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Fractional cointegration, long memory, and exchange rate dynamics
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Abstract
In this article, we examine empirically whether fractional cointegration exists in the system of seven exchange rates and this form of cointegration is associated with long memory. The results indicate that fractional cointegration in exchange rates is a feature for only the 1980–1984 sample, not for the entire post-1973 float, the subperiod before 1980, and the sample after 1984. The results show a significant long memory, mean-reverting behavior in equilibrium errors for the subperiod 1980–1984. The results also suggest that the exchange rates are cointegrated in the usual way for the 1985–1992 sample data. The current findings suggest a conjecture that the fractional cointegration feature of exchange rates, if exists, could be changing across varying time spans.

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1. Introduction
One of the observed empirical regularities in the finance literature is that the dynamics of individual nominal exchange rate series in the post-Bretton Woods era are well characterized by a time-series model with one unit root, or more closely, by a martingale. Using Johansen’s (1988) multivariate cointegration test, Baillie and Bollerslev (1989), however, find the existence of cointegration among a set of seven exchange rates, suggesting that the set of exchange rates possess a long-run equilibrium relationship. The
presence of cointegration implies the existence of Granger-causal orderings among cointegrated exchange rates. That is, it is possible to predict one exchange rate on the basis of the others.

More recently, Diebold, Gardeazabel, and Yilmaz (1994) provide evidence showing that vector autoregressive models with varying numbers of cointegrating relationships do not outperform a martingale in out-of-sample forecasting. Furthermore, Diebold et al. (1994) apply the Johansen (1991) procedure to the Baillie and Bollerslev (1989) sample and yet observe a strikingly different result—no cointegration. Diebold et al. (1994) argue that the contradiction arises because the Johansen test for cointegration is sensitive to whether a drift term is allowed for in the estimated model. Koop (1994) also obtains the same result of no cointegration in exchange rates by using Bayesian methods.

In view of the mixed evidence on cointegration in exchange rates, Baillie and Bollerslev (1994) employ the Engle and Granger (1987) residual-based approach to re-examine the issue and confirm the results of Diebold et al. (1994) that no cointegration exists in the seven nominal exchange rates. While reversing their earlier findings, Baillie and Bollerslev (1994) argue with a new perspective that the seven exchange rates are not cointegrated in the usual sense; they are fractionally cointegrated. In specific, they suggest that the error correction term is a fractionally-integrated series with an order of 0.89. Nonetheless, whether fractional cointegration is indeed a feature of nominal exchange rates has yet to be substantiated with further evidence.

In this study, we empirically examine whether fractional cointegration exists in the system of seven exchange rates and this form of cointegration is associated with long memory. The concept of fractional cointegration (Granger, 1986) permits us to measure the size of the common stochastic trend of a system of economic variables on a continuous scale. Conventional cointegration tests, such as the Johansen (1988, 1991) test and the Engle and Granger (1987) residual-based test, do not allow for fractional cointegration. For the residual-based test, the null hypothesis of a unit root in the cointegrating residuals against stationarity is arbitrarily restrictive. Fractional cointegration analysis relaxes this restriction by allowing the order of integration in the cointegrating residuals to be noninteger. Particularly, in the context of exchange rate dynamics, fractional cointegration implies that its equilibrium errors follow a fractionally-integrated, long-memory process. That is, short-run deviations from the equilibrium point tend to respond more slowly to shocks so that the influence of shocks to the equilibrium exchange rates may only vanish at very long horizons. It also implies that the equilibrium errors exhibit a mean reversion, although they do not dismiss rapidly in the short run. More specifically, if exchange rates are fractionally cointegrated then any improvement in reducing forecasting errors from using error correction terms may be useful only for long horizons.¹

The rest of the article is organized as follows. Section 2 discusses briefly the data and preliminary cointegration test results. Section 3 outlines Geweke and Porter-Hudak’s (1983) method for testing fractional cointegration based on the residuals from cointegrating regression and reports empirical results. In Section 4, we examine further whether the error correction term follows a non-random, mean-reverting
Table 1
Johansen cointegration tests

