Capacity allocation and outsourcing in a process industry

Ton G. de Kok

School of Technology Management, Eindhoven University of Technology, Room F4, P.O. Box 513, 5600 MB Eindhoven, The Netherlands

Received 1 April 1998; accepted 1 November 1999

Abstract

In this paper we consider a production facility producing a uniform product that ships a number of different package sizes to a (number of) stockpoint(s). The packaging capacity owned by the facility is finite and the actually used capacity must be reserved sometime before actual use. Furthermore, the packaging capacity must be allocated among the different package sizes, such that a target fill rate for each package size is achieved. We propose two different capacity reservation strategies, both derived from a periodic review order-up-to-policy. One strategy assumes that excess capacity needs compared with the owned capacity cannot be filled and are postponed to the future. The other strategy assumes that excess capacity needs are outsourced. The objective of the paper is to find cost-optimal policies within each of the two classes of policies and then select the best capacity reservation policy. We present some managerial insights into the impact of various process and cost parameters and the choice of the capacity reservation strategy.

Keywords: Capacity allocation; Inventory management; Outsourcing

1. Introduction

In this paper we consider a continuous processing plant, the output of which consists of a uniform product, which is packaged in barrels of different sizes before shipment to company-owned geographically dispersed stockpoints. Demand for packaged products is only occurring at these stockpoints. It is assumed that the uniform product can be stored in sufficient amounts before packaging so as to guarantee supply of product to be packaged. Thus the main problem remaining is the planning of the packaging department. The planning of the packaging department can be seen as a hierarchical planning problem consisting of two phases. In phase 1 the total packaging capacity in working hours is determined. This decision is taken a fixed lead time before actual packaging. This lead time is required to ensure that the planned capacity, i.e. number of people, is available in the production period planned for. This phase is called the capacity reservation phase. In the second phase the available capacity is allocated to specific barrel sizes in order to satisfy customer demand. This phase is called the capacity allocation phase.

In this paper we propose a simple hierarchical planning framework that ensures that predefined customer service levels are met against minimal cost. In fact we discuss two different strategies with respect to the capacity reservation in phase 1 combined with a single capacity allocation strategy in phase 2. The first capacity reservation strategy is to install a fixed capacity \( C \). If the capacity demand exceeds \( C \) then this capacity demand cannot be filled and is postponed to future periods. The second capacity reservation strategy also assumes that
a fixed capacity $C$ is available, but in this case additional capacity can be obtained by hiring additional personnel at some labour agency. The first capacity reservation strategy buffers against future uncertainty through inventory buffers and fixed excess capacity. The second capacity reservation strategy in addition exploits short-term availability of external capacity.

Clearly a trade-off must be made in order to decide which strategy to employ. For a proper trade-off between the two strategies we must compare cost-optimal strategies within each class satisfying the service level constraints. So within each class we have to consider all costs involved and then determine the optimal capacity reservation policy parameters. In both cases the policy parameter to be determined is the fixed capacity $C$. However, the cost functions to be minimized under the two strategies differ, since only the second strategy implies costs for outsourcing. We show that, if we employ the outsourcing strategy, then the optimal $C$ satisfies a Newsboy equation. Through numerical experimentation we provide managerial insights as to when to employ which strategy.

The literature on stochastic multi-item finite capacity planning problems is rather limited. Glasserman [1] considers systems consisting of stockpoints in series, where shipments from a stockpoint to its successor are constrained by some upper bound. Such serial systems relate to the production of a single product consisting of a single subassembly, which in turn consists of another single subassembly, etc. Such capacitated systems do not occur in practice. Glasserman [2] studies a similar problem to ours. The main difference lies in the fact that a fixed amount of capacity is allocated to each package size. In our model capacity is allocated based on current capacity requirements, thereby providing more flexibility. Fransoo et al. [3] present a single machine multi-product system with set-up times. The focus of the paper is on the decision hierarchy leading to an aggregate stochastic planning model to determine optimal cycle times for each product and a run-out time based operational planning model to schedule the subsequent production runs. Fransoo’s paper differs from ours in that he considers a continuous review system with lost-sales, whereas we consider a periodic review system with backlogging. The literature on deterministic finite capacity problems is huge.

