can easily be checked at options and features available in the product family.

It should be noted that in the sales process described above, it is assumed that all the required parts are available; no explicit check has been made on the actual availability of all the parts needed for this product variant. However, it makes no sense to select a product variant and promise it to the customer if we are not certain that all parts needed for this variant are available. Therefore, this check should take place concurrently with the selection of the product variant and not after the selection. Especially if the customer order lead times are short and assembly is organized as a flow process (to accommodate a short lead time), it is essential to integrate the product selection with a materials availability check. For that purpose the parts should be grouped in sets according to the options and features that can be selected, in order to check materials availability per set.

Thus, we need to group the individual parts used for the product family in disjoint sets which are linked to the options and features offered in the product family. Each set consists of the parts that are uniquely identified as being part of the individual BOM of the product variant, at a certain node in the option and feature choice tree set of the generic BOM. All parts (and their quantities) that are used by each product variant in the product family are clustered in one set and linked to the starting node in the decision tree. These are the common parts which together form a pseudoitem with a unique identification code number. At the next nodes in the choice tree one or more features and options are chosen. For each node this results in the identification of the sets of parts that are required to realize these features or options. Thus with each node there can be associated one or more sets of parts that are completely identified by the choice made at this node. Each such set is a pseudoitem which is identified by a unique identification code number.

Fig. 2 shows the choice tree that is associated with the generic BOM in Fig. 1.

The solid lines in Fig. 2 indicate compulsory relationships between the options, while the dashed lines represent the optional relationships. The boxes represent the options a customer can select. To an option, a pseudoitem can be related that represents the parts that are tied up by selecting this option. In Table 1, the parts that are attached to a particular option are presented.

Note that in the desk chair example the back and the seat are the only parts that are common to all chairs. Thus, in this example the common pseudoitem consists of the set of these two parts. Also note that the “no wheels” choice is associated with an empty set. Finally, note that in a choice tree, there may be nodes where no parts are uniquely identified. These parts are only uniquely identified by more than one parameter value. For
instance suppose that the desk chair of Fig. 1 is available in three different widths and three different heights. Then in one node the decision is taken about the color of a chair (red), whereas the dimensions of the chair width (12", 14", 16"), and height (20', 30') still have to be determined at next nodes. Then no pseudo item is defined at the decision node for the color. However, a number of sets is linked to the last node where the selection of the other parameter values is completed, (e.g. in this case at maximum $2 \times 3 \times 2 = 12$ different sets, each representing the specific parts list that identifies a chair with a specific color, a specific width and a specific height).

Each of these parts must be made available in accordance with the expected demand and the required delivery performance per option and feature. However availability of functional options like color, chair width and chair height cannot be controlled independently. We only can control
availability in terms of physical parts or groups of physical parts. The generic bill of materials efficiently relates functional options or features to pseudos which represent physical parts. Each pseudoitem is a group of parts which together provide the critical materials for an option, a feature or a combination of options and features. These pseudoitems can be used to check material availability and to set coordinated material requirement levels.

The use of this principle requires that a list of all pseudoitems is available and that each pseudoitem is associated with a node in the choice tree. The choice tree and the associated sets of pseudoitems together, we refer to as an hierarchical pseudoitem BOM; it is a hierarchically structured set of pseudoitems or modules; as such it is a variant of the modular BOM. It is structured such that it is directly coupled with the generic BOM that is used for product variant selection. In the rest of this paper we consider the situation where a generic BOM and the associated hierarchical pseudoitem BOM is available in the product structure data base and discuss the use of the pseudoitem BOM for customer order acceptance and the planning of materials availability.

4. Customer order acceptance

Customer order acceptance involves matching the customer’s functional requirements with the options and features provided by the product family, and checking the materials availability of each of the parts required for the manufacturing of the selected product variant. This section deals with the materials availability checking part of this process.

We assume that materials are made available by placing replenishment orders for individual parts, which each can have an individual supplier, a replenishment lead time and a replenishment batch size. At any point in time, the time phased inventory position of a part is made up of the number of items physically in stock, the schedule receipts over the lead time and the reservations. The reservations can be subtracted from the inventory in order to calculate the time phased net inventory position.

We further assume that the net time phased inventory is available per part.

For checking the materials availability during the acceptance of a customer order we should check whether the pseudos related to the selected product variant can be delivered. A pseudo i consists of a set of parts $S_i$ where with each part j in the set is associated a quantity per: $q_{ij}$.