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( r \leq 6 )</td>
<td>1.554</td>
<td>0.873</td>
<td>1.914</td>
<td>0.236</td>
<td>3.76</td>
</tr>
<tr>
<td>( r \leq 5 )</td>
<td>3.504</td>
<td>5.472</td>
<td>5.750</td>
<td>5.394</td>
<td>15.41</td>
</tr>
<tr>
<td>( r \leq 4 )</td>
<td>12.787</td>
<td>11.612</td>
<td>13.254</td>
<td>11.026</td>
<td>29.68</td>
</tr>
<tr>
<td>( r \leq 3 )</td>
<td>24.455</td>
<td>21.608</td>
<td>28.090</td>
<td>20.656</td>
<td>47.21</td>
</tr>
<tr>
<td>( r \leq 2 )</td>
<td>42.427</td>
<td>45.470</td>
<td>45.600</td>
<td>34.865</td>
<td>68.52</td>
</tr>
<tr>
<td>( r \leq 1 )</td>
<td>67.927</td>
<td>72.043</td>
<td>78.475</td>
<td>50.791</td>
<td>94.16</td>
</tr>
<tr>
<td>( r = 0 )</td>
<td>104.488</td>
<td>108.108</td>
<td>132.790</td>
<td>88.204</td>
<td>124.24</td>
</tr>
</tbody>
</table>

This table reports the cointegration results of the Johansen trace statistics for the seven foreign exchange rates. The error-correction model used is \( \Delta s_t = \alpha + B\Delta s_{t-1} - \Pi s_{t-1} + e_t \), where \( S \) denotes a log exchange rate. The Johansen trace test statistic of the null hypothesis that there are at most \( r \) cointegrating vectors \( 0 \leq r \leq 6 \) (i.e., there are \((7-r)\) common stochastic trends) is

\[
\text{trace} = -T \sum_{i=r+1}^{7} \ln(1 - \lambda_i),
\]

where \( \lambda_i \)'s are the \((7-r)\) smallest squared canonical correlations of lagged exchange rates with respect to differenced exchange rates corrected for lagged differences and \( T \) is the sample size. \( T \)'s are 1736, 1245, 2003, and 4984 for the periods of 1973–1979, 1980–1984, 1985–1992, and 1973–1992, respectively. The critical values are obtained from Diebold et al. (1994).

process using a joint variance ratio test. Section 5 provides further tests on whether the error correction term is a long-memory, mean-reverting process using Lo’s (1991) rescaled range analysis. The final section concludes the article.

2. Data and cointegration test results

We use the same seven nominal exchange rate series, namely German mark, British pound, Japanese yen, Canadian dollar, French franc, Italian lira, and Swiss franc vis-à-vis the U.S. dollar, as in Baillie and Bollerslev (1989, 1994) and Diebold et al. (1994). The data are bid rates (U.S. dollar per currency unit) at the close of trading in London, obtained from a data tape maintained by Ministry of Education of Taiwan.


The results of the Johansen cointegration tests are reported in Table 1. The trace statistics show that the null hypothesis of no cointegration \( (r = 0) \) cannot be rejected for three of the four periods, including 1973–1979, 1980–1984, and 1973–1992. The result of non-cointegration reenforces the findings reported in Baillie and Bollerslev
3. Fractional cointegration analysis

While the usual notion of cointegration has the strict I(0) and I(1) distinction, fractional cointegration allows the equilibrium error from a system of variables to be fractionally integrated. A system of I(1) variables $S = \{s_j, j = 1, \ldots, n\}$ is said to be fractionally cointegrated if a cointegrating vector $\beta$ exists such that $\beta' S$ is integrated of order $d$ with $0 < d < 1$. A fractionally integrated process has long memory since its autocorrelations decay hyperbolically, in contrast to a faster, geometric decay of a finite order ARMA process (Granger & Joyeux, 1980; Hosking, 1981). Furthermore, an I($d$) process with $d < 1$ is mean-reverting (Cheung & Lai, 1993). Thus, if exchange rates are fractionally cointegrated, then an equilibrium relationship among the exchange rates in the system will prevail in the long run.