For an introduction to this subject we refer to Graves et al. [4].

The paper is organized as follows. In Section 2 we describe the model under consideration in detail and we define the capacity reservation strategies. In Section 3 we discuss the relation of this model with multi-echelon models. From this discussion we derive a near-optimal allocation policy. In Section 4 we combine the results of Sections 2 and 3 to derive expressions for the operating characteristics, i.e. service levels and costs. Procedures for deriving cost-optimal strategies are given in Section 5. Using these procedures we are able to provide in Section 6 managerial insights into the question when to use which policy and how much fixed access capacity must be installed. Finally Section 7 discusses further research opportunities.

2. The model

The production facility under consideration satisfies the demand for $N$ different package sizes from stockpoints. In turn these stockpoints satisfy external customer demand for these package sizes. Without loss of generality we assume that each stockpoint stores only one package size. The lead time from the production facility to stockpoint $i$ equals $L_i$, $i = 1, 2, \ldots, N$. The lead time equals the order fulfillment lead time consisting of order processing, production planning, production and distribution. Let $D_i$ denote the generic random variable for the demand for package size $i$ in an arbitrary time unit. We assume that the demand for package size $i$ in subsequent time periods is i.i.d. Demands for package size $i$ and $j$ in the same period may be correlated. Stockpoint $i$ has to achieve a fill rate $\beta_i$, i.e.

\[
\beta_i := \text{fraction of demand satisfied directly from stock on hand at stockpoint } i, \ i = 1, 2, \ldots, N.
\]

The stockpoints control their stocks according to periodic review order-up-to-policies.

Define

\[
S_i := \text{order-up-to-level of stockpoint } i,
\]
with the reserved capacity. If the aggregate capacity requirements are computed and compared with the reserved capacity, then all requirements are satisfied. Otherwise, the reserved capacity is rationed among the stockpoints.

Let us define the following generic random variables:

- \( D_i \): capacity demand from stockpoint \( i \) in an arbitrary time unit, \( i = 1, 2, \ldots, N \).
- \( D_0 \): aggregate capacity demand in an arbitrary time unit.
- \( c_i \): time needed for the production of one package size \( i \), \( i = 1, 2, \ldots, N \).
- \( D_i(s, t) \): capacity demand from stockpoint \( i \) during time interval \((s, t]\), \( i = 1, 2, \ldots, N \).

Then we have the following relations:

- \( D_i = c_i D_i, \quad i = 1, 2, \ldots, N \),
- \( D_0 = \sum_{j=1}^{N} c_j D_j \),
- \( D_i(s, t) = c_i D_i(s, t), \quad i = 1, 2, \ldots, N \),
- \( D_0(s, t) = \sum_{j=1}^{N} c_j D_j(s, t) \).

Capacity is reserved according to periodic review echelon order-up-to-policy. The echelon capacity is defined as the sum of all capacity reserved plus all capacity associated with the inventory positions of the stockpoints. Define

- \( S_0 \): capacity order-up-to-level.

The capacity reservation lead time equals \( L_0 \). This lead time is necessary to schedule the workforce and plan the production to enquire sufficient uniform product to be packaged. We assume that the production facility owns a fixed packaging capacity \( C \). Suppose that the order-up-to-policy prescribes to reserve a capacity \( Q_c \).

Now we distinguish between two capacity reservation strategies:

(i) Fixed capacity reservation strategy: if \( Q_c \leq C \), then reserve \( Q_c \), otherwise reserve \( C \).

(ii) Capacity reservation strategy with outsourcing: If \( Q_c \leq C \), then reserve \( Q_c \), otherwise reserve \( C \) and outsource \( Q_c - C \).