Let $I_j(t), t = 1, \ldots, T$, denote the time phased net inventory of part j. Since for $t > T$, any amount can still be made available by placing replenishment orders, we may assume that $I_j(t) = \infty$ for $t > T$.

Checking of material availability is done in the period related to the requested customer order due date. Given the high pressure for fast delivery, for most customer orders material checking is done in the first period.

Note that a part can be member of more than one pseudoitem. For instance, it can be member of the pseudoitem which represents the common parts, and it can be member of the pseudoitem that is associated with a specific option. In that case checking the availability of the common parts pseudo item should be done first (and it is; in the logic of the choice process we start by “deciding” that a product from the product family shall be selected, so the common parts are needed anyhow). If, after this first check, the common parts pseudoitem turns out to be available, then a temporary reservation must be made for all parts that are in the common parts pseudoitem. When checking the next pseudoitem in the choice tree, it is checked at the net inventory positions of all parts in this pseudoitem. Thus if many parts are member of many pseudoitems (high commonality) then the order in which the option and features in the decision tree are selected should reflect the commercial importance of these options and features. Otherwise it might be that a less important option is accepted and a more important option is later on confronted with insufficient materials availability.

If the parts for all pseudoitems needed for a specific product variant are available the order can be accepted and the temporary reservations are made definitive. If not all parts of all pseudoitems are available within the required time frame then the product cannot be delivered in time. We then can go back through the decision process and “undo”
a number of features and options. When doing so, the temporary reservation for the parts in the associated pseudoitems are canceled. Next, we can investigate the sufficient availability of materials for another product variant, with less attractive options and features. Another possibility is to investigate the future availability of sufficient materials for the preferred product variant. This requires that we check the cumulative availability of the parts for the earliest period in which all the parts for the associated pseudoitems can be made available. If the customer accepts the incurred delay in delivery, then the materials reservations for this customer order must be shifted to the period in which the assembly of this order is planned to take place. In this way we guarantee availability of materials for the assembly of this order, and create maximum availability of materials for accepting other customer orders; otherwise, the materials reservations for this order would unnecessarily block acceptance of other orders. Using the above procedure full use is made of the delivery flexibility that exists at any point in time given the time-phased availability of items at the parts level.

The parts that are consumed by the customer order acceptance process must be replenished. Replenishment takes place based on a requirements level for pseudo items, taking into account replenishment lead times, on-hand inventory, and scheduled receipts. In the next section we show how the pseudoitem definition introduced in Section 3 is used to coordinate and balance the parts requirement levels.

5. Coordinated and balanced part availability

In ATO-manufacturing part replenishments is decoupled from customer orders. To this end, stated in MRP-terminology, a master production schedule is needed which drives the part of the supply chain that is independent of the customer order. A master production schedule reconciles market needs with manufacturing and materials supply possibilities. In the situation considered in this paper the MPS should be expressed in terms of the pseudoitems. For each pseudoitem, which represents the set of parts that is uniquely needed for specific (combination of) options or features, a master schedule regarding future availability should be stated. Overplanning, needed to cope with demand level uncertainty, must be incorporated in the pseudoitem master schedule. The bill-of-material relationship between a pseudoitem and its parts automatically coordinates the availability of the individual parts to the master schedule of the pseudos.

In ATO-manufacturing with many options and features, demand uncertainty for specific products is extremely high (which is why ATO is used). We model this high uncertainty by assuming that the sales department can express its knowledge of future sales in a probability density function of the demand level for options and features. The demand level is the average demand per period, over a certain horizon, equal to or larger than the largest part replenishment lead time.

We consider the situation where the sales department is responsible for generating forecasts of total demand and of demand for options and features, and where this department determines for each pseudo item the amounts per period that should be available to serve the market. This includes estimating the uncertainty in demand, both at the product family level as well as the option and feature level, and translating this into coverage of this uncertainty by safety margins or overplanning in the pseudo item master schedules. The implication of this statement is that the sales department is directly responsible for the shortages and stocks that will emerge. This makes sense, since the sales department is in the best position to balance the possible stocking costs with the commercial losses that result from out-of-stock occurrences.