When testing fractional cointegration, we can adopt the Engle and Granger (1987) two-step cointegration approach, though a different method than the standard unit root test is preferred in the second step. Cheung and Lai (1993) suggest the use of Geweke and Porter-Hudak’s (GPH) (1983) method to detect the order of integration in the error correction term estimated from the ordinary least squares regression.

Consider the error correction term, $Z_t$, derived from the cointegrating system of seven exchange rates, and its first difference, $X_t$, denoted $X_t = (1-L)Z_t$, where $L$ is the lag operator. Time series $Z_t$ follows an autoregressive fractionally-integrative moving-average (ARFIMA) process of order $(p, d, q)$ if

$$
\Phi(L) (1 - L)^d Z_t = \Phi(L) (1 - L)^d X_t = \Theta(L) \epsilon_t, \quad \epsilon_t \sim \text{iid}(0, \sigma^2),
$$

(1)

where $\Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$, $\Theta(L) = 1 + \theta_1 L + \cdots + \theta_q L^q$, $d = 1 + d'$, and $(1 - L)^d$ is the fractional differencing operator defined by $(1 - L)^d = \sum_{k=0}^{\infty} \Gamma(k - d) L^k / \Gamma(k + 1) \Gamma(-d)$ with $\Gamma(.)$ denoting the gamma function. The parameter $d$ is allowed to assume any real value. Geweke and Porter-Hudak (1983) demonstrate that the fractional differencing parameter $d$ can be estimated consistently from the least squares regression at frequencies near zero:

$$
\ln(I(\omega_j)) = \alpha + \beta \ln(4 \sin^2(\omega_j/2)) + \psi_j, \quad j = 1, \ldots, J,
$$

(2)

where $\alpha$ is the constant term, $\omega_j = 2\pi j/T (j = 1, \ldots, T - 1), J = T^{\eta} \ll T$, $T$ is the number of observations, and $I(\omega_j)$ is the periodogram of time series $X_t$ at frequency $\omega_j$.

With a proper choice of $J$, the negative of the OLS estimate of $\beta$ coefficient gives a consistent and asymptotically normal estimate of the order of integration $d$. Moreover, this is true regardless the orders and the estimates of the parameters of the ARMA process. While $\eta = 0.5$ is suggested in the empirical analysis, we set $\eta$ also equal to 0.475 and 0.525 for checking the sensitivity of the results to the selection of $\eta$.

To ensure that stationarity and invertibility are achieved, we conduct the GPH test on the first-differenced series, i.e., $X_t$. As $d$ of the level series equals $1 + d'$, a value
Table 2
Results of the Geweke-Porter-Hudak test for fractional cointegration

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{d} (\eta = .475)$</td>
<td>0.915</td>
<td>0.666</td>
<td>0.858</td>
<td>0.863</td>
</tr>
<tr>
<td>$p$ value</td>
<td>(0.254)</td>
<td>(0.010)</td>
<td>(0.125)</td>
<td>(0.069)</td>
</tr>
<tr>
<td>$\hat{d} (\eta = .500)$</td>
<td>0.944</td>
<td>0.702</td>
<td>0.847</td>
<td>0.894</td>
</tr>
<tr>
<td>$p$ value</td>
<td>(0.321)</td>
<td>(0.011)</td>
<td>(0.082)</td>
<td>(0.110)</td>
</tr>
<tr>
<td>$\hat{d} (\eta = .525)$</td>
<td>1.001</td>
<td>0.759</td>
<td>0.862</td>
<td>0.921</td>
</tr>
<tr>
<td>$p$ value</td>
<td>(0.473)</td>
<td>(0.019)</td>
<td>(0.085)</td>
<td>(0.152)</td>
</tr>
</tbody>
</table>