It is our objective to select the cost-optimal capacity reservation policy within these two classes. To formulate an expression for the expected total relevant costs per time unit we introduce the following parameters:

- \( \gamma_F \): cost of owned capacity per time unit,
- \( \gamma_O \): cost of outsourced capacity per time unit,
- \( v_i \): value of package size \( i \), \( i = 1, 2, \ldots, N \),
- \( r \): interest rate per time unit.

Note that the interest rate may not only include cost of capital, but may also account for, e.g., obsolescence risks, storage costs. Hence \( rv_i \) equals the holding cost of package size \( i \) per time unit. Furthermore, we assume that the value of the package size is based on the market price, so that we do not distinguish between the value of products made by owned and outsourced capacity. We exclude fixed costs for outsourcing, although it is quite straightforward to include it into the analysis.

The expression for the expected total relevant cost per time unit \( TRC(C, S_0, \{S_i\}_{i=1}^{N}) \) is given by

\[
TRC(C, S_0, \{S_i\}_{i=1}^{N}) = \gamma_F C + \gamma_O E[(D_0 - C)^+] \zeta + \sum_{j=1}^{N} rv_j E[X_j^+(C, S_0, \{S_i\}_{i=1}^{N})],
\]

where

\[
\zeta = \begin{cases} 
1 & \text{if excess capacity requirements are outsourced}, \\
0 & \text{otherwise}, 
\end{cases}
\]

and \( X_j^+(C, S_0, \{S_i\}_{i=1}^{N}) := \text{net stock at stockpoint } j \text{ at the end of an arbitrary review period} \).

In the sequel we will drop the dependence of \( X_j^+ \) and \( TRC \) on \( C, S_0, \{S_i\}_{i=1}^{N} \). Our objective is to minimize \( TRC \) subject to fill rate constraints. Hence, we want to solve the following problem:

\[
\min_{C, S_0, \{S_i\}_{i=1}^{N}} TRC \quad \text{s.t.} \quad \beta_i \geq \beta_i^*. 
\]
Here \( \beta^*_i \) denotes the target fill rate. Note that \( \beta_i \) also depends on the choice of the capacity reservation policy, \( S'_0 \) and \( \{S'_i\}_{i=1}^N \). In the next section we derive expressions for TRC and \( \beta_i \) for the two capacity reservation policies under the assumption of the so-called Balanced Stock rationing policy introduced by Van der Heijden [5] for the allocation of reserved capacity among the stockpoints. Based on these expressions we propose procedures for determination of the cost-optimal policies.

3. Analysis of the model

In this section we derive expressions for long-term average costs and customer service levels for each of the two strategies proposed. First, we analyse the capacity reservation strategies and thereafter we analyse the common capacity allocation strategy.

3.1. Capacity reservation strategy with fixed capacity

Under the capacity reservation strategy with fixed capacity \( C \) we have to modify the order-up-to-strategy with order-up-to-level \( S'_0 \). Let us define

\[
I^+_0 := \text{echelon capacity of overall system immediately after review moment } n,
\]

\[
I^-_0 := \text{echelon capacity of overall system immediately before review moment } n,
\]

Taking into account the finite capacity \( C \) we obtain the following relation between \( I^-_0,n \) and \( I^+_0,n \):

\[
I^+_0,n = \min(I^-_0,n - C, S'_0).
\]

Since \( I^-_0,n \) equals \( I^-_0,n - 1 \) minus the total capacity demand during \( (n-1)R, nR \], we obtain

\[
I^+_0,n = \min(I^-_0,n - 1 - D'_0((n-1)R, nR], C, S'_0).
\]

Now let us introduce the random variable \( Y_n \) defined by

\[
Y_n = S'_0 - I^+_0,n, \quad n \geq 0.
\]

Then it is easy to see that

\[
Y_n = \max(0, Y_{n-1} + D'_0((n-1)R, nR] - C), \quad n \geq 0.
\]

Following De Kok [6] we observe that the above equation is identical to Lindley's integral equation for the waiting times in a D/G/1-queue. In [6] a simple and efficient moment-iteration algorithm is given that computes excellent approximations for the first two moments of the waiting time of an arbitrary customer. Hence, we can use this algorithm to computer \( E[Y] \) and \( E[Y^2] \) where \( Y = \lim_{n \to \infty} Y_n \). Hence, we find that under the assumption that the system is in its steady state at time 0, \( I^+_0,0 = S'_0 - Y \). This result will be employed when we analyse the capacity allocation strategy.