In the literature, planning bills of material, or percentage bills of material are recommended for determining the module requirement levels in modular bills of material (see Mather [10]). The percentages can be estimated by various forecasting techniques and the resulting parts requirements levels are directly used to create materials availability to cover the requirements. This seems to be a logical approach, but it totally neglects the differences in stock keeping cost and prices that result from different product variants. Therefore, the percentage bill of material concept should be extended to be able to balance materials costs with sales.
 revenues under uncertain demand for options and features. In this section we propose an alternative to the percentage bill of material that serves this purpose. The alternative builds on the generic bill of material concept and the related hierarchical pseudo item bill of material structure developed in Section 3. We also present a model that can be used to optimize the pseudoitem master schedules, taking into account (probabilistic) information about future demand, sales prices and material stocking costs. The model is demonstrated using a quantitative example. Before doing this we first discuss the economic factors that are directly affected by the parts replenishment process.

We assume that, over a certain horizon, the master schedule of the pseudos is exploded via the bill-of-material relationship into part requirements. For the individual parts, using time-phased order point logic, these gross requirements are transformed into time-phased net requirements and replenishment batches. This is equivalent to the MRP logic, with the master production schedule stated in pseudos. In this paper we only consider the effect of different pseudo master schedules on costs and revenues, and neglect the effects of safety stocks and batch sizes. This is in line with the decision hierarchy in MRP systems, where batching and safety stock decisions are taken independently from the decision regarding the levels set in the MPS. The reason for this is that master schedule decisions are taken at a different organizational level of decision making than batching and safety stock decisions. Master schedule decisions are tactical decisions, where the revenues of possible sales are balanced against the costs of making these sales possible. Batching and safety stock decisions are more strategic decisions which balance ordering costs, out-of-stock costs, inventory costs and capacity utilization costs. Generally, these latter decisions are made earlier, and at a higher level of decision making, than the requirement level decisions, which can and will be changed more frequently (see Silver et al. [11]).

We consider the problem of setting a master schedule for pseudoitems in the following context:

- Each pseudoitem, $i$, consists of a set of parts $\{S_i\}$, a part $j$ belonging to $\{S_i\}$ has a quantity per, $q_{ij}$, and a replenishment lead time $l_j$.  
- We consider the problem over an horizon which is equal to the maximum of the replenishment lead times of all parts in $S_i$:
  \[ L_i = \max\{l_j; j \in S_i\}, \]  
  $L_i$ is the replenishment lead time of pseudoitem $i$.  
- We assume that part batch sizes and safety stocks are given, and neglect their impact on period-to-period costs. We only consider the effect of different pseudoitem requirement levels in the MPS, on the costs and revenues over the pseudoitem replenishment horizon.  
- We assume that our knowledge of the level of demand, that is the average demand per period over the pseudo item replenishment lead time, for the (combination of) option(s) related to a pseudo $i$, $D_i$, can be expressed as a probability density function, $f_i(D_i)$, and its associated distribution function $F_i(D_i)$.  
- We assume that a pseudoitem master schedule, $R_i$, determines how many (combinations of) options related to that pseudoitem can be sold per period (as part of complete products) over the horizon $L_i$.  
- Over the horizon $L_i$, sales is restricted by the master schedule $R_i$; at maximum $L_i R_i$ products that require pseudo $i$ can be sold. If, over the horizon $L_i$, the demand for products that require pseudo $i$ turns out to be larger than, $R_i$, the unfilled demand for the product variants concerned will be lost. After each period the pseudoitem master schedule can be increased, but this will only affect the sales possibilities after the horizon $L_i$.  
- If demand for products that require pseudo $i$ turns out to be smaller than $R_i$, for each part $j$ belonging to the pseudoitem $i$ this will result in the build-up of stocks of parts during the part replenishment lead time $l_j$. The costs of keeping stock of parts is equal to $c_j$ per period. After each period the pseudoitem master schedule can be decreased, but this will only affect the stock of a part after its replenishment lead time $l_j$.  
- We only consider benefits and costs that are influenced by the pseudoitem master schedule that currently has to be set. The impact of future master schedules on the stocks and sales are not considered since this would require
the development of a dynamic model of the system, which goes far beyond the scope of this paper.

The optimal pseudoitem master schedule, \(R_i\), can be determined by maximizing the contribution that results from filled demand for options related to pseudo \(i\), minus the costs that result from stocks of the parts in the pseudo \(i\). For that purpose we need to know the profit contribution of an option or combination of options. Now for a product family that is modelled by a generic bill of material, it would be natural that the price of the product variant is determined by adding up price components for the options and features selected. Thus a basic price exists for any product in the family, and price components are added at each node of the choice tree, depending on the choice made at that point. In this paper we may assume that prices exist for options, or combinations of options which together require a specific pseudoitem. The profit contribution that results from the price of the option or the combination of options that require(s) pseudoitem \(i\), is denoted by \(m_i\).