The fractional differencing parameter $d'$ is estimated from the least squares regression

$$\ln(I(\omega_j)) = \alpha + \beta \ln(4 \sin^2(\omega_j/2)) + \nu_j, \quad j = 1, \ldots, J,$$

where $\omega_j = 2\pi j/T \quad (j = 1, \ldots, T - 1), \quad J = T^h, \quad T$ is the sample size, and $I(\omega_j)$ is the periodogram of the time series of error correction changes at frequency $\omega_j$. The error correction term $\hat{d}$ is the residual from an OLS regression of the logarithm of the German mark on a constant and the logarithms of British pound, Japanese yen, Canadian dollar, French franc, Italian lira, and Swiss franc. The fractional-differencing parameter $d'$ is estimated as the negative of the OLS estimate of $\beta$. The order of integration $d = 1 + d'$ in the error correction term is reported in the main rows, with $p$ values given in parentheses immediately below each main row. The $p$ value reports the probability that the $d$ estimate from the bootstrap distribution is less than the sample $d$ estimate in 1,000 iterations. The $p$ value is for testing the unit-root null hypothesis ($d = 1$) against the mean-reverting, fractionally-integrated alternative ($d < 1$).

The inconsistent findings between the 1980–1984 period and the other three time spans cast doubt on the inference of fractional cointegration derived from solely the 1980–1984 observations.
It is noticed that there exists usual cointegration in the 1985–1992 sample data, although no fractional cointegration is documented.

4. Mean reversion, martingale and variance ratio test

Although concrete evidence is not obtained as to whether the exchange rates are fractionally cointegrated, the $d$ estimates of the error correction term suggest that the exchange rates might not follow a vector martingale at least for the 1980–1984 period. To examine further whether the error correction term follows a martingale, we employ a joint variance ratio test based on Lo and MacKinlay (1988). Diebold (1989) shows that the joint variance ratio test used in this study has good power against fractionally-integrated alternatives.

The variance ratio test makes use of the fact that the variance of the increments in a martingale is linear in the sampling interval. That is, if the time series $Z_t$ follows a martingale process, the variance of its $q$-th differences would be $q$ times the variance of its first differences. Using a daily interval as the base in our study, the variance ratio at lag $q$, $VR(q)$, is defined as

$$VR(q) = \frac{\sigma_q^2}{\sigma_1^2},$$

where $\sigma_q^2$ is an unbiased estimator of the variance of the $q$-th difference of $Z_t$, and $\sigma_1^2$ is an unbiased estimator of the variance of the first difference of $Z_t$. Thus, under the martingale hypothesis, $VR(q)$ equals one for any $q$ chosen.

The variance ratio estimate is closely related to the serial correlation of the series as the variance-ratio estimate for a given $q$ is approximately a linear combination of the first $q - 1$ autocorrelation coefficients (Cochrane, 1988). Accordingly, rejecting the null hypothesis of martingale with a variance ratio of less (greater) than one implies negative (positive) serial correlations, or mean-reversion (mean-aversion). Thus, the $VR$ statistic provides an intuitively appealing way to search for mean reversion, an interesting deviation from the martingale hypothesis.

The hypothesis is tested under the asymptotic distributions of both homoskedasticity- and heteroskedasticity-robust variance ratio estimators developed by Lo and MacKinlay (1988). Nevertheless, Lo and MacKinlay's asymptotic distribution might not be an accurate approximation when $q$ is large and sample size is small (Richardson & Stock, 1990). We thereby provide the $p$ value of the $VR$ statistics in testing the null by using the bootstrap method. 9