Let us define \( \text{CRC}(C) \) as

\[
\text{CRC}(C) := \text{expected capacity reservation costs per time unit.}
\]

Then it readily follows that

\[
\text{CRC}(C) = \gamma FC.
\]

3.2. Capacity reservation strategy with outsourcing

Under the capacity reservation strategy with fixed capacity \( C \) and infinite outsourcing capacity we are always able to raise the echelon capacity to its order-up-to-level \( S'_0 \) at review moments. Hence

\[
I^+_0,0 = S'_0.
\]

Yet the choice of \( C \) has immediate cost implications. If \( C \) increases then the fixed capacity cost increases, but the outsourcing cost decreases. It is easy to see that under an order-up-to-policy the capacity reserved at review moment \( n \) equals the capacity demand during time interval \( (n-1)R, nR \]. If this capacity demand exceeds \( C \) we outsource the excess demand, otherwise we need not outsource. Thus we find the following expression for the capacity reservation costs \( \text{CRC}(C) \):

\[
\text{CRC}(C) = \gamma FC + \gamma O E[(D'_0((n-1)R, nR] - C)^+]\]

Note that this expression does not depend on \( n \). Hence, we find under the assumption that the system is stationary at time 0,

\[
\text{CRC}(C) = \gamma FC + \gamma O E[(D'_0,R - C)^+]. \quad (3.1)
\]

Here \( D'_0,R \) denotes the aggregate capacity demand for package sizes during an arbitrary review period. But Eq. (3.1) is related to the well-known Newsboy equation (cf. [7]). This follows from rewriting the
above equation as follows:

\[ \text{CRC}(C) = \gamma_F C + \gamma_0 \int_c^\infty \frac{(y - C)}{c} dF_{D_e,s}(y) \]

\[ = \gamma_F C + \gamma_0 \int_c^\infty (1 - F_{D_e,s}(y)) dy. \]

The cost-optimal \( C \) follows by setting the first derivative of \( \text{CRC}(C) \) equal to 0 and verification of a positive second derivative in the optimum found. It easily follows that

\[ \frac{d\text{CRC}(C)}{dC} = \frac{\gamma_F - \gamma_0(1 - F_{D_e,s}(C))}{\gamma_0} \]

and

\[ \frac{d^2c_{\text{cap}}(C)}{dC} = \frac{dF_{D_e,s}(C)}{dC} > 0. \]

Since \( F_{D_e,s} \) is a monotone increasing function, the second derivative is positive for all \( C \) and the optimum \( C^* \) is found by solving for

\[ F_{D_e,s}(C^*) = \frac{\gamma_0 - \gamma_F}{\gamma_0} \]

which is a Newsboy equation. Note that we implicitly used the fact that the choice of \( C \) does not influence the holding costs incurred at the stockpoints, nor the customer service levels. This concludes our analyses of the capacity reservation strategies.

### 3.3. Capacity allocation strategy

The capacity allocation strategy is based on Van der Heijden [5], who proposes a so-called Balanced Stock (BS) rationing rule for the situation where a stockpoint has to ration available inventory among its succeeding stockpoints. Here we are confronted with a similar situation. After reserving capacity at time 0 we must ration available capacity among different package sizes at time \( L_0 \). To ration the available capacity we aim at achieving the order-up-to-level \( S_i \) of package size \( i \) for each \( i \). If we assume that

\[ S_0 = \sum_{j=1}^N c_j S_j, \]

then we can be sure that all available capacity is rationed among the package sizes. This follows from the fact that at time \( L_0 \) the sum of the reserved capacity at time and all echelon capacities of the individual package sizes equals \( I_0 - D(0, L_0) \). The target sum of all echelon capacities equals \( \sum_{j=1}^N c_j S_j \). Eq. (3.3) together with \( I_0 \leq S_0 \) implies that the total available echelon capacity is less than or equal to the target echelon capacity. Hence, no reserved capacity remains unused. The BS rationing rule is as follows. Let \( I_i \) be defined as

\[ I_i := \text{inventory position of stockpoint } i \text{ after rationing at time } L_0, i = 1, 2, \ldots, N. \]