Since the demand is a stochastic variable we now must find expressions for the expected contribution and the expected stock keeping costs as a function of the master schedule level, \(R_i\). These expressions can be found as follows.

If, during the pseudo item replenishment order lead time, \(L_i\), the demand for the related option, \(D_i\), is smaller than the master schedule \(R_i\), then all demand can be filled. The total contribution over the lead time is then equal to \(m_i L_i D_i\).

If, during the pseudo item replenishment order lead time, \(L_i\), the demand level for the related option, \(D_i\), is larger than the master schedule \(R_i\), then all demand up to the master schedule can be filled. The total contribution over the lead time is then equal to \(m_i L_i R_i\).

Thus for each possible value of the master schedule, \(R_i\), we can calculate the expected value of the contribution over the lead time \(L_i\) as:

\[
E[M_i|R_i] = L_i \int_0^{R_i} m_i D_i f(D_i) \, dD_i + L_i R_i \int_{R_i}^{\infty} m_i f(D_i) \, dD_i.
\]

(2)

If during the pseudoitem replenishment order lead time the option demand level is larger than the master schedule, no stocks will build up. However, if the option demand level is smaller than the master schedule of the related pseudoitem, stocks of parts will build up. For each individual part \(j\) in the pseudoitem stocks will cumulate over a horizon which is equal to the part’s replenishment lead time \(\ell_j\). Given master schedule, \(R_i\), the expected stock keeping cost for these parts is given by

\[
E[C_i|R_i] = \sum_{j \in S_i} c_j q_{ij} \int_0^{R_i} \frac{1}{2} \ell_j^2 (R_i - D_i) f_i(D_i) \, dD_i.
\]

(3)

Maximizing \(E[M_i|R_i] - E[C_i|R_i]\) leads to the following condition:

\[
\frac{d}{dR_i} \left( E[M_i|R_i] - E[C_i|R_i] \right) = 0;
\]

(4)

which after differentiation gives

\[
m_i L_i (1 - F_i(R_i)) - \sum_{j \in S_i} c_j q_{ij} \left[ \frac{1}{2} \ell_j^2 F_i(R_i) \right] = 0
\]

(5)

or

\[
F(R_i) = \frac{m_i L_i}{m_i L_i + \sum_{j \in S_i} c_j q_{ij} \ell_j^2}.
\]

(6)

where \(F(\bullet)\) is the distribution function of the demand level.

On the basis of estimates of the parameters \(m_i, L_i, c_j, q_{ij}\) and \(l_j\), and the demand level distribution function \(F(\bullet)\), the optimal value for each pseudoitem master schedule can be determined from (6).

Eq. (6) shows that, if stock keeping costs are zero \((c_j = 0)\) then \(R_i\) should be chosen such that \(F(R_i) = 1\), which implies that \(R_i\) should be set equal to the maximum possible value of the demand level. This makes sense. As the stock costs increase, the optimal master schedule, \(R_i\), decreases. If stocking costs are infinitely high, according to (6) \(F(R_i) = 0\), which leads to an optimal master schedule level equal to the minimal possible demand level; again this makes sense.

Note that Eq. (6) can also be used in the design phase of the product family. In this phase it has to be decided which option and features should be
available in the product family. Eq. (6) models a part of the operational costs of offering an option (which is often overlooked in the design phase), namely the cost related to stock keeping and obsolescence risks of items that are specifically needed for this option. Eq. (6) for instance shows that expensive long lead time parts have a negative impact on the master schedule level, \( R_i \), of the pseudoitems they belong to.

We will demonstrate the use of the procedure outline above at the first two levels of the choice tree of a somewhat more complex desk chair family than the one shown in Fig. 2.

We assume that the chair manufacturer has set up a price list where for each chair the price is built up of a common price component plus an additional price component per feature or option.

We start at the top level of the choice tree for the desk chair: the desk chair common parts. The demand level over the horizon is normally distributed with an average of 100 chairs per week and a standard deviation of 25.

The contribution of the “common option” is €200. There are now six common parts in the common parts pseudoitem having the following characteristics.