Furthermore, in past work (i.e., Liu & He, 1991), the martingale hypothesis is considered rejected when at least some of the $VR$ statistics provide evidence against it. Richardson (1993) notes that the failure of including a joint test that combines all of the information from several $VR$ statistics would tend to yield stronger results. To provide a joint test that takes account of the correlations between $VR$ statistics at various horizons, we consider the Wald test as follows (Goetzmann, 1993; Cecchetti & Lam, 1994):

$$W(q) = [VR(q) - E[VR(q)]]'\Omega^{-1}VR[(q) - E[VR(q)]] \sim \chi^2$$

(4)
Table 3
Variance-ratio analysis of the martingale hypothesis for the error correction term

<table>
<thead>
<tr>
<th>Number of base observations forming variance ratio</th>
<th>4</th>
<th>8</th>
<th>16</th>
<th>24</th>
<th>Wald statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample period</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.628)</td>
<td>(1.268)</td>
<td>(2.371)</td>
<td>(2.438)</td>
<td>q = .053</td>
</tr>
<tr>
<td></td>
<td>[.462]</td>
<td>[.918]</td>
<td>[1.765]</td>
<td>[1.870]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>p = .474</td>
<td>p = .105</td>
<td>p = .019</td>
<td>p = .016</td>
<td></td>
</tr>
<tr>
<td>1980–1984</td>
<td>0.792</td>
<td>0.763</td>
<td>0.772</td>
<td>0.764</td>
<td>16.802</td>
</tr>
<tr>
<td></td>
<td>(−3.925)</td>
<td>(−2.825)</td>
<td>(−1.825)</td>
<td>(−1.516)</td>
<td>q = .003</td>
</tr>
<tr>
<td></td>
<td>[−2.068]</td>
<td>[−1.796]</td>
<td>[−1.322]</td>
<td>[−1.159]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>p = .000</td>
<td>p = .000</td>
<td>p = .024</td>
<td>p = .055</td>
<td></td>
</tr>
<tr>
<td>1985–1992</td>
<td>0.540</td>
<td>0.402</td>
<td>0.350</td>
<td>0.322</td>
<td>120.354</td>
</tr>
<tr>
<td></td>
<td>(−10.994)</td>
<td>(−9.044)</td>
<td>(−6.604)</td>
<td>(−5.539)</td>
<td>q = .000</td>
</tr>
<tr>
<td></td>
<td>[−5.845]</td>
<td>[−5.353]</td>
<td>[−4.273]</td>
<td>[−3.762]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>p = .000</td>
<td>p = .000</td>
<td>p = .000</td>
<td>p = .000</td>
<td></td>
</tr>
<tr>
<td>1973–1992</td>
<td>0.936</td>
<td>0.931</td>
<td>0.985</td>
<td>1.018</td>
<td>9.818</td>
</tr>
<tr>
<td></td>
<td>(−2.404)</td>
<td>(−1.654)</td>
<td>(−0.246)</td>
<td>(0.232)</td>
<td>q = .041</td>
</tr>
<tr>
<td></td>
<td>[−1.531]</td>
<td>[−1.135]</td>
<td>[−0.182]</td>
<td>[0.179]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>p = .007</td>
<td>p = .045</td>
<td>p = .430</td>
<td>p = .391</td>
<td></td>
</tr>
</tbody>
</table>

The error correction term is the residual from an OLS regression of the logarithm of the German mark on a constant and the logarithms of British pound, Japanese yen, Canadian dollar, French franc, Italian lira, and Swiss franc. The sample period and the number of observations are as noted in Table 1. One day is taken as a base observation interval. The variance ratio $VR(q)$ estimates are given in the main rows, with homoskedasticity- and heteroskedasticity-robust test statistics given in parentheses and brackets, respectively. Under the martingale hypothesis, the value of $VR(q)$ is 1, and the test statistics are asymptotically distributed $N(0,1)$. The $p$ value reports the probability that the variance ratio from the bootstrap distribution is less (larger) than the sample variance ratio if the sample value is less (larger) than the median of the bootstrap distribution in 1,000 iterations. The Wald statistic is computed by using the covariance matrix that describes the dependencies among the four $VR(q)$ estimates as derived from the bootstrap distribution. The Wald statistic distributes as a $\chi^2$ variate with 4 degrees of freedom, with a critical value of 9.488. The $q$ value reports the probability that the Wald statistic from the bootstrap distribution is larger than the sample Wald statistic.