Then we have

\[ I_i = S_i - \frac{p_i}{c_i} (D(0, L_0) + Y) \] (3.4)

with

\[ p_i = \frac{\sigma^2(D_{i,R})}{2 \sum_{j=1}^N \sigma^2(D_{j,R})} + \frac{1}{2N}, \quad i = 1, 2, \ldots, N. \]

The random variable \( D_{i,R} \) denotes the capacity demand for package size \( i \) during an arbitrary review period. Furthermore, we assume that \( Y \) is identically equal to 0 for the capacity reservation strategy with outsourcing. This BS rationing rule aims at minimizing the probability of imbalance. Imbalance occurs when the above allocation rule leads to negative capacity allocated to one or more package sizes. Van der Heijden et al. [8] show the robustness of BS rationing for a wide range of multi-echelon inventory systems under periodic review order-up-to-policies.

From this expression for \( I_i \) we can derive expressions for both costs and service levels for the different package sizes. It is easy to see that the net stock \( X_i \) at the end of period \( L_0 + L_i + R \) equals

\[ X_i = I_i - D_i(L_0, L_0 + L_i + R), \quad i = 1, 2, \ldots, N, \]

and thus we find for the average on-hand stock

\[ E[X_i^+] = E[I_i - D_i(L_0, L_0 + L_i + R)]^+. \] (3.5)

Analogously, we find for the fill rate of package size \( i \),

\[ \beta_i = 1 - \frac{[E[D_i(L_0, L_0 + L_i + R) - I_i]^+] - E[D_i(L_0, L_0 + L_i) - I_i]^+] / E[D_{i,R}]. \] (3.6)
Now note that
\[
D_i(L_0, L_0 + L + R) - I_i \\
= D_i(L_0, L_0 + L_i + R) \\
+ \frac{p_i}{c_i} (D_0(0, L_0] + Y) - S_i,
\]
\[
D_i(L_0, L_0 + L_i) - I_i \\
= D_i(L_0, L_0 + L_i) + \frac{p_i}{c_i} (D_0(0, L_0] + Y) - S_i.
\]

Thus, we need to know the probability distributions of \( D_i(L_0, L_0 + L_i) \) and \( D_i(L_0, L_0 + L_i + R) \), and \( D_i(L_0, L_0 + L + R) \). We can easily compute the first two moments of each random variable involved. Furthermore, the random variables added together are independent of each other. Hence, it is easy to compute the first two moments of the random variables in (3.5) and (3.6). As in [8] we can fit mixtures of Erlang distributions to the above random variables so that these expressions can be easily computed.

### 3.4. Total relevant costs

From the definition of TRC and CRC we find
\[
\text{TRC}(C) = \text{CRC}(C) + \sum_{j=1}^{N} r_{ij} E[X_j^+(C)]. \tag{3.7}
\]

Here we emphasized the dependence of TRC and CRC on the owned capacity \( C \). We want to minimize TRC subject to the service level constraints \( \beta_j \geq \beta_j^* \) for all \( j \). It follows from the analysis in Section 3.1 that the optimal capacity reservation policy with outsourcing follows from solving for \( C^* \) in the Newsboy equation (3.1) and the associated costs follow from the expressions for CRC(\( C^* \)) and \( E[X_j^+(C^*)] \). Note that in this case \( E[X_j^+(C)] \) is independent of \( C \). However, in the case of the fixed capacity reservation policy \( E[X_j^+(C)] \) depends on \( C \) through the random variable \( Y \). It is easy to find for each \( C \) the unique policy satisfying the service level constraints. Define
\[
\text{TRC}(C, \beta^*) := \text{expected total relevant cost associated with the unique policy that satisfies the service level constraints with owned capacity } C.
\]

From numerical experiments we found that \( \text{TRC}(C, \beta^*) \) is convex in \( C \) under the fixed reservation policy. Thus we can compute the optimal \( C^* \) by a simple bisection procedure.