<table>
<thead>
<tr>
<th>Part no.</th>
<th>Price per</th>
<th>Quantity</th>
<th>Lead time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

The lead time of the common pseudoitem is the maximum of the lead times of the items in the set, which is 8 weeks in this case. The manufacturer uses stock keeping costs of 20% per € per year. Applying these data to Eq. (6) leads to

\[
F(R) = \frac{(200 \cdot 8/52)\left[200(8/52) + 1/2(2 \cdot 2 \cdot (3/52)^2)\right] + 3(6/52)^2 + 4(4/52)^2 + 6(6/52)^2 + 8(8/52)^2 + 10(4/52)^2}{200} = 0.948.
\]

Given the average and standard deviation of the demand for the product family, the level of the master production schedule for the common pseudo item should be equal to \( 100 + 1.6449 \times 25 = 141 \) common pseudoitem sets per week.

On the next level in the choice tree of this product family, the master schedules for two types of underframes should be determined.

The first underframe has a demand over the horizon of 60 per week with a standard deviation of 25, the second underframe has a demand over the horizon of 40 per week with a standard deviation of 15.

The contribution of a first underframe is €20, the contribution for the second underframe is €10. The pseudoitem first contains three parts with the following characteristics.

<table>
<thead>
<tr>
<th>Part no.</th>
<th>Price per</th>
<th>Quantity</th>
<th>Lead time</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>4</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

leading to

\[
F(R) = \frac{20 \cdot 6/52\left[20.6/52 + 1/2(0.8(6/52)^2) + 0.6(2/52)^2 + 0.6(4/52)^2\right]}{50} = 0.995.
\]

Thus \( R_{first} \) must be set equal to 50 + 2.57 \times 20 = 101 pseudoitem sets per week.

The pseudoitem (second) contains parts with the following characteristics:

<table>
<thead>
<tr>
<th>Part no.</th>
<th>Price per</th>
<th>Quantity</th>
<th>Lead time</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>4</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>16</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

leading to

\[
F(R) = \frac{10 \cdot 6/52\left[10 \cdot 6/52 + 1/2(0.8(6/52)^2) + 0.6(2/52)^2 + 3.2(10/52)^2\right]}{101} = 0.947
\]
Thus $R_{\text{second}}$ must be set equal to $40 + 1.62 \times 15 = 64$ pseudoitem sets per week.

These optimal individual pseudoitem master schedules are calculated for each pseudoitem independently from the other pseudoitems in the decision tree. To avoid overstocking for the desk chair options, the optimal individual pseudoitem master schedule should be adjusted according to the following additional master schedule specification rules:

1. the master schedule of each pseudoitem should be smaller than the sum of the master schedules of the next higher level pseudoitems it is related to in the decision tree;
2. the sum of the master schedules of pseudoitems on a certain level in the decision tree, should be larger than the sum of the master schedules of pseudoitems on the next higher level in the decision tree;
3. the sum of the master schedules of pseudo items on a level in the decision tree that are related to the same higher level options, should be equal or larger than the master schedules of that particular next higher level pseudoitem.

We see that in the above example, the calculated master schedules all satisfy these constraints. Now suppose that for the second underframe option the contribution would be much lower and that the second underframe would require an even more expensive component with an even longer lead time than item 12, such that the optimal master planning level for the pseudoitem would be equal to 31. Then rule 3 would be violated and the master schedule of the common pseudoitem would have to be set to $101 + 31 = 132$ since it makes no sense to have more common item pseudosets available than can be sold due to constraints on the availability of materials for mandatory features. Note however, that an option with such a low contribution and such high stock-keeping costs should already have been very critically considered in the product design phase. Thus, the product family might be redesigned to either eliminate the expensive long lead time item or replace the second underframe option with a different one with a higher contribution.

6. Conclusion

In this paper we have studied customer order acceptance and materials requirements control in assemble-to-order manufacturing.

We have analyzed the materials control aspects of customer order acceptance and materials supply planning for non-modular product families, showing many interdependencies between options and features. Building on the generic bill of material concept as a tool for customer order specification for this type of product families, we have developed a variant of the modular bill of material, called the hierarchical pseudoitem BOM. Using a small-scale example we have shown how the hierarchical pseudo item bill of materials can be used for checking materials availability and materials allocation concurrently with the customer order specification. Furthermore, we have shown how the hierarchical pseudoitem BOM can be used for setting optimal master production schedule levels for pseudoitems, assuming probabilistic knowledge of the future demand for options and features, assuming knowledge of the contributions to profit of the options, and assuming knowledge of the lead times and stock keeping costs of the parts.

References