where $E$ is the expectation operator, $VR$ is a vector of sample $VR$ statistics $VR(q) = [VR(2), \ldots, VR(q)]'$, and $\Omega$ is a measure of the covariance matrix of $VR(q)$ estimated with the bootstrap

$$\Omega_{ij} = \sigma^2(VR, VR).$$

However, the simulation results given in Cecchetti and Lam (1994) indicate that the empirical distributions of $VR$ statistics have a large degree of positive skewness, suggesting that inference based on the $\chi^2$ distribution will be misleading. Accordingly, we calculate the Wald statistic for each bootstrapped $VR$ estimator vector and use the bootstrapped distribution of Wald statistics for hypothesis testing.

The results from performing the variance ratio test on the error correction term
are given in Table 3. The variance ratio estimates, homoskedasticity- and heteroskedasticity-robust standard normal Z statistics, and the bootstrap percentiles are reported for \( q = 4, 8, 16, \) and 24. The bootstrapped percentiles show that at least one of the variance ratio estimates for each of the four periods are significantly different from unity at the 5% level. Therefore, the deviations from the cointegrating relationship for all the four sample periods do not follow a martingale. Furthermore, the Wald statistics for the joint test reported in Table 3 also reject the martingale null hypothesis.

It is worth noting that, for the periods of 1980–1984 and 1985–1992, the VR statistics about the individual horizons decrease when \( q \) increases, meaning that the deviations from the cointegrating relationship follow a mean-reverting behavior. Given the fractional integration results reported in the previous section, it appears that the error correction term for the 1980–1984 sample takes a much longer time to reverting back to the mean than the 1985–1992 period. That the error correction term is a long-memory, mean-reverting process supports Baillie and Bollerslev’s (1994) contention that the seven exchange rates are fractionally cointegrated. This feature of long-memory, mean-reversion, however, exists only in the 1980–1984 data.

5. Long memory, mean reversion and rescaled range analysis

To further examine the characteristics of long-memory and mean-reversion in the error correction term of the system of the seven exchange rates, we employ the rescaled range (R/S) test. The R/S statistic is formed by measuring the range between the maximum and minimum distances that the cumulative sum of a stochastic random variable has deviated from its mean and then dividing this by its standard deviation. An unusually small (large) R/S statistic signifies mean-reversion (mean-aversion). Mandelbrot (1972) demonstrates that the R/S statistic can uncover not only periodic dependence but also nonperiodic cycles. He further shows that the R/S statistic is a more general test of long-memory in time series than examining either autocorrelations (i.e., the variance ratio test) or spectral densities.

Lo (1991) points out that the original version of the R/S analysis (which may be termed as classical R/S test) has limitations in that it cannot distinguish between short- and long-term dependence, nor is it robust to heteroskedasticity. Lo modifies the R/S statistic by replacing the standard deviation with a weighted autocovariances up to a finite number of lags:

\[
V^*(q) = \frac{1}{\sqrt{T \hat{\sigma}^q (q)}} \left\{ \max_{1 \leq k \leq T} \sum_{i=1}^{k} (X_i - \bar{X}) - \min_{1 \leq k \leq T} \sum_{i=1}^{k} (X_i - \bar{X}) \right\},
\]

(6)

where

\[
X_t = (1 - L) Z_t,
\]

\[
\hat{\sigma}^q (q) = \sigma^2 + 2 \sum_{i=1}^{q} \gamma(i) \gamma(1),
\]

and, respectively, \( \sigma^2 \) and \( \gamma \) denote the usual sample variance and autocovariance estimators. Thus, the modified R/S statistic detects long memory by controlling for
Table 4
Rescaled range analysis of long memory