In the next section we provide managerial insights into the problem of choosing the optimal capacity reservation policy. We observe that the optimal policy depends on the pdf. of \( D_i \), as well as on the parameters \( v_i, c_i, L_i, \beta_i, j = 1, 2, \ldots, N \) and \( L_0, \gamma_F \) and \( \gamma_O \).

We normalize the cost-optimization problem as follows. Define
\[
\gamma := \text{cost of owned capacity as a fraction of the cost of outsourced capacity},
\]
\[
\alpha := \text{cost of capacity per package size } i \text{ as a fraction of the value of package size } i \text{ in case the optimal capacity reservation policy with outsourcing is applied, } i = 1, 2, \ldots, N.
\]

Clearly,
\[
\gamma = \frac{\gamma_F}{\gamma_O}.
\]

The meaning of \( \alpha \) can be explained as follows.
Let us assume that the optimal capacity reservation policy with outsourcing is used. Define
\[
v_i := \text{cost of capacity per package size, } i = 1, 2, \ldots, N.
\]

The overall capacity cost per time unit equals CRC,
\[
\text{CRC} = \gamma_F C^* + \gamma_O E[(D_0^* - C^*)^+].
\]

Since \( E[D_i] \) equals the capacity requirements per time unit for package size \( i \), it is good practice to assume that total capacity cost per time unit associated with package
\[
\text{size } i = \frac{E[D_i]}{E[D_0^*]} (\gamma_F C_i^* + \gamma_O E[(D_0^* - C_i^*)^+]).
\]

By dividing this cost by the expected demand for package size \( i \) per time unit, we find
\[
v_i = \frac{c_i}{E[D_0^*]} (\gamma_F C_i^* + \gamma_O E[(D_0^* - C_i^*)^+]). \tag{3.8}
\]

Now we use the definitions of \( \alpha \) and \( v_i \) to obtain
\[
v_i = \alpha v_i, \quad i = 1, 2, \ldots, N.
\]
Given $x$ and $v^i_t$ we thus compute $v_i$ through

$$v_i = \frac{v^i_t}{x}, \quad i = 1, 2, \ldots, N.$$  

In the numerical experiments presented in Section 4 we first determine $C^*$ for the capacity reservation strategy with outsourcing given the value of $\gamma$ from Eq. (3.1). Next we compute $v^i_t$ from Eq. (3.8) and $v_i$ from Eq. (3.9). Finally, we compute the total relevant costs from Eq. (2.1) (or equivalently Eq. (3.7)) for both capacity reservation strategies and compute the optimal $C^*$ for both to decide which capacity reservation strategy is optimal.

4. Managerial insights

In this section we obtain managerial insights into the impact of a number of process characteristics on the choice of the capacity reservation policy. Since we deal with a complex problem with a large number of process characteristics interacting in some a priori unknown way, we consider only marginal effects of differences in one particular process characteristic. In order to make these marginal effects clear we use the following procedure. For each value of the process characteristic under consideration we determine all combinations of $x$ and $c$ values that yield identical total relevant costs for both capacity reservation policies. Knowing this iso-cost curve in the $(x, c)$-plane it is easy to determine the optimal capacity reservation policy for a given combination of $x$ and $c$. The impact of changes in the value of the process characteristics is determined by comparing the associated iso-cost curves in the $(x, c)$-plane.

Note that $x$ denotes the capacity cost per product as a percentage of the total value of the product and $c$ denotes the fixed capacity cost per time unit as a percentage of the outsourced capacity cost per time unit. It is intuitively clear that the question whether to outsource or not highly depends on the values of $x$ and $c$. If $x$ is extremely small than capacity costs are only a very small portion of the overall relevant costs. In that case one should focus on keeping holding costs as low as possible. This implies that one should choose for outsourcing. If on the contrary capacity costs are a large part of the total relevant costs, i.e. $x$ is close to 1, then one should choose the fixed capacity reservation policy. Similarly, if $c$ is close to 1, then the cost of outsourcing is only marginally higher than the cost of owned capacity. In that case outsourcing is cost-beneficial. If $c$ is close to 0 than the cost of outsourcing is extremely high, so one chooses the fixed capacity reservation policy at the expense of higher holding costs.

Although this reasoning is quite intuitive, it does not help very much for decision-making in practical situations, because typically in practice the above extreme $(x, c)$ combinations do not exist. As indicated above the iso-cost curve in the $(x, c)$-plane provides us a powerful decision tool. Especially it is easy to determine the sensitivity of the decision taken with respect to errors in the estimation of $x$ and $c$.

The Pareto base case: In order to determine iso-cost curves for relevant practical cases we constructed a situation that obeys Pareto’s law, i.e. a small number of package sizes accounts for the majority of turnover. The Pareto base case is described in Table 1.

For each of the package sizes we assume that the lead time to the local stockpoints equals two time units, i.e. $l_t = 2$ for all $i$. Furthermore, we assume that the required fill rate $\beta_i = 0.90$. The capacity reservation lead time $L_h$ equals three time units, the review period is one time unit.

4.1. Impact of interest rates

Using the optimization procedures described in Section 3 we determine for each $(x, c)$-combination the optimal policies within each of the two classes of capacity reservation policies, that yield the same

<table>
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<tr>
<th>$i$</th>
<th>$E(D_i)$</th>
<th>$\sigma(D_i)$</th>
<th>$c_i$</th>
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expected overall relevant costs within a tolerance of 0.005. We varied the interest rate as 0.004, 0.008, 0.016 and 0.032. The interest rates 0.004 and 0.008 are comparable to a 20% and 40% interest rate on a yearly basis, when the time unit equals one week. The interest rates 0.016 and 0.032 are comparable to an interest rate of 20% and 40% on a yearly basis, when the time unit equals one month. The resulting iso-cost curve is presented in Fig. 1.

As expected we find that as the interest rate increases, it becomes more favourable to outsource capacity, since the cost of holding inventory increases. The capacity outsourcing policy always yields minimal holding costs. Another typical phenomenon of the iso-cost curves is the steep ascent beyond some critical value of $\gamma$. This indicates the dominant effect on total relevant costs of the cost of outsourcing capacity as compared with the cost of owned capacity.

4.2. Impact of variability in demand

Another important process characteristic is demand variability. The measure we use for demand variability is the coefficient of variation. The coefficient of variation (cv) is defined as the quotient of the standard deviation and the mean. To measure the impact of cv on the iso-cost curves we simplified the Pareto base case with respect to the demand and capacity usage data. We consider four products with $E[D_i] = 100, c_i = 1, i = 1, 2, 3, 4$. Lead time and fill rate parameters remain unchanged. We assumed $r$ equal to 0.004. We varied cv as 0.5, 1 and 2. In Fig. 2 we show the resulting iso-cost curves.

We find that the cv has a considerable impact on the selection of the optimal capacity reservation policies. As cv increases it becomes more favourable to outsource the excess need for capacity. The intuitive explanation is probably two-fold. First of all holding costs increase due to higher safety stocks to maintain the same service level. Secondly, with increasing cv of demand for different package sizes, the cv of the demand for capacity also increases. If one chooses a fixed capacity reservation strategy then increasing variety implies that more and more periods of low capacity demand alternate with some periods with excessive capacity demand. Such a demand pattern implies either a lot
4.3. Impact of capacity usage distribution

Another process characteristic is the capacity usage per package size. It is not a priori clear whether differences in capacity usage have an impact on the choice of the optimal capacity reservation policy. To understand the impact of differences in capacity usage we consider the situation described above, i.e. four different package sizes each with $E[D_i] = 100$ and $cv_i = 1$. We assume that the average capacity usage equals 1. We assume that $c_i = 1 + \delta(i - \frac{1}{2})$, $i = 1, 2, 3, 4$. We vary $\delta$ as 0, 0.25 and 0.5. Increasing the value of $\delta$ implies greater differences in capacity usage. With each value of $\delta$ a different capacity usage profile is associated. Fig. 3 shows the resulting iso-cost curves.