<table>
<thead>
<tr>
<th>Sample period</th>
<th>$V$</th>
<th>$V^*(4)$</th>
<th>$V^*(8)$</th>
<th>$V^*(16)$</th>
<th>$V^*(24)$</th>
<th>Wald statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p = .273$</td>
<td>$p = .326$</td>
<td>$p = .449$</td>
<td>$p = .461$</td>
<td>$q = .086$</td>
<td></td>
</tr>
<tr>
<td>1980–1984</td>
<td>0.793</td>
<td>0.902</td>
<td>0.920</td>
<td>0.910</td>
<td>0.918</td>
<td>8.690</td>
</tr>
<tr>
<td></td>
<td>$p = .107$</td>
<td>$p = .117$</td>
<td>$p = .097$</td>
<td>$p = .097$</td>
<td>$q = .096$</td>
<td></td>
</tr>
<tr>
<td>1985–1992</td>
<td>0.501</td>
<td>0.723</td>
<td>0.807</td>
<td>0.856</td>
<td>0.887</td>
<td>16.476</td>
</tr>
<tr>
<td></td>
<td>$p = .005$</td>
<td>$p = .043$</td>
<td>$p = .068$</td>
<td>$p = .079$</td>
<td>$q = .013$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$p = .327$</td>
<td>$p = .342$</td>
<td>$p = .290$</td>
<td>$p = .253$</td>
<td>$q = .535$</td>
<td></td>
</tr>
</tbody>
</table>

Rescaled range (R/S) analysis of the error correction term for long-term memory using the modified R/S $V^*(q)$ and the classical R/S $V$ statistics. The sample period and the number of observations are as noted in Table 1. The critical values of 5% significance level for left and right tails are 0.809 and 1.862, respectively. The $p$ value reports the probability that the R/S statistics from the bootstrap distribution are less (larger) than the sample R/S statistic if the sample value is less (larger) than the median of the bootstrap distribution in 1,000 iterations. The Wald statistic is computed by using the covariance matrix that describes the dependencies among the four R/S estimates as derived from the bootstrap distribution. The Wald statistic distributes as a $\chi^2$ variate with 4 degrees of freedom, with a critical value of 9.488. The $q$ value reports the probability that the Wald statistic from the bootstrap distribution is larger than the sample Wald statistic.

The possible short-term dependence in the time series and hence is more reliable in terms of providing evidence of long memory. Lo also derives the asymptotic distribution of the modified R/S statistic under the hypothesis of no long-term dependence.

As with the variance ratio test, the R/S test that is based on an arbitrarily chosen autocovariance adjustment tends to reject the no long-memory null hypothesis too often. For a joint test, we compute the Wald statistic in a similar way as in the variance ratio test. Because the distribution of the R/S statistics is right-skew and leptokurtotic, the hypothesis testing is properly done by comparing the sample Wald statistic with the bootstrapped distribution of Wald statistics.

Both the classical and modified rescaled range analyses are performed, and the results are contained in Table 4. Except for the 1985–1992 period, the findings that none of the modified R/S $V^*(q)$ statistics are significant at the 5% level disclose no long-range dependence in the error correction term. Nevertheless, under a conservative significance level such as 10%, the two higher order $V^*$ statistics for the 1980–1984 period are significant. Additionally, the joint tests based on the bootstrapped distribution of the Wald statistics suggest that the error correction term for the three sub-periods displays a long-run dependence at a 10% significance level. This long-run dependence is associated with the observation that the deviations from cointegrating relationship follow a mean-reverting behavior. The evidence of significant classical R/S $V^*$ in the 1980–1984 period and especially the 1985–1992 sample might simply partially reflect the presence of short-run dependence in the error correction term, as echoed by the significant variance-ratio test results.
Although the R/S test-based evidence of long-memory, mean-reverting characteristic in the error correction term is only marginal even for the 1980–1984 period, we should caution the readers that the use of the R/S statistic is, at best, the second best alternative, since it is developed for detecting general long-run memory. To estimate a time series as the long-memory process (i.e., the fractionally-integrated model), a semiparametric approach such as the GPH test is more preferable. Therefore, although the evidence of long-memory from the R/S test is weak, the GPH test of fractional integration does suggest the presence of long-memory for the 1980–1984 period.

6. Conclusions

Contrary to the findings of Baillie and Bollerslev (1989) that nominal dollar spot exchange rates are cointegrated, both Diebold et al. (1994) and Baillie and Bollerslev (1994) detect no cointegration among the system of seven nominal exchange rates. Although the conflicting findings can be attributed to the estimation model with or without an intercept, Baillie and Bollerslev (1994) infer that the contradiction in results could be due to the existence of fractional cointegration.

In this article, we empirically examine whether the seven nominal exchange rates are fractionally cointegrated by employing three testing methods, each of which gives a different perspective to examine the long-memory, mean-reversion hypothesis. The results generally indicate that the deviations from the cointegrating relationship of the seven exchange rates follow a fractionally-integrated process for only the 1980–1984 sample, but not for the 1973–1979, 1985–1992, and 1973–1992. However, supporting evidence of usual cointegration is obtained for the more recent sample period, 1985–1992. During the 1980–1984 period, the results also show that the equilibrium errors exhibit a long memory, mean-reverting behavior.

In sum, the findings seem to suggest that if fractional cointegration is a feature of the exchange rate dynamics, such a feature could be changing across varying time spans. The possible changing nature of the fractional cointegrating relationships might be linked to structural breakpoints such as central bank interventions and periodic exchange rate realignments. It is also plausible to suggest that the mixed results are due to the fact that short-run deviations from the equilibrium point are attributed to random shocks. Certainly, it is very likely that short-run deviations may be attributable to other economic factors. Nevertheless, it would be desirable for future research to developing nonlinear models of cointegration in which short-run deviation are conditioned on other factors that may affect the strength of the cointegration in nonlinear ways.

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Notes

1. For example, Barkoulas and Baum (1997) find that eurocurrency deposit rates exhibit long-term memory. Furthermore, they show that the long-memory models provide more reliable out-of-sample forecasts, especially over long horizons, than those from benchmark linear models.


3. We also performed the Engle and Granger (1987) two-step cointegration test and the results are qualitatively the same as those from the Johansen test.

4. Certainly, fractional cointegration does not require individual series to be an $I(1)$ process. A system of fractionally integrated series (i.e., $I(d)$ with $d < 1$) could be fractionally cointegrated. Our interest, however, is confined to exchange rates that contain unit roots, we focus on the special case of fractional cointegration for a system of $I(1)$ process.

5. Diebold and Rudebusch (1991) demonstrate that the standard unit root tests have low power against fractional alternatives. Similarly, cointegration test based on the standard unit root tests might have low power when the cointegrating relationship is fractionally integrated.

6. Under the hypothesis of fractional cointegration for a two-variable case, Cheung and Lai (1993) show that the OLS estimate of the cointegrating vector is consistent and converges at the rate of $T^{1-d}$. The result for a more than two-variable case is still unknown but we expect the same results would hold as for the two-variable case.

7. Agiakloglou et al. (1993) suggest that the estimate of the order of fractional integration from the GPH method could be biased for a model with large ARMA parameters. However, if the estimates $d$ remain stable for different $\eta$’s, there is no hint of a bias due to ARMA parameters (Hassler & Wolters, 1995).

8. The bootstrap method is conducted by first shuffling (with replacement) the change of the error correction term, then estimating $d'$ through the GPH method for a replication of 1,000 times. The $p$ value for the $d (= 1 + d'$) estimate is determined from the frequency table of the bootstrapped distribution.

9. The bootstrap method is conducted in a similar way as we compute the $p$ value of $d$ estimate in Section 3.

References