It is clear that the difference in capacity usage profile hardly impacts the choice of the capacity reservation policy. We have no intuitive explanation for this robustness phenomenon. This insensitivity
phenomenon shows even stronger when comparing the three cases with four package sizes with the case of one package size having the same aggregate demand and capacity usage characteristics, i.e. $E[D_1] = 400$ and $cv_1 = \frac{1}{2}$, and $c_1 = 1$. Again we find more or less the same iso-cost curve. Hence, the selection of the capacity reservation policy depends on the capacity usage profile only through its aggregate average usage.

4.4. Impact of number of package sizes

Another process characteristic considered here is the number of package sizes $N$. Again we assume identical demand and lead time characteristics for all $N$ package sizes. Although it is clear that the larger the $N$, the larger the average capacity usage, the iso-cost curves in the $(\alpha, \gamma)$-plane are a means of comparing situations with different numbers of package sizes, since the iso-cost curves do not change if all demands are multiplied by a fixed constant. We varied $N$ as 4, 8 and 16. In Fig. 4 we show the iso-cost curves.

As expected we find a similar effect as observed in Fig. 2, where we have shown the impact of demand variability. Due to the increase in $N$ the $cv$ of the aggregate demand and aggregate capacity usage decreases. Due to this decrease in $cv$ the outsourcing option becomes less attractive as $N$ increases. Note again the steep increase in the iso-cost curve beyond $\gamma$ equal to 80–90%.

This concludes our discussion of the impact of different process characteristics on the choice of the capacity reservation policy. Our intention was to present some generic managerial insights. Of course, the final choice for one capacity reservation policy or the other depends on the particular process characteristics in the situation under consideration.

5. Conclusions and further research

In this paper we studied a continuous processing plant, the output of which consists of a uniform product, which is shipped in different package sizes to one or more, possibly geographically dispersed, stockpoints. We proposed a simple hierarchical planning framework that ensures that predefined customer service levels are met against minimal cost. We found optimal policies within two classes of simple, practically useful, capacity reservation policies. These capacity reservation policies are combined with a single capacity allocation policy. This allocation policy is the so-called linear allocation policy as described in [8,9]. Diks [9] gives...
evidence that such linear allocation policies are near-optimal. We used the so-called Balanced Stock rationing rule, because that policy ensures a minimal probability of imbalance.

The focus of this paper was on the choice between two types of capacity reservation policies: a fixed capacity reservation policy and a policy that outsources excess capacity demand beyond the available packaging capacity. Through appropriate normalization through the introduction of $\gamma$, the cost of owned capacity as a percentage of the cost of outsourced capacity, and $\alpha$, the cost of capacity as a percentage of the value of a product, it was possible to show some generic managerial insights.

The most important observation is that there is some critical $\gamma$ above which it is beneficial to outsource capacity and below which it is beneficial not to outsource capacity. The value of the critical $\gamma$ depends on the process characteristics. Qualitative insights are obtained into the impact of various process characteristics on the choice of the capacity reservation policy. One of the observations is that this choice depends mainly on the aggregate capacity usage characteristics.

The analysis presented in this paper can be easily extended to multi-stage systems, where, after the capacity allocation phase the packaged products are shipped via a number of intermediate stockpoints to the stockpoints that satisfy customer demand. This analysis combines the results presented in this paper with the results in [10] for divergent multi-echelon systems. The key assumption made in this paper is the availability of sufficient product to be packaged. Further research is required to find practically useful policies for the situation where this assumption is no longer valid.

References